A Regulation of Bids for Dual Class Shares.

Implication: Two Shares – One Price

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Abstract

This paper examines the consequences of a certain regulatory restriction on bids for dual class shares. Shares of different classes are often argued to have different prices because a premium will be paid to the superior voting shares in the case of a tender offer. This paper takes as given a setup where the shares in a firm are widely held and regulations require that a tender offer pays the same relative premium to all share classes. In this setup, it is shown that the shares of different classes will sell at the same price as long as there is a strictly positive probability that either the current management is sufficiently strong or that a sufficiently strong rival will show up. Furthermore, under this condition the regulation is socially optimal in the sense that the management that gives the highest total firm value will be the management of the firm. Finally, the regulation is shown to favor (or protect) the holders of restricted voting shares and this is not necessarily at the expense of the holders of superior voting shares.

If the weak condition above is not satisfied, the paper demonstrates the existence of a whole range of possible price equilibria. These equilibria can be decisive for whether the current management will continue or the rival will take over.

The practical interest of this paper derives from the fact that some European countries have adopted regulatory restrictions on bids for dual class shares. This has more or less occurred due to proposed EU Directives. The regulation examined in this paper applies for example to tender offers in Denmark and empirical results on the voting premium in Denmark are shown to be consistent with the theoretical results in this paper.

1 Introduction

By dual class shares is meant that a firm has issued two different types of shares. These shares usually differ with respect to the number of votes that are attached to each share. The shares might also differ with respect to the dividend rights.[1] The difference in votes means that the shares are divided into superior voting shares (or A-shares) and restricted voting shares (or B-shares).

Given that the two types of shares receive the same dividends, superior voting shares are expected to be worth at least as much as restricted voting shares. In addition, the possibility of a takeover might imply that superior voting shares will be worth more than restricted voting shares. In takeover situations investors will in particular be interested in voting rights. In the debate it is argued that a dual class structure can make it too easy for a competing management to obtain the majority of the votes by only buying superior voting shares. The takeover might then happen at the expense of the non-selling shareholders naturally including all the holders of restricted voting shares. This potential problem associated with dual class shares has led to a discussion about the extent to which tender offers and dual class shares should be regulated by law. In order to consider the relevance of and need for such regulations it is important to know the consequences of the different possible regulations.

These issues are especially becoming of importance as it now appears more and more likely that the European Council will adopt the *13th Directive on company law concerning takeover bids.*[2] According to this Directive all member states of the European Union must have rules that provide protection of minority shareholders in the case of takeovers. However, the Directive will probably leave the member states with considerable latitude on exactly how to achieve this protection. As stated in the Directive: "*Member States should ensure that rules or other mechanisms or arrangements are in force which either oblige this person* [who has obtained control of a company] *to make a bid in accordance with Article 10* [i.e. to all shareholders for all or for a substantial part of their holdings at a price which meets the objective of protecting their interests] *or offer other appropriate and at least equivalent means in order to protect the minority shareholders of that company.*"[3] With this latitude in the Directive it is natural to expect discussions in the different countries about the pros and cons of the different regulations

[1] In some cases firms have issued more than two different types of shares.

[2] The Directive was discussed on a meeting December 7, 1999 where the member states seem to have approached a common agreement about the Directive.

[3] Article 3(1).

by which minority shareholders can be protected. In the case of takeovers holders of restricted voting shares are normally among the minority shareholders. Therefore, the Directive also requires regulation on how restricted voting shares should be treated relative to holders of superior voting shares in the case of takeovers.

Grossman and Hart (1988) examined the outcome of tender offers under two different regulations of tender offers for a firm financed with widely held dual class shares. The first regulation examined is the case where the tender offer can be restricted to only a fraction of a share class. In this case it will be possible for the bidder to discriminate between shareholders within the same class and between share classes. The second regulation examined is the case where a restricted tender offer within a class is not possible but where the bidder is allowed to discriminate freely between share classes.

However, as the proposed EU directives illustrate, it is possible to make the regulation of tender offers even more restrictive. In some countries a tender offer cannot discriminate freely between share classes. Examples of such countries are Austria, Denmark, Finland, Great Britain, Sweden, and Switzerland. In these countries a person obtaining control of a firm has to make an offer to all classes of shares and the prices offered either have to give the "same premium" to all classes of shares or the prices offered to the different classes of shares have to be "reasonable" (see Clausen and Sørensen (1998)). In Denmark, for example, a tender offer is required to give a class of restricted voting shares the same relative premium as offered to a class of superior voting shares.[4] This means that if the two share classes trade at 100 and 50 respectively, a tender offer giving 150 to the first class is required to offer 75 to the second class. Furthermore, partial bids are not allowed.

This paper examines the consequences of the regulation adopted in Denmark, i.e. the paper considers the case where a tender offer is required to give the same relative premium to all classes of shares. The main theoretical result of the paper is that the regulation under weak conditions implies that there will be no price difference between share classes. We will later see that the condition for this to be the case is robust with respect to changes in the model's assumptions.

A consequence of identical prices for A- and B-shares is that the management under which the firm will have the highest total value will be able to offer the highest price for the A-shares

[4]This follows from the Law on Security Trade Number 1072, December 20, 1995, §§31–32, and Fondsrådets legal notice no. 333, April 23, 1996, §§1–10. The introduction of the regulation is further described in section 6.

and thereby control the company.[5] Therefore, the regulation leads to social optimality in the sense that the company will be controlled by the management that can contribute with the highest total value. Finally, the results show that when there is no price difference between share classes the regulation will always favor (or protect) the holders of restricted voting shares. Whether the regulation favors the holders of A-shares and/or the shareholders as a whole depends on the behavior of the losing management team. It is especially possible to have cases where the holders of A-shares and the shareholders as a whole also benefit from the regulation.

Dual class shares have been considered in a number of papers.[6] From the empirical literature follows that superior voting shares sell at higher prices than restricted voting shares but also that the premiums differ across countries.[7] Several papers including Lease, McConnell, and Mikkelson (1983), DeAngelo and DeAngelo (1985), Megginson (1990), Smith and Amoako-Adu (1995), and Rydqvist (1996) explained this price difference between share classes by a premium paid to superior voting shares in the case of tender offers and provided evidence for this explanation. A different explanation for the price difference between share classes is offered by Bergström and Rydqvist (1992). In a model with a pivotal blockholder and a bidder that wants (has) to buy all the shares, Bergström and Rydqvist point to the fact that it is the ability to price discriminate (not the voting power) between share classes that gives rise to different prices. If and only if the blockholder is endowed primarily with A-shares, the A-shares will sell at a premium relative to the B-shares. In addition, the wealth consequences of a regulation requiring that the same offer has to be given to both classes of shares are discussed in the paper. The effect of a change in regulations on the price difference between share classes has been examined empirically in Maynes (1996). Maynes (1996) considered the introduction of a so-called coattail requirement for the Toronto Stock Exchange in 1984. According to the coattail requirement, holders of restricted voting shares must be given an offer equivalent to the offer made for the superior voting shares. Consistent with the results in the present paper, Maynes found a significant decline in the premium paid for superior voting shares at the announcement of the coattail requirement.

[5]Here total value refers to the sum of the value of the shares (the security value) and the value derived by the management from having control over the firm (the private benefit of control).

[6]Rydqvist (1992) provided a review of the theory and empirical evidence on dual class shares.

[7]Some of the premiums found in the literature are: Canada 10% (Smith and Amoako-Adu (1995)), France 54% (Muus (1998)), Israel 46% (Levy (1983)), Italy 80% (Zingales (1994)), Norway 10% (Ødegaard (1998)), Sweden 12% (Rydqvist (1996)), Switzerland 10% (Horner (1988)), UK 13% (Megginson (1990)), and USA 5% (Lease, McConnell, and Mikkelson (1983)).

This paper is organized as follows. Section 2 describes the basic model and establishes the main results. Section 3 illustrates a case with multiple price equilibria. Section 4 derives the distributional consequences of the regulation. Section 5 shows that the main results are robust to changes in the basic model. Section 6 demonstrates that the changes over time in the voting premium in Denmark are consistent with the theoretical results in this paper. The conclusions are given in section 7.

2 The basic model

The basic model in this paper is similar to the model in Grossman and Hart (1988). We assume the existence of a firm which under the current (incumbent) management, I, has a total value given as $y_I + z_I$. $y_I > 0$ is the security value and $z_I \geq 0$ is the private benefit of control derived by the current management. The firm is financed with a dual class share structure. It is assumed that one of the classes denoted class A has the majority of the votes. In order to change the management of the firm, the majority of the votes is required. The other class is denoted class B. Without loss of generality, we assume that the total sum of the number of A-shares (s_A) and the number of B-shares (s_B) is equal to one, i.e. that $s_A + s_B = 1$. The security value of the firm, y_I, is distributed between the two classes with the amount $s_A y_I$ to class A and $s_B y_I$ to class B.

At some time denoted time one, it is possible that a competing management (a rival), R, shows up. If the competing management takes over the firm, the total value of the firm will change to $y_R + z_R$. The security value $y_R > 0$ is again distributed between the two classes of shares, and $z_R \geq 0$ is the private benefit of control derived by the competing management when the competing management is controlling the firm.

It is assumed that the shares are widely held, i.e. that all shareholders are small. This means that neither the incumbent nor the competing management own large blocks of shares before the potential bidding contest. Finally, it is assumed that I, R, and the shareholders are all risk neutral, that the interest rate is zero, and that there is no cost associated with a tender offer.[8] At time one when the competing management shows up, the values of y_I, z_I, y_R, and z_R will be realized but seen from time zero, they can all be stochastic variables.

[8]Section 5 considers the case where the two competing management teams own shares before the takeover contest and the rival incurs a cost of bidding.

We will let p_A and p_B denote the price per share at time zero for the class A- and class B-shares respectively.[9] Because both types of shares have equal claim to the security value and the class A-shares determine control, we should expect $p_A \geq p_B$. Therefore, in the rest of this paper we will restrict the analysis to the natural case with $\frac{p_A}{p_B} \geq 1$.

Similarly, we will let p_A^1 and p_B^1 denote the price per share offered in a tender offer at time one for the class A- and class B-shares respectively. In this setup, the requirement that the same relative premium should be offered to both classes of shares can be written as follows

$$\frac{p_A^1}{p_A} = \frac{p_B^1}{p_B} \qquad \Leftrightarrow \qquad p_B^1 = \frac{p_B}{p_A} p_A^1. \tag{1}$$

The main theoretical result of the paper is that the regulation under weak conditions implies that there will be no price difference between share classes. Before going into the technical details we will give the basic intuition for this result. Just before time one, there will be a competition for control of the company. The outcome of this competition is either that the incumbent management stays in control or that a rival (competing management) takes over. The prices for an A- and a B-share at time zero (p_A and p_B) are given as the expected value of the A- and the B-share respectively at time one. Assume now that there exists an equilibrium characterized by $\frac{p_A}{p_B} > 1$. In order to control the company at time one, the winner has to acquire the controlling A-shares – i.e. at time one the winner has to give an offer that is accepted by the atomistic holders of the A-shares. In addition, because of the regulation the winner will have to give a pro rata lower offer to the holders of the B-shares ($\frac{p_B}{p_A}\times$the offer given to the holders of the A-shares). If the holders of the B-shares accept this offer, the price of an A-share at time one will be precisely $\frac{p_A}{p_B}\times$the price of a B-share at time one. If all the winning offers are characterized by that the holders of B-shares accept the pro rata lower offer, we will have that the price of an A-share at time one always will be $\frac{p_A}{p_B}\times$the price of a B-share at time one. Therefore, $\frac{p_A}{p_B} > 1$ will in this case be a rational equilibrium. On the other hand, if the holders of the B-shares do not always accept the pro rata lower offer (because it is optimal for them instead to get their share of the firm's security value) we have that the price of an A-share is less than $\frac{p_A}{p_B}\times$the price of a B-share at time one. Therefore, in such a case $\frac{p_A}{p_B} > 1$ cannot be a rational equilibrium. This implies that if there for all $\frac{p_A}{p_B} > 1$ is just a small probability that the winning offer is rejected by the holders of the B-shares, then we will have that the only

[9] That means that the whole class of A-shares (B-shares) will be worth $s_A p_A$ ($s_B p_B$) at time zero.

equilibrium is $p_A = p_B$. By free riding the holders of B-shares will instead obtain the higher security value of their shares. It turns out that this condition will be fulfilled if there is just a slight chance that a sufficiently strong winner exists.

2.1 The analysis

The analysis will now proceed in the following way. Firstly, for $\frac{p_A}{p_B}$ given, we calculate the maximum prices that the two competing management teams will be able to offer for the two share classes at time one. The management with the highest maximum price for the A-shares (and thereby also for the B-shares) will win the takeover contest. Secondly, we use this to derive the possible prices at time one for the class A- and the class B-shares respectively. Finally, this will allow us to go back to time zero and derive the share prices, if any, that are consistent both with the value of the shares at time one and the $\frac{p_A}{p_B}$ that was taken as given.

The following lemma states the maximum prices that the two competing management teams are able to offer at time one.

Lemma 1 (The incumbent's and the rival's maximum prices).
a) Incumbent:
There exist maximum prices (p_A^I, p_B^I) *given by*

$$p_A^I = \frac{y_I + z_I}{s_A + s_B \frac{p_B}{p_A}} \qquad \text{for } \frac{p_A}{p_B} \leq 1 + \frac{z_I}{s_A y_I} \tag{2a}$$

$$p_A^I = y_I + \frac{z_I}{s_A} \qquad \text{for } \frac{p_A}{p_B} > 1 + \frac{z_I}{s_A y_I} \tag{2b}$$

$$p_B^I = \frac{p_B}{p_A} p_A^I$$

such that the incumbent only will launch winning offers p *for the* A*-shares characterized by* $y_I \leq p \leq p_A^I$.

b) Rival:
There exist maximum prices (p_A^R, p_B^R) *given by*

$$p_A^R = \frac{y_R + z_R}{s_A + s_B \frac{p_B}{p_A}} \qquad \text{for } \frac{p_A}{p_B} \leq 1 + \frac{z_R}{s_A y_R} \tag{3a}$$

$$p_A^R = y_R + \frac{z_R}{s_A} \qquad \text{for } \frac{p_A}{p_B} > 1 + \frac{z_R}{s_A y_R} \tag{3b}$$

$$p_B^R = \frac{p_B}{p_A} p_A^R$$

such that the rival only will launch winning offers p *for the* A*-shares characterized by* $y_R \leq p \leq p_A^R$.

Proof. The proof is given in the appendix. □

Since the results in the lemma are symmetric with respect to the incumbent and the rival, we will only give comments to the results for the incumbent. We note that the incumbent's maximum price for the controlling A-shares is continuous in $\frac{p_A}{p_B}$ and strictly increasing in $\frac{p_A}{p_B}$ until $\frac{p_A}{p_B}$ reaches the critical value $1+\frac{z_I}{s_A y_I}$. For $\frac{p_A}{p_B}$ larger than this critical value, the maximum price is independent of $\frac{p_A}{p_B}$. The explanation is as follows. For low values of $\frac{p_A}{p_B}$ the holders of A- and B-shares will all accept the maximum offer given by the incumbent. Thereby, the incumbent will lose money on buying both the A- and the B-shares because the shareholders are offered a price higher than the security value. The loss of money is covered by the private benefit. For larger values of $\frac{p_A}{p_B}$, the offer given to the holders of B-shares is lowered. This will decrease the incumbent's loss on the B-shares and make it possible for him to pay a higher price for the A-shares. For a sufficiently high $\frac{p_A}{p_B}$ $(> 1+\frac{z_I}{s_A y_I})$ the holders of B-shares will not accept the incumbent's offer because it instead will be optimal for them to receive the security value of the shares. In this case, the incumbent can use all his private benefit to cover the loss from buying the A-shares $\left(z_I = s_A(p_A^I - y_I) \quad \Leftrightarrow \quad p_A^I = y_I + \frac{z_I}{s_A}\right)$.

We will now choose a fixed $\frac{p_A}{p_B} \geq 1$. At time one we can then use lemma 1 to determine which management that will win the takeover contest. The incumbent will stay in control if and only if $p_A^I \geq p_A^R$. In order to determine the precise size of the winning offer, we must know something about the behavior of the losing management. Let us consider the case where $p_A^I > p_A^R$. If the rival does not give any offer the rival will have zero profit. If the rival gives an offer $\hat{p}_A^R > p_A^I$, the incumbent will prefer to let the rival win. This will lead to a negative profit for the rival. If instead the rival gives an offer $\hat{p}_A^R \in]p_A^R; p_A^I]$, the rival will get a zero profit if the incumbent matches his offer and the rival will get a negative profit if the incumbent mistakenly (has a trembling hand and) lets R win. If R gives an offer $\hat{p}_A^R \leq p_A^R$, R will get zero profit if his offer is matched by the incumbent and a positive profit if the incumbent mistakenly (has a trembling hand and) lets R win. Thereby, we have argued that R's loser reply, $\hat{p}_A^R$, must be given by $\hat{p}_A^R \leq p_A^R$ and correspondingly, that I's loser reply, $\hat{p}_A^I$, must be given by $\hat{p}_A^I \leq p_A^I$.[10]

We will later in this paper show that the main theoretical results are independent of the precise specification of the loser's behavior. However, when we in the following discuss the dis-

[10]This hinges on the assumption that the loser does not already own any A-shares. If the loser already owns some shares, the loser will have a strategic incentive to offer above his own maximum price but below the winner's maximum price. The loser will have this incentive because this will force the winner to buy his shares at a higher price. This fact will be further discussed in section 5.

tributional consequences of the regulation for the holders of A- and B-shares, it will be useful to have a more specific description of the behavior of the losing management. Therefore, we will here briefly describe the behavior where R does not give any reply (R is passive, $\hat{p}_A^R = 0$) but I gives a reply corresponding to his maximum price (I is maximally aggressive, $\hat{p}_A^I = p_A^I$). The reason for such a behavior could be as follows. When R knows that he will never win, R will never give an offer if for example he incurs just a small cost of bidding. In contrast, it can be argued that I will not incur the same cost of bidding and that I at the same time is more loyal to the existing shareholders than to R. If these relations are known to both of the competing management teams, R's losing behavior is never to give an offer while R as winner always will have to offer at least I's maximum price.

For a fixed $\frac{p_A}{p_B} \geq 1$, an arbitrary realization of (y_I, z_I, y_R, z_R) at time one, and an arbitrary loser reply $\hat{p}_A^I, \hat{p}_A^R$, table 1 lists the winner of the control contest, the winner's offer for the A-shares (and thereby also for the B-shares), and the condition for the holders of B-shares to accept the offer from the winning management. The table will later be used to obtain the prices of the A- and B-shares at time one and based on rational expectations this will enable us to calculate the prices at time zero.

We will now briefly explain table 1. Assume that we at time one (for a fixed $\frac{p_A}{p_B}$) have a realization of (y_I, z_I, y_R, z_R) fulfilling condition a) in table 1. In this case I's and R's maximum prices are given by (2a) and (3a) respectively. This gives that R will win if and only if $y_R + z_R > y_I + z_I$. If R offers less than y_I for the A-shares, the shareholders will not tender to R. Hence, we can say that I always gives an implicit reply of y_I and accordingly we will have $\hat{p}_A^I \in [y_I; p_A^I]$. Because R must buy the A-shares, R has to: i) match I's reply and ii) ensure that his offer is at least y_R such that the holders of the A-shares will prefer to sell the shares rather than free riding. Thereby, R's offer for the A-shares has to be given by $\max\{y_R, \hat{p}_A^I\}$. The holders of the B-shares then receive an offer of $\frac{p_B}{p_A}\max\{y_R, \hat{p}_A^I\}$. They will accept this offer if and only if the offer is above y_R, because they by rejecting the offer instead will receive their share of the security value, y_R, under R's management. For $\frac{p_A}{p_B} > 1$ or $\frac{p_A}{p_B} = 1$ and $\hat{p}_A^I > y_R$ the condition for the holders of B-shares to accept is equivalent to $\frac{p_B}{p_A}\hat{p}_A^I \geq y_R$. For $\frac{p_A}{p_B} = 1$ and $\hat{p}_A^I \leq y_R$ we will just assume that the holders of B-shares reject the offer and instead receive their share of the security value, which gives the same result.

		The winner's offer for the A-shares	Condition for the holders of B-shares to accept the offer
$a)$	$\frac{p_A}{p_B} \leq \min\{1+\frac{z_I}{s_A y_I}, 1+\frac{z_R}{s_A y_R}\}$		
$a_R)$	R wins: $y_R + z_R > y_I + z_I$	$\max\{y_R, \hat{p}_A^I\}$ $\hat{p}_A^I \in \left[y_I; \frac{y_I+z_I}{s_A+s_B\frac{p_B}{p_A}}\right]$	$\frac{p_B}{p_A}\hat{p}_A^I \geq y_R$
$a_I)$	I wins: $y_I + z_I \geq y_R + z_R$	$\max\{y_I, \hat{p}_A^R\}$ $\hat{p}_A^R \in \left[0; \frac{y_R+z_R}{s_A+s_B\frac{p_B}{p_A}}\right]$	$\frac{p_B}{p_A}\hat{p}_A^R \geq y_I$
$b)$	$1+\frac{z_I}{s_A y_I} < \frac{p_A}{p_B} < 1+\frac{z_R}{s_A y_R}$ $\left(\frac{z_I}{y_I} < \frac{z_R}{y_R}\right)$		
$b_R)$	R wins: $\frac{y_R+z_R}{s_A+s_B\frac{p_B}{p_A}} > y_I + \frac{z_I}{s_A}$	$\max\{y_R, \hat{p}_A^I\}$ $\hat{p}_A^I \in \left[y_I; y_I + \frac{z_I}{s_A}\right]$	$\frac{p_B}{p_A}\hat{p}_A^I \geq y_R$
$b_I)$	I wins: $y_I + \frac{z_I}{s_A} \geq \frac{y_R+z_R}{s_A+s_B\frac{p_B}{p_A}}$	$\max\{y_I, \hat{p}_A^R\}$ $\hat{p}_A^R \in \left[0; \frac{y_R+z_R}{s_A+s_B\frac{p_B}{p_A}}\right]$	$\frac{p_B}{p_A}\hat{p}_A^R \geq y_I$
$c)$	$1+\frac{z_R}{s_A y_R} < \frac{p_A}{p_B} < 1+\frac{z_I}{s_A y_I}$ $\left(\frac{z_R}{y_R} < \frac{z_I}{y_I}\right)$		
$c_R)$	R wins: $y_R + \frac{z_R}{s_A} > \frac{y_I+z_I}{s_A+s_B\frac{p_B}{p_A}}$	$\max\{y_R, \hat{p}_A^I\}$ $\hat{p}_A^I \in \left[y_I; \frac{y_I+z_I}{s_A+s_B\frac{p_B}{p_A}}\right]$	$\frac{p_B}{p_A}\hat{p}_A^I \geq y_R$
$c_I)$	I wins: $\frac{y_I+z_I}{s_A+s_B\frac{p_B}{p_A}} \geq y_R + \frac{z_R}{s_A}$	$\max\{y_I, \hat{p}_A^R\}$ $\hat{p}_A^R \in \left[0; y_R + \frac{z_R}{s_A}\right]$	$\frac{p_B}{p_A}\hat{p}_A^R \geq y_I$
$d)$	$\frac{p_A}{p_B} \geq \max\{1+\frac{z_I}{s_A y_I}, 1+\frac{z_R}{s_A y_R}\}$		
$d_R)$	R wins: $y_R + \frac{z_R}{s_A} > y_I + \frac{z_I}{s_A}$	$\max\{y_R, \hat{p}_A^I\}$ $\hat{p}_A^I \in \left[y_I; y_I + \frac{z_I}{s_A}\right]$	$\frac{p_B}{p_A}\hat{p}_A^I \geq y_R$
$d_I)$	I wins: $y_I + \frac{z_I}{s_A} \geq y_R + \frac{z_R}{s_A}$	$\max\{y_I, \hat{p}_A^R\}$ $\hat{p}_A^R \in \left[0; y_R + \frac{z_R}{s_A}\right]$	$\frac{p_B}{p_A}\hat{p}_A^R \geq y_I$

Table 1: For a fixed $\frac{p_A}{p_B} \geq 1$, an arbitrary realization of (y_I, z_I, y_R, z_R), and an arbitrary reply of the losing management given by $\hat{p}_A^I$ and $\hat{p}_A^R$, the table shows which management that will win the takeover contest, the winning offer for the A-shares, and the condition for the holders of the B-shares to accept the offer given by the winning management.

Similarly, we can explain all the other cases in table 1. The only important thing to remember is the difference between the cases where I wins and the cases where R wins. If R wins, he has to offer at least y_I because I otherwise will stay in control, i.e. I always gives an implicit offer of y_I. In the cases where I wins, there is no such implicit offer from R because R will have to give an explicit offer in order to obtain control.

2.2 Share prices at time zero

At time one, the value of (y_I, z_I, y_R, z_R) will be realized. We will denote these possible realizations by $(y_I^i, z_I^i, y_R^i, z_R^i)$, $i = 1, \ldots, n$. The corresponding probabilities, known at time zero, are denoted α^i. We will now for a given $\frac{p_A}{p_B} \geq 1$ define the following disjoint sets:

$$A_R = \{i \mid R \text{ wins control and the } B\text{-shareholders reject } R\text{'s offer}\}$$

$$AB_R = \{i \mid R \text{ wins control and the } B\text{-shareholders accept } R\text{'s offer}\}$$

$$A_I = \{i \mid I \text{ wins control and the } B\text{-shareholders reject } I\text{'s offer}\}$$

$$AB_I = \{i \mid I \text{ wins control and the } B\text{-shareholders accept } I\text{'s offer}\}.$$

From risk neutrality, an interest rate of zero, and the results in table 1, the share prices at time zero are given as

$$\begin{aligned}
p_A &= \sum_{i \in A_R} \alpha^i \max\{y_R^i, \hat{p}_A^I(i)\} + \sum_{i \in AB_R} \alpha^i \max\{y_R^i, \hat{p}_A^I(i)\} \\
&\quad + \sum_{i \in A_I} \alpha^i \max\{y_I^i, \hat{p}_A^R(i)\} + \sum_{i \in AB_I} \alpha^i \max\{y_I^i, \hat{p}_A^R(i)\} \\
&= \sum_{i \in A_R} \alpha^i \max\{y_R^i, \hat{p}_A^I(i)\} + \sum_{i \in AB_R} \alpha^i \hat{p}_A^I(i) + \sum_{i \in A_I} \alpha^i \max\{y_I^i, \hat{p}_A^R(i)\} + \sum_{i \in AB_I} \alpha^i \hat{p}_A^R(i) \qquad (4) \\
p_B &= \sum_{i \in A_R} \alpha^i \max\{y_R^i, \tfrac{p_B}{p_A} \hat{p}_A^I(i)\} + \sum_{i \in AB_R} \alpha^i \max\{y_R^i, \tfrac{p_B}{p_A} \hat{p}_A^I(i)\} \\
&\quad + \sum_{i \in A_I} \alpha^i \max\{y_I^i, \tfrac{p_B}{p_A} \hat{p}_A^R(i)\} + \sum_{i \in AB_I} \alpha^i \max\{y_I^i, \tfrac{p_B}{p_A} \hat{p}_A^R(i)\} \\
&= \sum_{i \in A_R} \alpha^i y_R^i + \sum_{i \in AB_R} \alpha^i \tfrac{p_B}{p_A} \hat{p}_A^I(i) + \sum_{i \in A_I} \alpha^i y_I^i + \sum_{i \in AB_I} \alpha^i \tfrac{p_B}{p_A} \hat{p}_A^R(i). \qquad (5)
\end{aligned}$$

We will now examine if a given $\frac{p_A}{p_B} \geq 1$ is consistent with (4) and (5). First we observe that $\frac{p_A}{p_B} = 1$ is consistent with (4) and (5) (can be seen directly by inserting in (4) and (5)). The intuition is that there in this case will be given the same offer for both A- and B-shares.

Since the offer is accepted by the holders of A-shares it will also be accepted by the holders of B-shares. Therefore, the value of an A-share at time one will be equal to the value of a B-share at time one realization for realization. This gives $p_A = p_B$.

We will now assume that $\frac{p_A}{p_B} > 1$ and examine when this can be consistent with (4) and (5). For $\frac{p_A}{p_B} > 1$ we have that

$$i \in A_R \quad \Rightarrow \quad \tfrac{p_B}{p_A} \max\{y_R^i, \hat{p}_A^I(i)\} < y_R^i \quad \Leftrightarrow \quad \max\{y_R^i, \hat{p}_A^I(i)\} < \tfrac{p_A}{p_B} y_R^i \tag{6}$$

$$i \in A_I \quad \Rightarrow \quad \tfrac{p_B}{p_A} \max\{y_I^i, \hat{p}_A^R(i)\} < y_I^i \quad \Leftrightarrow \quad \max\{y_I^i, \hat{p}_A^R(i)\} < \tfrac{p_A}{p_B} y_I^i. \tag{7}$$

By using this together with (4) and (5) we obtain

$$p_A \leq \tfrac{p_A}{p_B} \left\{ \sum_{i \in A_R} \alpha^i y_R^i + \sum_{i \in AB_R} \alpha^i \tfrac{p_B}{p_A} \hat{p}_A^I(i) + \sum_{i \in A_I} \alpha^i y_I^i + \sum_{i \in AB_I} \alpha^i \tfrac{p_B}{p_A} \hat{p}_A^R(i) \right\} = p_A.$$

For $\frac{p_A}{p_B} > 1$ we have that the weak inequality above will hold as a strict inequality (leading to a contradiction) if and only if there is a realization i in one of the two cases in (6) and (7). Therefore, we have that $\frac{p_A}{p_B} > 1$ will be an equilibrium if and only if $\sum_{i \in AB_R \cup AB_I} \alpha^i = 1$, i.e. when the holders of B-shares always accept the winning offer. The intuition for this is fairly simple. Assume for example that the price of an A-share is 30% above the price of a B-share at time zero. In the cases where the holders of the B-shares also accept the winning offer at time one, the price of an A-share will be 30% above the price of a B-share at time one. However, in the cases where the holders of B-shares do not accept the winning offer at time one, the price of an A-share will be less than 30% above the price of a B-share at time one. The price of a share at time zero is the expected value of the share at time one. Therefore, if there are cases where the holders of B-shares do not accept the winning offer, the price of an A-share will be less than 30% above the price of a B-share. Hence, $\frac{p_A}{p_B} = 1.3$ cannot be an equilibrium in this case.

In the case where $\frac{p_A}{p_B} = 1$ is the only possible equilibrium, all realizations of (y_I, z_I, y_R, z_R) will be in region $a)$ in table 1. This gives that R will obtain control if and only if $y_R + z_R > y_I + z_I$. Therefore, the management that can provide the highest total value of the firm will end up managing the firm, i.e. the regulation implies social optimality.

We summarize the above in the following theorem.

Theorem 1 (Price equilibria).

Independent of the reply of the losing management we have the following results:

a) Multiple equilibria:

There will be multiple equilibria if and only if there exists a $\frac{p_A}{p_B} > 1$ *such that* $\sum_{i \in AB_R \cup AB_I} \alpha^i = 1$, *i.e. if and only if the holders of* B*-shares always accept the winning offer. Furthermore, we can say that*

- $\frac{p_A}{p_B} = 1$ *will always be one equilibrium.*

b) Unique equilibrium:

If there does not exist a $\frac{p_A}{p_B} > 1$ *such that* $\sum_{i \in AB_R \cup AB_I} \alpha^i = 1$ *then*

- $\frac{p_A}{p_B} = 1$ *is the only equilibrium.*
- *There is social optimality in the sense that the management giving the highest total firm value* $(y_j + z_j)$ *will be the management after the takeover contest.*
- *The prices at time zero for the two classes of shares will be given by*

$$p_A = p_B = \sum_{i \in A_R} \alpha^i y_R^i + \sum_{i \in AB_R} \alpha^i \hat{p}_A^I(i) + \sum_{i \in A_I} \alpha^i y_I^i + \sum_{i \in AB_I} \alpha^i \hat{p}_A^R(i),$$

where the reply of the losing management is characterized by

$$\hat{p}_A^I(i) \geq y_R^i \qquad \qquad \hat{p}_A^R(i) \geq y_I^i.$$

Theorem 1 does not specify the exact reply of the losing management. We will now examine how the reply of the losing management influences the possibility of multiple price equilibria. Thereafter, we will simplify the condition above for excluding multiple price equilibria.

Definition: A reply $\hat{\hat{p}}_A^I, \hat{\hat{p}}_A^R$ is said to be more aggressive than $\hat{p}_A^I, \hat{p}_A^R$ if and only if $\hat{\hat{p}}_A^I(i) \geq \hat{p}_A^I(i)$ and $\hat{\hat{p}}_A^R(i) \geq \hat{p}_A^R(i)$ for all i.

Lemma 2 (Equilibrium under different replies).

If $\frac{p_A}{p_B} > 1$ *is an equilibrium under one reply* $\hat{p}_A^I, \hat{p}_A^R$, *the same* $\frac{p_A}{p_B}$ *will also be an equilibrium for all replies that are more aggressive than* $\hat{p}_A^I, \hat{p}_A^R$.

Proof. If $\frac{p_A}{p_B} > 1$ is an equilibrium under the reply $\hat{p}^I_A, \hat{p}^R_A$, it follows from table 1 and theorem 1 that we for every realization $(y^i_I, z^i_I, y^i_R, z^i_R)$ either have that

$$i) \qquad R \text{ wins and } \frac{p_B}{p_A}\hat{p}^I_A(i) \geq y^i_R$$

or that

$$ii) \qquad I \text{ wins and } \frac{p_B}{p_A}\hat{p}^R_A(i) \geq y^i_I.$$

But if R (I) wins for a given realization under the reply $\hat{p}^I_A$ $(\hat{p}^R_A)$, then for fixed $\frac{p_A}{p_B} > 1$ R (I) will also win under the more aggressive reply $\hat{\hat{p}}^I_A$ $(\hat{\hat{p}}^R_A)$ (see table 1). Because $\frac{p_B}{p_A}\hat{\hat{p}}^{I(R)}_A(i) \geq \frac{p_B}{p_A}\hat{p}^{I(R)}_A(i) \geq y^i_R$ (y^i_I), we have that the holders of B-shares also will accept the winner's offer under the more aggressive reply.

□

From lemma 2 it follows that a more aggressive reply from the loser will make it more likely that there exist multiple price equilibria. Therefore, if we can exclude multiple price equilibria for the maximally aggressive reply, we will have excluded multiple price equilibria for all replies.

The condition for multiple price equilibria given in theorem 1 is difficult to use because the determination of whether a realization $(y^i_I, z^i_I, y^i_R, z^i_R)$ belongs to a certain region or not will depend on $\frac{p_A}{p_B}$. The following theorem will give the necessary and sufficient condition for one realization $(y^i_I, z^i_I, y^i_R, z^i_R)$ to exclude multiple price equilibria under the maximally aggressive reply. From lemma 2 follows that this condition will also be a sufficient condition to exclude multiple price equilibria for less aggressive replies.

Theorem 2 (Excluding multiple price equilibria).

a) For all replies, we have that $\frac{p_A}{p_B} = 1$ will be the unique equilibrium if there is a strictly positive probability for a realization $(y^i_I, z^i_I, y^i_R, z^i_R)$ fulfilling one of the following conditions:

i)	$z^i_R \geq s_B z^i_I$	$y^i_R > y^i_I + z^i_I$	*(R wins)*
ii)	$z^i_R < s_B z^i_I$	$y^i_R + \frac{z^i_R}{s_A} > y^i_I + \frac{z^i_I}{s_A}$	*(R wins)*
iii)	$z^i_I \geq s_B z^i_R$	$y^i_I \geq y^i_R + z^i_R$	*(I wins)*
iv)	$z^i_I < s_B z^i_R$	$y^i_I + \frac{z^i_I}{s_A} \geq y^i_R + \frac{z^i_R}{s_A}$	*(I wins)*

b) For the maximally aggressive reply the conditions above are in addition the necessary conditions for one single realization to exclude multiple price equilibria.

Proof. The proof is given in the appendix. □

From theorem 2 it follows that multiple price equilibria will be excluded if there is just a small probability for a realization where there exists a sufficiently strong winner. For the maximally aggressive reply, the condition in the theorem is also the necessary condition for one realization to exclude multiple price equilibria. However, it is possible for several realizations together to exclude multiple price equilibria even though each realization alone is not able to exclude multiple price equilibria (see the example in section 3). Therefore, the theorem can be strengthened in the general case with respect to the necessary conditions.

3 A numerical example with multiple price equilibria

The following numerical example will illustrate the possibility of multiple price equilibria and go through the economic arguments leading to these equilibria. In addition we will also illustrate how several realization of (y_I, z_I, y_R, z_R) together can reduce the set of multiple price equilibria. Especially, we will see how several realizations together can exclude multiple price equilibria even though each realization alone is not able to exclude multiple price equilibria.

In the example we will assume that the behavior of the losing management is as discussed in subsection 2.1, i.e. that R is totally passive and that I is maximally aggressive.[11]

We assume that $s_A = s_B = \frac{1}{2}$. Furthermore, we will start by assuming that there is only one possible realization given by $(y_I^1, z_I^1, y_R^1, z_R^1) = (120, 30, 100, 45)$. We observe that this realization does not satisfy the condition in theorem 2 – i.e. we must have that there exist multiple price equilibria under some replies.

From lemma 1 we get that I's maximum price is higher than R's maximum price if and only if $\frac{p_A}{p_B} \leq 1.6364$. The multiple price equilibria are given in the following table.

[11]Here we note that if we have multiple price equilibria under this behavior, we will also have multiple price equilibria if R has a more aggressive behavior (lemma 2).

$\frac{p_A}{p_B}$	p_A	p_B	Winner
1	120	120	I
$\in]1.6364; 1.8]$	180	$\frac{p_B}{p_A} \cdot 180$	R

If the market sets $\frac{p_A}{p_B} = 1$, I will win the takeover contest. If the market sets $1.6364 < \frac{p_A}{p_B} \leq 1.8$, R will win. In addition these prices are the only possible rational prices. On the other hand, if the market expects I to win, we will have $\frac{p_A}{p_B} = 1$, and I will win. Similarly, if the market expects R to win, we will have $1.6364 < \frac{p_A}{p_B} \leq 1.8$ and R will actually win. The explanation for the price equilibria goes as follows:

1. If $\frac{p_A}{p_B} \leq 1.6364$ I will be able to give the highest offer and thereby I will win. R is assumed to be passive so R will not give any counter offer and I will just stay in control. Therefore, we will have that $p_A = p_B = 120$ and $1 < \frac{p_A}{p_B} \leq 1.6364$ will not be a rational equilibrium.

2. If $1.6364 < \frac{p_A}{p_B} \leq 1.8$ R will be able to give the highest offer and thereby R will win. Because I is assumed to be maximally aggressive, I's reply is $(\hat{p}_A^I, \hat{p}_B^I) = (180, \frac{p_B}{p_A} \cdot 180)$. Since $\frac{p_B}{p_A} \cdot 180 < 120$ for all $\frac{p_A}{p_B}$ in the interval, we have that the holders of B-shares will reject the offer. Hereby, I will buy only the A-shares and his profit will be $30 + \frac{1}{2} \cdot 120 - \frac{1}{2} \cdot 180 = 0$. This shows that it will be possible for I to give such an offer. In order to win, R has to match I's offer. Since $\frac{p_B}{p_A} \cdot 180 \geq 100$ for all $\frac{p_A}{p_B}$ in the interval, the holders of B-shares will accept R's offer. Therefore, we have that $p_A = 180$ and $p_B = \frac{p_B}{p_A} \cdot 180$ and thereby that $\frac{p_A}{p_B}$ in the interval will be a rational equilibrium.

3. If $\frac{p_A}{p_B} > 1.8$ we will still have that R wins and that R must match I's offer. However, in this case R's offer to the B-shareholders will be $\frac{p_B}{p_A} \cdot 180 < 100$ for all $\frac{p_A}{p_B} > 1.8$. Therefore, the holders of B-shares will reject the offer given by R and we get that $p_A = 180$ and $p_B = 100$. But this leads to $\frac{p_A}{p_B} = 1.8$ why $\frac{p_A}{p_B} > 1.8$ cannot be a rational equilibrium.

We now change the example by assuming that we have two possible realizations:

$$(y_I^1, z_I^1, y_R^1, z_R^1) = (120, 30, 100, 45) \qquad \text{with probability } \alpha^1 = \tfrac{1}{2}$$

$$(y_I^2, z_I^2, y_R^2, z_R^2) = (120, 30, 42.941, 100) \qquad \text{with probability } \alpha^2 = \tfrac{1}{2}.$$

By going through this new example we get that the price equilibria are now characterized by

$$\left\{\frac{p_A}{p_B} \mid \frac{p_A}{p_B} = 1 \ \vee \ \frac{p_A}{p_B} \in]1.7; 1.8]\right\}.$$

We observe that the extension of the example with another realization has decreased the set of price equilibria. Furthermore, we note that realization 2 is not alone able to exclude multiple price equilibria.

Finally, we now change realization 2 to

$$(y_I^2, z_I^2, y_R^2, z_R^2) = (120, 30, 37.368, 100) \qquad \text{with probability } \alpha^2 = \tfrac{1}{2}.$$

By going through the calculations again we now get that $\frac{p_A}{p_B} = 1$ is the only equilibrium. This shows that realization 1 and 2 together can exclude multiple price equilibria even though none of the two realizations fulfill the conditions in theorem 2. For realization 1 we have already seen that we for this realization alone ($\alpha^1 = 1$) will have multiple price equilibrium. For realization 2 alone ($\alpha^2 = 1$) we can show that we as price equilibria in addition to $\frac{p_A}{p_B} = 1$ will have $\frac{p_A}{p_B} \in]1.9; 4.817]$.

4 Distributional consequences of the regulation

Section 2 showed that the regulation under weak conditions leads to $\frac{p_A}{p_B} = 1$ being the unique price equilibrium. From this followed that the regulation also implies social optimality. Based on $\frac{p_A}{p_B} = 1$ we will in this section in further detail discuss the consequences of the regulation (in this section called regulation 1) with respect to share prices and the total value of the shares. The consequences will be compared to the less restrictive regulation (called regulation 2) that allows tender offers to discriminate between share classes but not within the same class.[12]

Under regulation 1, R will take over if and only if $y_R + z_R > y_I + z_I$. Under regulation 2, I and R will only compete about the A-shares to which they are able to offer $y_j + \frac{z_j}{s_A}$, $j = I, R$. Therefore, R will win if and only if $y_R + \frac{z_R}{s_A} > y_I + \frac{z_I}{s_A}$. Hence, regulation 2 does not in general imply social optimality since too much weight is put on the private benefit. In order to further discuss the distributional consequences, it is advantageous to make an assumption regarding

[12]For regulation 2 we also assume that partial bids within a class is not allowed (as also assumed for regulation 1).

the behavior of the losing management. As in the previous section we will assume that R's losing behavior is passive while I's losing behavior is maximally aggressive. The results that will follow from assuming another behavior of the losing management will also be listed, and we will see that there is no qualitative difference in the results.

From the losing behavior assumed it follows that R will not give any offer under regulation 1 when $y_I + z_I \geq y_R + z_R$. Therefore, in this case I will just stay in control. When $y_R + z_R > y_I + z_I$, R will have to offer $\max\{y_I + z_I, y_R\}$ for both the A- and the B-shares due to the losing behavior of I. Similarly, under regulation 2 we have that R will not give any offer when $y_I + \frac{z_I}{s_A} \geq y_R + \frac{z_R}{s_A}$ why I will just stay in control. When instead $y_R + \frac{z_R}{s_A} > y_I + \frac{z_I}{s_A}$, R will have to offer $\max\{y_I + \frac{z_I}{s_A}, y_R\}$ for the A-shares due to the losing behavior of I. From this we obtain the following lemma.

Lemma 3 (Results from a tender offer).
Assume that we have given an incumbent management with (y_I, z_I) and a rival with (y_R, z_R) both known at time zero. Furthermore, assume that the losing behavior of I is to be maximally aggressive while the losing behavior of R is to be passive. Under the two different regulations, the share prices and the market value of the firm at time zero will then be as follows.
a) If the same relative premium has to be offered to both share classes (regulation 1), we will have:

- *The management with maximum $y_j + z_j$ will win the takeover contest.*

i) If I is the winner ($y_I + z_I \geq y_R + z_R$):

$$p_A = y_I$$
$$p_B = y_I$$
$$V_{firm} = y_I.$$

ii) If R is the winner ($y_I + z_I < y_R + z_R$):

$$p_A = \max\{y_I + z_I, y_R\}$$
$$p_B = \max\{y_I + z_I, y_R\}$$
$$V_{firm} = \max\{y_I + z_I, y_R\}.$$

b) If the tender offer can discriminate freely between share classes (regulation 2), we will have:

- *The management with maximum* $y_j + \frac{z_j}{s_A}$ *will win the takeover contest.*

i) If I *is the winner* ($y_I + \frac{z_I}{s_A} \geq y_R + \frac{z_R}{s_A}$)*:*

$$p_A = y_I$$

$$p_B = y_I$$

$$V_{firm} = y_I.$$

ii) If R *is the winner* ($y_I + \frac{z_I}{s_A} < y_R + \frac{z_R}{s_A}$)*:*

$$p_A = \max\{y_I + \tfrac{z_I}{s_A}, y_R\}$$

$$p_B = y_R$$

$$V_{firm} = s_A \max\{y_I + \tfrac{z_I}{s_A}, y_R\} + s_B y_R.$$

By using lemma 3 we can now compare the distributional consequences of the two regulations.

Theorem 3 (Distributional consequences).
If we under the assumptions stated in lemma 3 compare the case where tender offers are required to give the same relative premium to both classes of shares (regulation 1) to the case where tender offers can discriminate freely between share classes but not within the share classes (regulation 2), we have the following:

a) Regulation 1 will always favor the holders of the class B*-shares.*

b) The effect of regulation 1 on the value of the class A*-shares is mixed.*

c) The effect of regulation 1 on the market value of the firm is mixed.

Proof. The proof is given in the appendix. □

Further analysis shows that the holders of the B-shares independently of the losing behavior will prefer regulation 1. Only when both management teams are passive (maximally aggressive) as losing management will the holders of A-shares unambiguously prefer regulation 1 (regulation 2). Therefore, it is only in the unlikely case where both management teams are passive losers that both groups of shareholders in all cases will prefer regulation 1 rather than regulation 2. Furthermore, only in this case will the conclusion under *c)* in theorem 3 be that the market value of the firm is unambiguously highest under regulation 1.

All in all we can conclude that regulation 1 is socially optimal in the sense that regulation 1 ensures that the management of the firm will be the one under which the firm has the highest total value. In addition regulation 1 favors the holders of B-shares. However, the conclusion regarding the holders of the A-shares and the total market value of the firm depends on the specific circumstances.

In the theorem above, we have not said anything about how s_A will be chosen under regulation 2. Let us assume that an entrepreneur (E) is considering to start up a new firm and hence, E wants to choose s_A such that his expected profit from the firm is maximized. Furthermore, we will assume that I's losing behaviour is to be maximally aggressive, that R's losing behaviour is passive, and that E can appropriate I's private benefit either when hiring I or by employing himself as I. Independently of which s_A E chooses, E will not be able to appropriate all R's private benefit in the cases where R wins control.[13] If the probability for $z_R^i z_I^i > 0$ is small i.e. that it is unlikely that both management teams have large private benefit, then E will optimally choose $s_A = 1$. Hence, under this condition and with regulation 2 applying, it will be optimal for E voluntarily to enforce regulation 1. However, if instead the probability for $z_R^i z_I^i > 0$ is large,[14] E will choose $s_A < 1$ under regulation 2. The reason for this is that E then will be able appropriate a larger fraction of R's private benefit in the cases where R obtains control. Because it under regulation 2 can be optimal for E to choose $s_A < 1$, it is no longer certain that the firm will be controlled by the management providing the highest total value. Hence, for existing firms regulation 1 is socially preferred relative to regulation 2. However, in the case of upstart of a firm it may be socially optimal only to enforce regulation 2. This is because regulation 2 can make it possible for E to appropriate more of R's private benefit compared to regulation 1. Hereby, we can come in situations where E will start up a firm under regulation 2 but not under regulation 1. This disadvantage of regulation 1 has to be weighted against the advantage regulation 1 will have for existing firms. Finally, it should be noted that it is still not possible to appropriate all R's private benefit under regulation 2, why regulation 2 also gives a too small incentive to start up firms.

[13] The proofs for this result and the remaining results in this section are omitted. However, the proofs can be obtained from the authors.

[14] Compare with situation 3 in Grossman and Hart (1988).

5 Discussion of robustness

In this section we will examine and discuss if the results above also hold when the incumbent and the rival already own some of the A-shares before the takeover contest.[15] Furthermore, this section will discuss the consequences of the case where in addition to ownership of A-shares the rival incurs a fixed cost when bidding for the shares. Finally, this section will briefly discuss the distributional aspects of these changes in the model.[16]

5.1 Incumbent/Rival owns A-shares before the takeover contest

We will now assume that the two possible management teams own a number of A-shares before the takeover contest. However, we will assume that none of the two control the majority of the votes.

The maximum prices that the two management teams are able to offer are independent of their shareholdings and are therefore still given by lemma 1. For a price below a management's maximum price the management will prefer to buy and in this way obtain control. For an offer above the maximum price, the management will prefer to sell his A-shares. Given that it is costless to make an offer, the losing management will now always have an incentive to make an offer at least corresponding to his own maximum price. This way the losing management will increase the price that the winning management will have to pay for the loser's position in A-shares. The loser will prefer an offer that is as close to the winner's maximum price as possible because he in this way will obtain the highest price for his A-shares. The loser cannot hope to press the price above the winner's maximum price because such an offer will make the winner sell his shares. Therefore, we must in general expect that the losing behavior leads to a winning offer between the maximum price of the loser and the winner. We will not model exactly what the price will be.

With respect to the earlier results, we will also have that lemma 2 holds unchanged. Furthermore, we also have that $\frac{p_A}{p_B} = 1$ always is an equilibrium and that there will be multiple price equilibria if and only if there exists a $\frac{p_A}{p_B} > 1$ such that the winner always wins with an offer that is accepted by the holders of the B-shares. Therefore, it is still most difficult to exclude multiple price equilibria when the loser's reply is equal to the winner's maximum price.

[15] This will among other things be of importance in the following empirical section. This is because the ownership of shares in Denmark is not in general public information. Therefore, it will not be possible to divide the data-set into groups dependent on how many A-shares the incumbent or a potential rival owns.

[16] All the results in this section will be given without proof. The proofs can be obtained from the authors.

Using this we obtain the following theorem regarding the exclusion of multiple price equilibria.

Theorem 4 (Excluding multiple price equilibria – no cost of bidding).
Consider the case where I and R both have a position in the A-shares before the takeover contest and where there is no cost associated with bidding. In this case the following gives the necessary and sufficient condition for one realization to exclude multiple price equilibria when the loser's reply is equal to the winner's maximum price (maximally aggressive). The condition is at the same time a sufficient condition for excluding multiple price equilibria for all weaker replies.

The condition is that there exist a realization $(y_I^i, z_I^i, y_R^i, z_R^i)$ such that either

$$a) \qquad z_I^i = 0 \text{ and } y_I^i \geq y_R^i + \frac{z_R^i}{s_A} \qquad \text{(I wins)}$$

or

$$b) \qquad z_R^i = 0 \text{ and } y_R^i > y_I^i + \frac{z_I^i}{s_A} \qquad \text{(R wins).}$$

Before giving the comments to the theorem we will first briefly sketch the proof of part b) in the theorem: When $y_R^i > y_I^i + \frac{z_I^i}{s_A}$, R will for all $\frac{p_A}{p_B}$ be able to match any offer given by I and therefore R will win. It is most difficult to exclude multiple price equilibria when I is maximally aggressive. When $z_R^i = 0$, it is only possible for I to press R to offer y_R^i for the A-shares. For $\frac{p_A}{p_B} > 1$ we will have $\frac{p_B}{p_A} y_R^i < y_R^i$ why the holders of B-shares will reject the offer and both classes of shares will be worth y_R^i. If $z_R^i > 0$ it will be possible for I to press R to give an offer higher than y_R^i for the A-shares. But in that case, there will exist a $\frac{p_A}{p_B} > 1$ such that the holders of the B-shares also will accept the offer. Therefore, we have that $z_R^i = 0$ is a necessary condition.

□

Consider now a company, where the incumbent always derives positive private benefit, has a position in the A-shares, and is maximally aggressive when losing. In order to exclude multiple price equilibria for this company there must be a positive probability for a sufficiently strong rival without any private benefit. We also note that if the incumbent does not own any shares, condition b) above is replaced with theorem 2's conditions i) and ii). The reason for this is simply that the incumbent will not have any incentive to offer above his own maximum price when he does not own any shares. However, if the incumbent has just a small position in the shares, his maximum reply will increase discontinuously to R's maximum price. Similar arguments holds for R.

Compared to theorem 2 where it nearly seems too easy to exclude multiple price equilibria, we are when both management teams always own A-shares in a situation where it is much more difficult to exclude equilibria with $\frac{p_A}{p_B} > 1$.

In the following subsection we will see that the existence of cost of bidding again will make it easier to exclude price equilibria with $\frac{p_A}{p_B} > 1$.

5.2 Cost of bidding

Now assume that it costs a fixed amount c for R to bid for the shares. The maximum price for the rival is still given by lemma 1 if we change the definition of the rival's private benefit to being private benefit net of costs, i.e. $z'_R \equiv z_R - c$. We note that it is now possible for the private benefit net of costs to be negative.[17] If we let e_I and e_R denote the fractions of the A-shares owned by the incumbent and the rival respectively, we can show the following theorem.

Theorem 5 (Excluding multiple price equilibria – R incurs cost of bidding). *Consider the case where I and R both have a position in the A-shares before the takeover contest and where R incurs a cost of c when bidding. In this case the following gives the necessary and sufficient condition for one realization to exclude multiple price equilibria when the loser's reply is equal to the winner's maximum price. The condition is at the same time a sufficient condition for excluding multiple price equilibria for all weaker replies.*

The condition is that there exist a realization $(y_I^i, z_I^i, y_R^i, z_R^i)$ satisfying one of the following five conditions:

$$a) \qquad {z_R^i}' \geq 0, \quad z_I^i \geq s_B {z_R^i}', \quad y_I^i \geq y_R^i + {z_R^i}', \quad e_R z_I^i \leq c$$

(I wins and $p_A^1 = p_B^1 = y_I^i$).

$$b) \qquad {z_R^i}' \geq 0, \quad z_I^i < s_B {z_R^i}', \quad y_I^i + \frac{z_I^i}{s_A} \geq y_R^i + \frac{{z_R^i}'}{s_A}, \quad e_R z_I^i \leq c$$

(I wins and $p_A^1 = p_B^1 = y_I^i$).

[17] We will, when R obtains control, assume that I's unmodeled reply is known by R. Alternatively, we can assume that R does not incur the cost c until after I has given his last reply. In both cases, it will not be profitable for I to make an offer above R's maximum price.

c) $\quad {z_R^i}' = 0, \quad y_R^i > y_I + \frac{z_I^i}{s_A}$

(R wins and $p_A^1 = p_B^1 = y_R^i$*).*

d) $\quad {z_R^i}' < 0, \quad {z_R^i}' \leq e_R s_A (y_I^i - y_R^i), \quad y_I^i + \dfrac{z_I^i}{e_I s_A + e_R s_A} \leq y_R^i + \dfrac{{z_R^i}'}{e_I s_A + e_R s_A}$

(I allows R to win and $p_A^1 = p_B^1 = y_R^i$*).*

e) $\quad {z_R^i}' < 0, \quad y_I^i + \dfrac{z_I^i}{e_I s_A + e_R s_A} > y_R^i + \dfrac{{z_R^i}'}{e_I s_A + e_R s_A}, \quad e_R z_I^i \leq c$

(I wins and $p_A^1 = p_B^1 = y_I^i$*).*

We observe that if we in theorem 5 set $c = 0$ (and have $e_R > 0$), the theorem is reduced to theorem 4. Therefore, as expected we have that theorem 4 is just a special case of theorem 5.

If we compare theorem 4 case a) with theorem 5 cases a), b), and e) we see that it is easier for the incumbent to prevent multiple price equilibria when the rival incurs cost of bidding. This is caused by the fact that in this case R will not be able to press I so aggressively. When I wins independently of $\frac{p_A}{p_B}$, R's reason for pressing I is that he will obtain a higher price for his A-shares. If R does not press I, R's A-shares will be worth $e_R s_A y_I^i$. The maximum price that R will be able to press I to pay is I's maximum price, in which case the shares will be worth at most $e_R s_A \left(y_I^i + \frac{z_I^i}{s_A}\right)$. Thereby, R's profit on his shares from pressing I is at most $e_R z_I^i$. Therefore, if we have that R's position in the A-shares is small relative to the cost of bidding, R will choose to be a passive loser and I will stay as management without any competition.

As in theorem 4 we have that a rival with a (net) private benefit of zero will exclude multiple price equilibria. Because of the cost of bidding we can have that the (net) private benefit now becomes negative. In this case it is possible to come in a situation where the rival is not able to make an offer above the security value of the A-shares. Given such an offer neither of the two groups of atomistic shareholders will accept such an offer. However, it is possible that it will be profitable for the incumbent to accept such an offer. By accepting the offer I will lose his private benefit but on the other hand I may receive a higher price for his shares.[18] When R has bought all I's A-shares we have two possible cases. First, R may now own the majority of the votes and has thereby obtained control. Second, R may still not own the majority of the votes, but he will still obtain control because none of the atomistic shareholders will vote against R. This is because case d) implies that $y_R^i > y_I^i$.

[18]More precisely as an implication of d) in the theorem, I will accept such an offer if $e_R z_I^i \leq -e_I {z_R^i}'$.

Based on theorems 2, 4, and 5 the model seems fairly robust with respect to excluding other equilibria than $\frac{p_A}{p_B} = 1$. Before we in the next section examine this result empirically, we will end this section by listing the distributional consequences of $\frac{p_A}{p_B} = 1$ in this more general setup.

Based on $z'_R \geq 0$, and $\frac{p_A}{p_B} = 1$ under regulation 1, we have[19]

a) The management that can contribute with the highest total value net of bidding cost will end up managing the company. Therefore, we have that regulation 1 in contrast to regulation 2 is socially optimal in the sense that regulation 1 ensures that the management of the firm will be the one under which the firm has the highest total value.

b) The price of the B-shares will be (weakly) higher under regulation 1 than under regulation 2.

c) It is not possible to say anything general about the price of the A-shares and the total market value of the firm under regulation 1 versus regulation 2.

6 The time pattern in the voting premium in Denmark

The requirement that the same relative premium should be offered to all classes of shares was first mentioned in Denmark in the Ethic Rules for the Copenhagen Stock Exchange (Børsetiske regler) on November 3, 1987. However, there were no sanctions or punishments for breaking these rules. Therefore, it was not until the Law on Security Trade No. 1072, December 20, 1995, §§31–32, and Fondsrådets legal notice No. 333, April 23, 1996, §§1–10, that the regulation became required by law.

The results in this paper imply that the regulation under weak conditions leads to equal prices for class A- and class B-shares. The following will examine if this result can be seen in the time pattern of the price ratio between the different share classes in Denmark.

Only a few other studies have looked at the price ratio between A- and B-shares in Denmark and none of these examined whether there is a change in this ratio over time.[20]

[19] When $z'_R < 0$ and $y_R + z'_R > y_I + z_I$ it is not certain that R will obtain control over the company. If e_I is sufficiently small, it will not be possible for R to buy the A-shares from I. This is because I will lose z_I and will only receive a small profit on his own position in A-shares. Therefore, R must obtain control based on the atomistic shareholders. Because of free riding this requires an offer of at least y_R for the shares. If R gives such an offer, he will obtain control and his profit will be $z'_R + e_R s_A y_R$. If R does not give such an offer I will stay in control and R's profit will be $e_R s_A y_I$. Therefore, the incremental profit from giving such an offer is $e_R s_A(y_R - y_I) + z'_R = e_R s_A(y_R - y_I) + z_R - c$. But because c is assumed to be larger than R's private benefit, we have that if R only has a small position in the A-shares it will not be possible for R to earn enough on his own shares to take over.

[20] Examples of studies are Kjærgaard and Jensen (1997) and Damstrup (1997).

As data-set we use all firms in Denmark that have a dual class share structure with the same dividend rights and where both types of shares are traded on the Copenhagen Stock Exchange. The time period considered is from January 2, 1985 to December 30, 1999. The number of firms varies over time but consists in total of 61 different firms.[21] Daily prices for the time period for the shares traded on the Copenhagen Stock Exchange are obtained from the DSD database. DSD (Danish Stock Data), also known as Børsdatabasen, contains a range of financial data from the Danish stock market starting January 2, 1985.

Using these share prices, we calculate the voting premium, i.e. the ratio between the price for a class A share and a class B share for dates where prices are available for both classes. This provides us with a time pattern in the price ratio for the individual firms.[22] Figure 1 plots the time pattern of the average voting premium, while figure 2 plots the quartiles.

Insert figures 1 and 2 here.

From the figures we observe that the voting premium has been changing over time and that these changes at least to some extent are consistent with what should be expected based on the results in this paper. The figures show a decrease in the price ratio in the period from 1990 to 1996. This steady decrease in the price ratio is consistent with the discussion in the European Union on the use of restricted voting shares and the introduction of the regulation examined in this paper in late 1995/early 1996. On December 20, 1990 the European Commission came up with a proposal (5th Directive) according to which restricted voting shares should more or less be banned. This proposal (and it is still only a proposal) was very influential on the Danish debate concerning A- and B-shares. Hence, the data are seen to be consistent with the following. The regulation of bids for dual class shares was mentioned in the Ethic Rules for the Copenhagen Stock Exchange in 1987, but there were no sanctions or punishments for breaking these rules. However, the proposal by the European Commission in December 1990 made it more likely that the Danish regulation could become required by law. As mentioned, this actually happened December 20, 1995 and April 23, 1996.

[21] Ideally, we would like to omit all firms where the incumbent management controls the majority of the votes. Unfortunately, this is not possible because the ownership of shares in Denmark is not in general public information as mentioned earlier. However, it has been possible to omit three firms fulfilling all of the following criteria: i) There is almost no trade in the A-shares, ii) There is almost no quotations of bid and ask prices for the A-shares, and iii) The incumbent management controls almost all the A-shares (i.e. there are no atomistic shareholders).

[22] Graphs of the time patterns in the price ratio for the individual firms are available from the authors upon request.

From 1996 and onwards we observe a $\frac{p_A}{p_B}$ price ratio of approximately 1.05. However, this is an average price ratio and from figure 2 it follows that only very few firms have a price ratio substantially higher than 1. A common characteristic of these few firms is that their A-shares are thinly quoted and traded. Hence, a guess might be that these shares are closely held. In general the Danish regulation seems hard to circumvent. However, one possibility is the following. Suppose a firm is a potential or actual target and that its A-shares are closely held. In a thin market in which almost no atomistic holders of A-shares exist the holders of A-shares can collude/cooperate and thereby trade up the price of an A-share. If the takeover bid arrives the holders of A-shares will this way receive a larger part of the bid. Hence, such an attempt to circumvent the regulation may explain the few firms having a price ratio above 1.

7 Conclusion

This paper has examined the case where a tender offer is required to give the same relative premium to all share classes in a setup similar to the one in Grossman and Hart (1988). If there is just a small probability that either the current management will continue or that a sufficiently strong rival will show up, we have shown that there will not be any price difference between the different share classes. Therefore, in countries with such a regulation or similar regulations of tender offers, we should expect to see only small price differences between share classes.

The result that we in general will have 'Two Shares - One Price' was shown to imply social optimality in the sense that the regulation ensures that the management of the firm will be the one under which the firm has the highest total value. In addition, the regulation will always be in favor of the holders of restricted voting shares and can in some cases also be to an advantage of the holders of superior voting shares. Therefore, the results also demonstrate that regulations can have important consequences for the size of the voting premium and for the distribution of the total firm value between the management and the different classes of shares.

The regulation examined was adopted by law in Denmark in 1995/1996, and the time pattern in the voting premium in Denmark was by and large shown to be consistent with the theoretical results in this paper.

A Proofs

Proof of Lemma 1

Proof. Due to symmetry, we only have to prove the results for the incumbent.

If the incumbent does not give a winning offer at time one, his profit at time one will be 0. If instead he gives a winning offer p for the A-shares his profit at time one will be

$$\pi_I^{Buy}(p) = z_I - s_A(p - y_I)^+ - s_B(\frac{p_B}{p_A}p - y_I)^+. \tag{8}$$

The incumbent will only give the offer if $\pi_I^{Buy}(p) \geq 0$. Furthermore, we must have that $p \geq y_I$ such that the holders of the A-shares will accept the offer. Finally, we note that $\pi_I^{Buy}(p)$ is continuous for all p and strictly decreasing for $p \geq y_I$.

A) Assume that $\frac{p_A}{p_B} \leq 1 + \frac{z_I}{s_A y_I}$.
Consider now an offer $p \geq p_A^I$ where $p_A^I \equiv \frac{y_I + z_I}{s_A + s_B \frac{p_B}{p_A}}$. Together with assumption A) this implies that $p \geq y_I$ and that $\frac{p_B}{p_A}p \geq y_I$ for all $\frac{p_A}{p_B}$ in the interval. That is, both the holders of the A- and the B-shares will accept the offer. From (8) we get

$$\pi_I^{Buy}(p) = y_I + z_I - p\big(s_A + s_B\frac{p_B}{p_A}\big) \begin{cases} = 0 & \text{for } p = p_A^I \\ < 0 & \text{for } p > p_A^I \end{cases}$$

Finally, because $\pi_I^{Buy}(p)$ is continuous and strictly decreasing for $p \geq y_I$ we have that $\pi_I^{Buy}(p) > 0$ for $p < p_A^I$. Therefore, the winning offers given by I must fulfill $y_I \leq p \leq p_A^I$.

B) Assume that $\frac{p_A}{p_B} > 1 + \frac{z_I}{s_A y_I}$.
Consider now an offer $y_I \leq p \leq p_A^I$ where $p_A^I \equiv y_I + \frac{z_I}{s_A}$. Together with assumption B) this implies that $p \geq y_I$ and that $\frac{p_B}{p_A}p < y_I$ for all $\frac{p_A}{p_B}$ in the interval. That is, the holders of the A-shares will accept the offer while holders of the B-shares will reject the offer. From (8) we get

$$\pi_I^{Buy}(p) = z_I + s_A y_I - s_A p \begin{cases} > 0 & \text{for } y_I \leq p < p_A^I \\ = 0 & \text{for } p = p_A^I \end{cases}$$

Finally, because $\pi_I^{Buy}(p)$ is continuous and strictly decreasing for $p \geq y_I$ we have that $\pi_I^{Buy}(p) < 0$ for $p > p_A^I$. Therefore, the winning offers given by I must fulfill $y_I \leq p \leq p_A^I$.

□

Proof of Theorem 2

Proof. One single realization will according to theorem 1 exclude multiple price equilibria if and only if we for all $\frac{p_A}{p_B} > 1$ have that the holders of the B-shares will reject the winning offer. We will now systematically examine how such a realization must be when the reply of the losing management is maximally aggressive.

We will start by looking at realizations characterized by $y_I + z_I \geq y_R + z_R$. Thereafter, we will consider realizations $y_I + z_I < y_R + z_R$.

A. Realizations characterized by $y_I + z_I \geq y_R + z_R$.

Let us first assume that I wins for all $\frac{p_A}{p_B}$. We must then ensure that the holders of B-shares reject I's offer – i.e. according to table 1 we must have that

$$\frac{p_B}{p_A}\hat{p}_A^R < y_I \text{ for all } \frac{p_A}{p_B} > 1.$$

It will be most difficult to have this condition satisfied when $\frac{p_A}{p_B}$ is close to 1. Therefore the necessary and sufficient condition is obtained by setting $\frac{p_A}{p_B} = 1$. Remembering that R is maximally aggressive we get

$$y_R + z_R \leq y_I \ (\leq y_I + z_I). \tag{9}$$

A.1 Assume that $z_I \geq s_B z_R$.

Together with (9) this assumption implies that $y_I + \frac{z_I}{s_A} \geq y_R + z_R + \frac{s_B z_R}{s_A} = y_R + \frac{z_R}{s_A}$. Therefore, we have that I will win in cases a, b, and d in table 1. For case c where $1 + \frac{z_R}{s_A y_R} < \frac{p_A}{p_B} < 1 + \frac{z_I}{s_A y_I}$ we have that

$$\frac{y_I + z_I}{s_A + s_B \frac{p_B}{p_A}} > \frac{y_I + z_I}{s_A + s_B \frac{s_A y_R}{s_A y_R + z_R}} = \frac{y_I + z_I}{y_R + z_R}\left(y_R + \tfrac{z_R}{s_A}\right) \geq y_R + \tfrac{z_R}{s_A},$$

where the last inequality follows from $y_I \geq y_R + z_R$. Under assumption A.1 we have that I will win for all $\frac{p_A}{p_B}$ and the condition for excluding multiple price equilibria is therefore given by (9). This gives us condition iii) in theorem 2.

A.2 Assume that $z_I < s_B z_R$.

We start by noting that we must have that $y_I \geq y_R + z_R$. This can be seen from the following. For $z_I = 0$ this follows directly from $y_I = y_I + z_I \geq y_R + z_R$. For $z_I > 0$ we have from the assumption that $z_R > 0$. But then there will exist a $\frac{p_A}{p_B} > 1$ such that we are in case a in table

1, in which case I will win. In order for the holders of B-shares to reject the offer, it follows from (9) that we must have $y_I \geq y_R + z_R$.

Because $z_I < s_B z_R \Leftrightarrow z_R < \frac{1}{s_A}(z_R - z_I)$ we get that

$$y_R + z_R < y_R + \tfrac{1}{s_A}(z_R - z_I).$$

This leads all in all to the following two cases:

$$\begin{array}{ll} \text{I} & y_R + z_R \leq y_I < y_R + \frac{1}{s_A}(z_R - z_I) \\ \text{II} & y_R + z_R < y_R + \frac{1}{s_A}(z_R - z_I) \leq y_I. \end{array}$$

Assume first that we are in case I, i.e.

$$y_I + \tfrac{z_I}{s_A} < y_R + \tfrac{z_R}{s_A} \quad \Leftrightarrow \quad y_I\big(1 + \tfrac{z_I}{s_A y_I}\big) < y_R\big(1 + \tfrac{z_R}{s_A y_R}\big).$$

Since $y_I \geq y_R$ we get that this case implies that

$$\frac{z_R}{y_R} > \frac{z_I}{y_I}.$$

For small values of $\frac{p_A}{p_B}$ we will be in case a in table 1. This gives that I will win and (9) will ensure that the holders of the B-shares will reject I's offer. For $1 + \frac{z_I}{s_A y_I} < \frac{p_A}{p_B} < 1 + \frac{z_R}{s_A y_R}$ we will be in case b in table 1. For $\frac{p_A}{p_B}$ close to $1 + \frac{z_R}{s_A y_R}$ we will have that R will win while I will win for $\frac{p_A}{p_B}$ close to $1 + \frac{z_I}{s_A y_I}$. Hence, there must exist a $\big(\frac{p_A}{p_B}\big)^* \in]1 + \frac{z_I}{s_A y_I}; 1 + \frac{z_R}{s_A y_R}[$ such that I wins for $1 + \frac{z_I}{s_A y_I} < \frac{p_A}{p_B} \leq \big(\frac{p_A}{p_B}\big)^*$ and R wins for $\big(\frac{p_A}{p_B}\big)^* < \frac{p_A}{p_B} < 1 + \frac{z_R}{s_A y_R}$. When I wins, we have that $y_I \geq y_R + z_R$ will ensure that his offer will be rejected by the holders of B-shares. We will now show that the holders of B-shares will accept R's offer when he wins.

First we note that $\big(\frac{p_A}{p_B}\big)^*$ must be given by

$$\frac{y_R + z_R}{s_A + s_B\big(\frac{p_B}{p_A}\big)^*} = y_I + \tfrac{z_I}{s_A} \quad \Rightarrow$$

$$s_B\big(\tfrac{p_B}{p_A}\big)^*\big(y_I + \tfrac{z_I}{s_A}\big) = y_R + z_R - s_A\big(y_I + \tfrac{z_I}{s_A}\big) > y_R + z_R - s_A\big(y_R + \tfrac{z_R}{s_A}\big) = s_B y_R \quad \Rightarrow$$

$$\big(\tfrac{p_B}{p_A}\big)^*\big(y_I + \tfrac{z_I}{s_A}\big) > y_R.$$

For a $\frac{p_A}{p_B} > \big(\frac{p_A}{p_B}\big)^*$ but close to $\big(\frac{p_A}{p_B}\big)^*$, we will have that the holders of the B-shares will accept R's offer. Therefore, in order to exclude multiple price equilibria, we must be in case II, i.e.

$$y_I + \tfrac{z_I}{s_A} \geq y_R + \tfrac{z_R}{s_A}.$$

It is now straightforward to see that I will win in all the cases (a–d) in table 1. Because $y_I \geq y_R + z_R$, I will win in such a way that the holders of the B-shares will reject I's offer. We have thereby shown condition iv) in theorem 2.

B. Realizations characterized by $y_I + z_I < y_R + z_R$.
Condition i) and ii) in the theorem follows directly from symmetry and the results above.

□

Proof of Theorem 3

Proof. Lemma 3 allows us to derive the following results under the two different regulations in the following cases.
Case 1:

$$y_I + z_I \geq y_R + z_R \text{ and } y_I + \tfrac{z_I}{s_A} \geq y_R + \tfrac{z_R}{s_A} \Leftrightarrow y_I - y_R \geq \max\{z_R - z_I, \tfrac{1}{s_A}(z_R - z_I)\}.$$

In this case, I will be the management of the firm under both regulations, and we have:

	Regulation 1		Regulation 2
p_A	y_I	$=$	y_I
p_B	y_I	$=$	y_I
V_{firm}	y_I	$=$	y_I

Case 2:

$$y_I + z_I < y_R + z_R \text{ and } y_I + \tfrac{z_I}{s_A} < y_R + \tfrac{z_R}{s_A} \Leftrightarrow y_I - y_R < \min\{z_R - z_I, \tfrac{1}{s_A}(z_R - z_I)\}.$$

In this case, R will be the management of the firm under both regulations, and we have:

	Regulation 1		Regulation 2
p_A	$\max\{y_I + z_I, y_R\}$	$\leq$	$\max\{y_I + \frac{z_I}{s_A}, y_R\}$
p_B	$\max\{y_I + z_I, y_R\}$	$\geq$	y_R
V_{firm}	$\max\{y_I + z_I, y_R\}$	?	$s_A \max\{y_I + \frac{z_I}{s_A}, y_R\} + s_B y_R$

Case 3:

$$y_I + z_I \geq y_R + z_R \text{ and } y_I + \tfrac{z_I}{s_A} < y_R + \tfrac{z_R}{s_A} \Leftrightarrow z_R - z_I \leq y_I - y_R < \tfrac{1}{s_A}(z_R - z_I).$$

In this case, I will be the management of the firm under regulation 1 while R will end up as management under regulation 2, and we have:

	Regulation 1		Regulation 2
p_A	y_I	$\leq$	$\max\{y_I + \frac{z_I}{s_A}, y_R\}$
p_B	y_I	$\geq$	y_R
V_{firm}	y_I	?	$s_A \max\{y_I + \frac{z_I}{s_A}, y_R\} + s_B y_R$

where $y_I \geq y_R$ follows from $z_R - z_I \leq y_I - y_R < \frac{1}{s_A}(z_R - z_I)$.

Case 4:

$$y_I + z_I < y_R + z_R \text{ and } y_I + \frac{z_I}{s_A} \geq y_R + \frac{z_R}{s_A} \Leftrightarrow \frac{1}{s_A}(z_R - z_I) \leq y_I - y_R < z_R - z_I.$$

In this case, R will be the management of the firm under regulation 1 while I will continue as management under regulation 2, and we have:

	Regulation 1		Regulation 2
p_A	$\max\{y_I + z_I, y_R\}$	$\geq$	y_I
p_B	$\max\{y_I + z_I, y_R\}$	$\geq$	y_I
V_{firm}	$\max\{y_I + z_I, y_R\}$	$\geq$	y_I

The results from these four different cases prove the theorem.

$\square$

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Figure 1: The voting premium in Denmark 1985-1999
(Average)

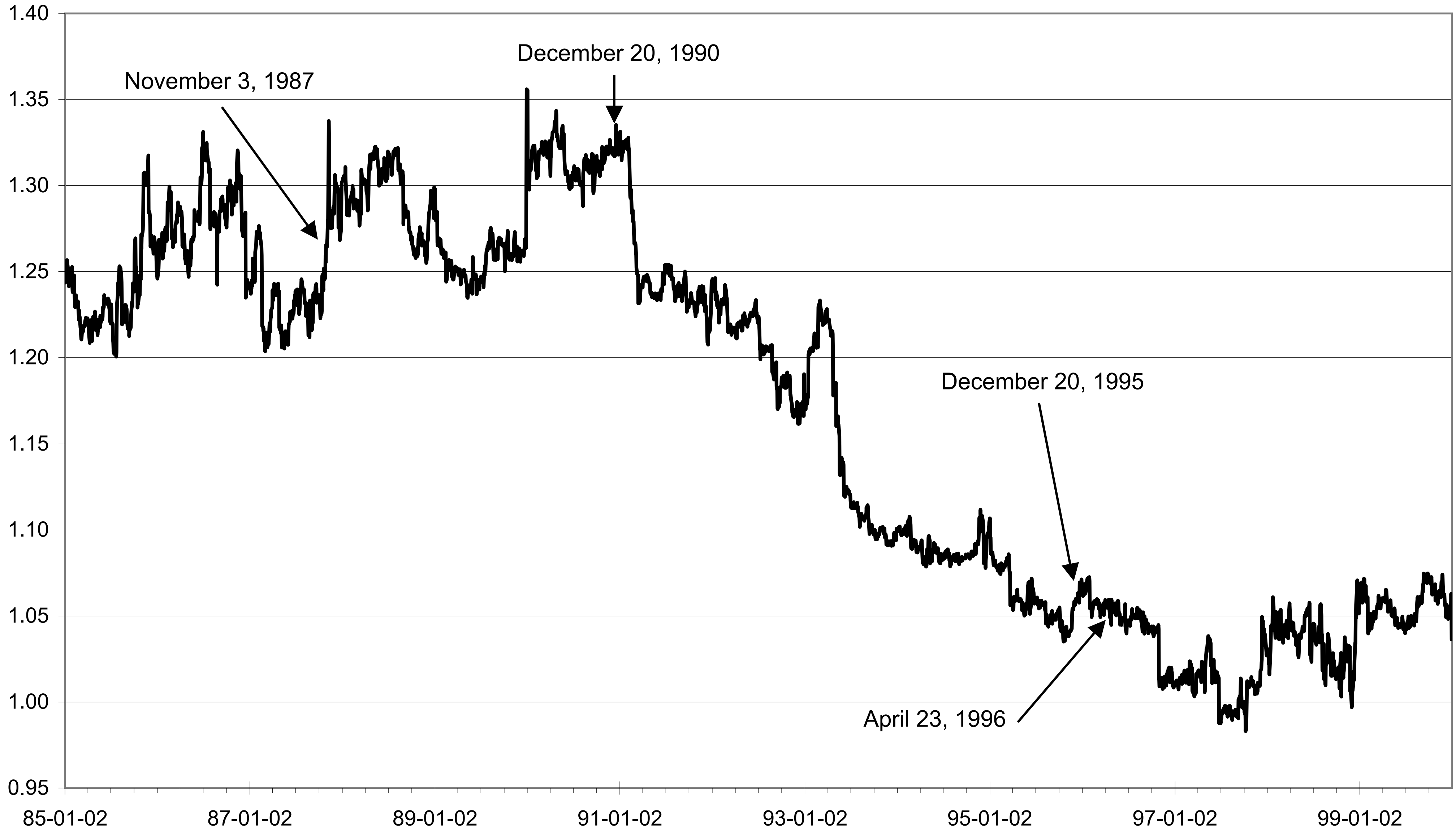

Figure 2: The voting premium in Denmark 1985-1999
(Quartiles)