Legal pre-emption rights as call-options, redistribution and efficiency loss

by

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Abstract*: We conduct an analysis of legal pre-emption rights in which a beneficiary has a right but not an obligation to acquire a specific good at a certain price. We analyse how such an option influences seller and other prospective buyers. Furthermore, we address the question of the efficiency loss if the option holder cannot use his option and then sell the asset to the person with the highest reservation price. We model a sealed second bid auction with uniformly distributed subjective values. We show that the option leads to an expected loss for the other bidders as well as the seller and a total efficiency loss for society. The efficiency loss is born by the other bidders and amounts to fifty percent of the redistribution from the seller to the person who gets the option. The results are almost similar when introducing bidders with subjective values drawn from a normal distribution.*

Introduction

Legislation sometimes contains provisions in which a beneficiary has a right but not an obligation to buy an asset or a specific good at a given price. Typically, this price is the price at which the asset has just been traded between two other parties, either at an auction or an ordinary trade. This is equivalent to a call option although with special features. The option holder (the beneficiary) normally has a short period of time to decide whether to use his option or not. Thus, contracts between private parties may also operate with such an option. In some situations the asset can be sold afterwards, sometimes not. Examples are numerous in both Common Law and Civil Law countries. A few examples from Danish law (Civil law) and from Danish contracts are shown below:

- When a rental housing property is sold, the seller is obliged to offer the tenants as a group that they can buy the property for the price the buyer should pay. They have a certain period of time to decide whether to use the option or not, in order to give them a possibility to find the necessary financing.
- When agricultural land is sold, the government can step in and buy the land at the price the buyer should pay.
- When a ”cultural artefact” is being wished exported from Denmark, the government has the right to forbid export. It then has to offer a fair price for the artefact. If the good has been sold at a Danish auction and the foreign buyer wants to take the artefact out of Denmark, the auction price is considered the ”fair price”. Technically this is not a call option, as the buyer doesn’t have to sell the artefact but can choose to keep it. But in 90%

of all cases the foreign buyer doesn’t want the asset when the government forbids him to take the asset out of Denmark.

- Legal entities owning an asset together (often in the form of owning shares in the same limited company) sometimes agree that if one of them wants to sell, he has to offer his part of the asset to the other owners at the same price as he can obtain from another party.

In this paper we analyse the following questions:

1. What is the value of such a “legal call option”?
2. To what extent does the creation of such an option influence the price at which the asset is sold, who gains and who loses?
3. What is the ”efficiency loss” if the asset cannot be sold by the option holder (as is often in practice the case, e.g. rental property and cultural artefacts in the above-mentioned examples)?

We have found no literature dealing directly with these problems. This is somewhat surprising as the subject combines two very large areas, option pricing theory and auction theory (besides law). We try to answer the above questions by analysing the result of an open outcry auction (or sealed second-bid auction) without and with a bidder with an immediate call-option to buy the asset at the highest bid, varying the number of bidders in the auction.

The disposition of the article is as follows. In section 2 we show that if the owner of the option is not allowed to sell the asset using his option, the effects of a law granting someone a call option under certain simplifying assumptions are the following:

- A redistribution from the owner of the asset to the option owner
- An efficiency loss for the economy as such, born by all other bidders

In section 3 we analyse a situation with n bidders which have uniformly distributed subjective values of the asset drawn from the same interval. We analyse the situation both with and without one of the bidders having a call option. We show that under these circumstances the value of the call option is twice the expected profit for the average bidder from participating in the auction in the situation without someone having a call-option. We furthermore show that the efficiency loss (which is borne by the other bidders) equals the expected profit of the average bidder from participating in the auction in the situation where nobody has a call option. The efficiency loss caused by the option thus is 33% of the total redistribution.

In section 4 we analyse a situation with n bidders which have subjective values of the asset drawn from the same normal distribution, using Monte Carlo simulation. We then compare the results for the normal distribution with the results obtained given a uniform distribution. The results are slightly different but not dramatically so. In section 5 we discuss possible ways of expanding the model, including the introduction of bidding costs, asymmetries and strategic bidding. Section 6 summarises the results.

2. A legal call-option: Redistribution and efficiency loss

Much option theory deals with options on financial assets with a well-defined market value and where the expiration date is fixed in advance. We are interested in options on real assets with only a limited number of potential buyers and consequently without a well-defined market price and where the option holder only has a very short time to decide to use his option. Our interest is in the dispersion of reservation prices and not in the time value of options.

Assume that a real asset is sold at an open outcry auction. It is a non-standardized or specific asset (a house, a painting) and consequently the asset does not have a well-defined market price.

We assume that we have n potential buyers. We assume (heroically) that they all can bid without any costs to ascertain the value of the asset from themselves. Without loss of generality we assume that the seller's reservation price is 0.

Now let us assume that the law grants one of the n potential buyers a call-option to buy the asset at the highest price bid at the auction. Obviously this means that the owner of the call-option does not bid at the auction. For the time being we assume no strategic bidding, i.e. the auction stops at the second highest reservation price among the *(n-1)* bidders which partake in the auction. In at strategic bid, the highest bidder might bid even higher than this price to lessen the risk that the option holder uses his option. It is obvious that if a bidder knows with very high precision the option owner's reservation price he will use this knowledge when bidding. But with limited knowledge and more than 2-3 other bidders strategic bidding is probably not worth while (see section 5 for arguments). Let us sort the potential bidders in order of ascending reservation price for the asset. With n participants the person with the highest reservation price has a reservation price $a_{(n)}$, the person with the second highest reservation price has a reservation price of $a_{(n-1)}$ and so forth. Given no strategic bidding, it is easy to see that the consequences of the option are:

- An expected loss for the owner of the asset.
- An expected gain of exactly the same size as the owner's loss for the receiver of the call-option
- An expected loss for the rest of the bidders which equals the efficiency loss for the community

The argument is as follows:

The only situations where the option influences the selling price are when the option holder has the highest or the second highest reservation price. The seller suffers a loss if the bidder has the highest or the second highest reservation price, as the seller in these situations gets an auction price equal to $a_{(n-2)}$ instead of $a_{(n-1)}$. This exactly equals the option holder's gain due to the option. The other bidders only suffer a loss due to the option when the option holder has the second highest reservation price(when he has the highest reservation price, he will get the asset anyhow). Their loss of course equals society's loss; the assets is being bought and used by a person with reservation price of $a_{(n-1)}$ instead of a person with a reservation price of $a_{(n)}$.

Below we show the result of the option in a situation where the option is given to a representative bidder but where we make no assumptions about which distribution the reservations prices is drawn from.

No bidder has an option:

The asset will be sold for $a_{(n-1)}$. The highest bidder has a gain of $a_{(n)} - a_{(n-1)}$. In other words:

- The seller gets $a_{(n-1)}$
- On average a bidder gets a profit of $\frac{a_{(n)} - a_{(n-1)}}{n}$

- Total profit for all participants (including the seller) is $a_{(n)}$. The asset is used optimally, being sold to the bidder with the highest reservation price.

One potential bidder has a legal call option to buy the asset at the auction price

Now let us assume that a random person among the *n* potential bidders by law gets a call option to buy at the highest bid. Therefore he doesn't bid and there is only be *(n-1)* bidders. We assume non-strategic bidding. If the option owner has one of the two highest reservation prices, he will get the asset, at a price of $a_{(n-2)}$. We get:

Expected sales price: $$\frac{2a_{(n-2)} + (n-2)a_{(n-1)}}{n} \qquad (1)$$

The probability that the option owner has one of the two highest reservation prices is *2/n* and the probability that he has a lower reservation price is *(n-2)/n*. The expected sales price is a weighted average of the probabilities multiplied with the relevant sales prices. Expected gain for all other bidders combined is:

$$\frac{(n-2)\left(a_{(n)} - a_{(n-1)}\right)}{n} \qquad (2)$$

The option owner gets an expected profit yielding:

$$\frac{a_{(n)} + a_{(n-1)} - 2a_{(n-2)}}{n} \qquad (3)$$

Thus, the expected loss of efficiency (given that the option owner cannot sell the asset) equals

$$\frac{a_{(n)} - a_{(n-1)}}{n} \tag{4}$$

We may now compute the distributive and efficiency effects of the law giving an option to a representative bidder by finding the differences caused by the option.

[INSERT TABLE 1]

We cannot from the above table say what the size of the redistribution to the option holder is compared to the efficiency loss. However, the option holder's gain equals the seller's loss and the efficiency loss to society equals the loss suffered by all other bidders than the person who receives the option.

Assuming the possibility of the option owner selling the asset to the person with the highest reservation price at a price equal to the option owner's reservation (that is, the second highest reservation price) will not change anything from the perspective of the option owner or the seller. However, there will be no loss of efficiency and the other bidders´ expected profit rises by this amount. That is, the effect of the option is only a redistribution from seller to option holder. The results are dependent on especially two assumptions.

1) We assume that there are n bidders with no costs to bid. A model with bidding costs would lead to fewer people bidding at the auction because the option would lessen the expected profit form participating. Fewer participants leads to lower prices and a loss for the seller, but raises profits for the remaining bidders (incl. the option holder).
2) We assume the absence of strategic bidding.

3. A model with n bidders and uniformly distribution of reservation prices

In this section we analyse a situation with n identical bidders which draw their reservation price from the same distribution. Seller's reservation price is 0. We assume that bidding costs are zero and that bidders know the value of the asset for themselves with certainty. In other words there is no winner's curse. A typical example could be the sale of a house in a village with only a few potentially interested buyers. We assume that all bidders draw their reservation prices from the interval (0;1). The reservation prices are independent and the probability distribution is uniform.

We only analyse situations with at least 3 bidders since with two potential bidders, one of which has an option, the highest bid is always zero, given the assumption of non-strategic bidding. We compute the average values of the highest, second highest and third highest reservations prices. With a uniform distribution we get simple results.

$$E(a_{(n)}) = n/(n+1) \tag{9}$$

$$E(a_{(n-1)}) = (n-1)/(n+1) \tag{10}$$

$$E(a_{(n-2)}) = (n-2)/(n+1) \tag{11}$$

We can now compute the expected auction prices dependent on n and on whether there is an option holder or not, the difference in price between these to situations (which equals the seller's loss or the option holder's gain) and the efficiency loss. We use the formulas in table 1.

[INSERT TABLE 2]

With n bidders the expected bid was $(n\text{-}1)/(n+1)$. With (n-1) bidders it is $(n\text{-}2)/n$. The difference is $2/n(n+1)$**.** The seller's loss due to the option (which equals the option owner's gain due to the option) equals two times the average bidder's gain from participating in an auction with no option holder.

What is the other bidders´ combined loss due to the option? The other bidders' loss equals the efficiency loss, as we saw above. There is only an efficiency loss when the option owner has the second highest valuation. The probability that the option owner has the second highest valuation is $1/n$. The efficiency loss is the average difference between the highest valuation and the second highest valuation, which is $1/(n+1)$. Thus, we have:

The expected efficiency loss: $1/n(n+1)$ (12)

Independent of the number of bidders the efficiency loss when the option is non-transferable in the sense that the asset cannot be sold by the option owner, equals 50% of the redistribution from asset owner to option owner, or 1/3 of the total redistribution.

4. A model with n bidders with a normal distribution of reservation prices

In this section we assume that the *n* potential bidders all draw their reservation price from a normal distribution with mean 0.5 and standard deviation 0.2887. This is the same mean and standard deviation as for the uniform distribution in section 3. Naturally, this formally gives the problem of the possibility of negative prices. An obvious alternative would be to draw from a log normal distribution or use a higher mean. We have not found it worth while. Sellers' reservation price is 0. We still assume that the option owner has no possibility of selling the asset after he has used his option. In this section, instead of finding analytical values, we use Monte Carlo simulation. Again we assume non-strategic bidding. We let the numbers of potential bidders vary from 3 to 100. For each number of potential bidders we made 100.000 Monte Carlo simulations. In addition, for each of the 100.000 times we have drawn *n* observations from the normal distribution, we compute the highest valuation, the second highest valuation and the third highest valuation and then the average for the 100.000 observations in each group. Whatever the number of potential bidders, only the 3 persons with the highest valuations are relevant for our analysis. Table 3 shows the average simulated values for $a_{(n)}$, $a_{(n-1)}$ and $a_{(n-2)}$, dependent on *n*.

[INSERT TABLE 3]

We now use our formulas from section 2 to compute expected auction prices with and without options and expected gains and losses to the different groups involved.

[INSERT TABLE 4]

If we compare with the results for the uniform distribution we get the following results:

- For the uniform distribution, the efficiency loss is always half of the redistribution from the seller to the option holder, that is we take 1 from the seller and 0.5 from the other bidders to give 1 to option holder, independent of *n*.

- For the normal distribution we have almost the same result for $n = 3$. But for n increasing, the efficiency loss gets higher and higher compared to the redistribution from seller to option holder, and for n large is almost equal to the redistribution.

Economists ordinarily use standard deviation as the measure of dispersion. However, in some connections the average absolute difference from the mean is more relevant. If we look at an option to buy an asset at the price today, the expected profit depends not necessarily on the standard deviation (SD). If the distribution is symmetric, what is relevant is the average absolute deviation (AD). For the uniform distribution AD/SD is $\sqrt{(3/4)}$. For the normal distribution AD/SD is $\sqrt{(2/\pi)}$.

Table 5 shows the results for both the normal distribution and the uniform distribution when we normalize with; the standard deviations of the distributions (SD) and the average absolute deviations of the distributions (AD)

[INSERT TABLE 5]

[INSERT FIGURE 1]

[INSERT FIGURE 2]

There seems to be no advantage of using AD instead of SD.

5. Bidding costs, asymmetries, strategic bidding and expansions of the model

The model may be expanded in several ways: asymmetries, bidding costs and strategic bidding. Of these expansions we find that asymmetries and bidding costs potentially are the most rewarding and that strategic bidding is more of theoretical than empirical interest in most cases.

Asymmetries and transaction costs: A large literature deals with the effects of asymmetries in auction theory. See Klemperer (1999) for an overview of the literature.

It is well known that a small advantage for one bidder may change the price and the gain for the highest bidder dramatically. If one bidder has a slight advantage compared with the other bidders (he can use the asset more efficiently) he will bid higher. The other bidders know that they have a "winner's curse" problem and therefore they will bid a little more carefully. This betters the situation for the bidder with a slight advantage as it lowers his "winner's curse" and so on. In a situation with transaction costs in connection with bidding this might lead to very few bidders and consequently very low prices. Consequently granting an option to a particular bidder is especially price pressing in a situation where there are transaction costs in connection with bidding and where the person who gets the option has an advantage already.

Milgrom (1981), Klemperer (1998), and Levin and Smith (1994) have analysed some of the problems relevant in this connection.

In Denmark the law granting the tenants a legal pre-emption right made a huge difference. It is not possible to judge whether and to what extent the law has changed the prices of rental properties as the price indices are of rather low quality and influenced by many other factors.

But the change in the percentage of properties sold to tenants has been startling. Before the law tenants bought very few properties (estimated less than 10 per cent of all trades). After the law was implemented the percentage rose to 60-75% (different sources have different estimates). This indicates that the law has given the tenants a rather large advantage in this area. The tenants probably have an edge compared with other buyers as to knowledge about the quality of the property. This edge probably is rather low as a seller of a property be law is forced to have an expert make a rather large report about the quality of the property with a description of what should be repaired or renewed, how long before the roof should be changed etc, with an estimation of costs. But the fact that they have a period of several weeks to decide themselves raises their advantage.

Strategic bidding: One may argue that the simple model does not incorporate bidders taking the option into account. The bidder with the highest reservation price might want to make a higher bid than necessary to win the auction, in order to lower the probability that the option holder will use his option. We will argue, however, that strategic bidding is only probable in very few cases and that the simplified model therefore works quite well. An “envelope” estimate may illustrate why

Let us initially look at our example with a uniform distribution in the interval (0;1) which we used in section 3. We have an option holder with an (unknown) reservation price of V.

Let us call the highest reservation price among the *(n-1)* bidders $a_{(n-1)}$ and the second highest reservation price reservation price $a_{(n-2)}$. What bid should the person with the highest reservation price bid? We assume that she has the option to overbid herself when the auction prices stops at $a_{(n-2)}$. She can then make a bid in the interval $(a_{(n-2)};a_{(n-1)})$. As a consequence, she will never offer more than her reservation price. Call the optimal bid Q. Given risk neutrality the bidder wants to maximize:

$$\Pr(Q > V))(a_{(n-1)}-Q) \qquad (13)$$

Given the uniform distribution we have $\Pr(Q > V) = Q$. Thus, the bidder wants to maximize.

$$(a_{(n-1)}-Q)Q. \qquad (14)$$

This function has maximum for

$$Q = a_{(n-1)}/2. \qquad (15)$$

In other words, the bidder will always want to bid half of his reservation price, even if the auction stops at a lower price. In an auction with 1-3 bidders (except the option owner) strategic bidding might be realistic. With more bidders the probability for at strategic bid is small, because the second highest reservation price normally will be higher than the strategic bid. In a situation with very few bidders or where the bidders know with a very high degree of precision the option holder's reservation price strategic bidding might be relevant.

Although it is difficult to imagine many situations where a bidder might know the reservation price of the option holder with any high degree of precision. Intuitively, it is obvious, that the conditions for at strategic bid being used are:

- Low probability that the option holder's reservation price is below $a_{(n-2)}$.

- Low spread on the estimate of *B*'s reservation price, given it is above $a_{(n-2)}$.

6. Summary

This article deals with "legal call options" where the law (or a private agreement) grants a beneficiary a pre-emptive right to a specific asset or good. A rather naive, but intuitively appealing argument for this kind of legislation is that it ”does not cost anybody anything to give someone this option”, the thought being that the seller gets the price he was willing to sell for. We cannot be sure that this is the argument. However, there is no doubt that the legislature sometimes gives agents such options, considering neither the costs nor the efficiency loss

In this article we have argued for the following conclusions:

- There is an efficiency loss associated with such options, of the size of 33-50% of the value transferred to the person who gets the option.
- The loss to the seller equals the gain of the option holder
- The efficiency loss equals the loss suffered by the other bidders
- The results do not depend that much on the exact form of the distribution assumed, the results for a normal distribution and a uniform distribution with the same SD (or AD) being more or less the same.

The results naturally dependent on the assumptions, especially the assumption of no transaction costs, no asymmetries and no strategic bidding., however, we believe that the last assumption is reasonably realistic. But there should be much room for more research in this – somewhat exotic – area. It should be possible to come to other, interesting conclusions. Our feeling is that taking transaction costs and asymmetries into account would raise both the value of the option and the efficiency loss, by “scaring” bidders away from participating in an “unfair” auction. The interesting question is, of course, how much.

Table 1. Distributive and efficiency effects of giving an option to a representative bidder

Change in profit due to option		
Seller	$\frac{-2\left(a_{(n-1)}-a_{(n-2)}\right)}{n}$	(5)
Option holder	$\frac{2\left(a_{(n-1)}-a_{(n-2)}\right)}{n}$	(6)
All other bidders	$\frac{-\left(a_{(n)}-a_{(n-1)}\right)}{n}$	(7)
Society as a whole	$\frac{-\left(a_{(n)}-a_{(n-1)}\right)}{n}$	(8)

Table 2. Effects of granting an option to a representative bidder, given a uniform distribution of reservation prices.

N	Highest bid with no option	Highest bid with option	Difference	Efficiency loss
	(n-1)/(n+1)	*(n-2)/n*	*2/n(n+1)*	*1/n(n+1)*
3	0.5	0.3333	0.1667	0.0833
4	0.6	0.50	0.10	0.05
5	0.6667	0.6	0.0667	0.0333
6	0.7143	0.6667	0.0476	0.0238
7	0.75	0.7143	0.0357	0.0179
8	0.7778	0.75	0.0278	0.0139
16	0.8824	0.875	0.0074	0.0037
99	0.98	0.9798	0.0002	0.0001
100	0.9802	0.98	0.0002	0.0001

Note: The average numerical difference between observations and the mean is, given the assumptions, 0.25. The standard deviation is 0.2887.

Table 3: Average values for the highest 3 valuations in a population of *n*

N	3	4	5	6	7	8	16	99	100
$a_{(n)}$	0.747	0.799	0.837	0.866	0.891	0.912	1.013	1.222	1.227
$a_{(n-1)}$	0.500	0.587	0.643	0.686	0.717	0.746	0.873	1.117	1.123
$a_{(n-2)}$	0.255	0.414	0.500	0.558	0.602	0.637	0.787	1.063	1.064

Table 4. Expected prices with and without options and expected redistribution and efficiency loss due to options.

N	Price without Option	Price with option	Difference	Efficiency loss
3	0.500	0.337	0.162	0.082
4	0.587	0.500	0.087	0.053
5	0.643	0.586	0.057	0.039
6	0.686	0.643	0.043	0.030
7	0.717	0.684	0.033	0.025
8	0.746	0.719	0.027	0.021
16	0873	0.862	0.011	0.009
99	1.117	1.116	0.001	0.001
100	1.123	1.122	0.001	0.001

Table 5. Redistribution relative to standard deviation and average absolute deviation of the distributions

N	Uniform distribution	Normal distribution	Uniform distribution	Normal distribution
	Redistribution/SD	Redistribution/SD	Redistribution/AD	Redistribution/AD
3	0.577	0.564	0.667	0.705
4	0.346	0.299	0.40	0.374
5	0.231	0.198	0.267	0.248
6	0.165	0.147	0.190	0.184
7	0.124	0.114	0.143	0.142
8	0.096	0.094	0.111	0.117
16	0.026	0.037	0.030	0.047
99 100	0.001	0.004	0.001	0.005
100	0.001	0.004	0.001	0.005

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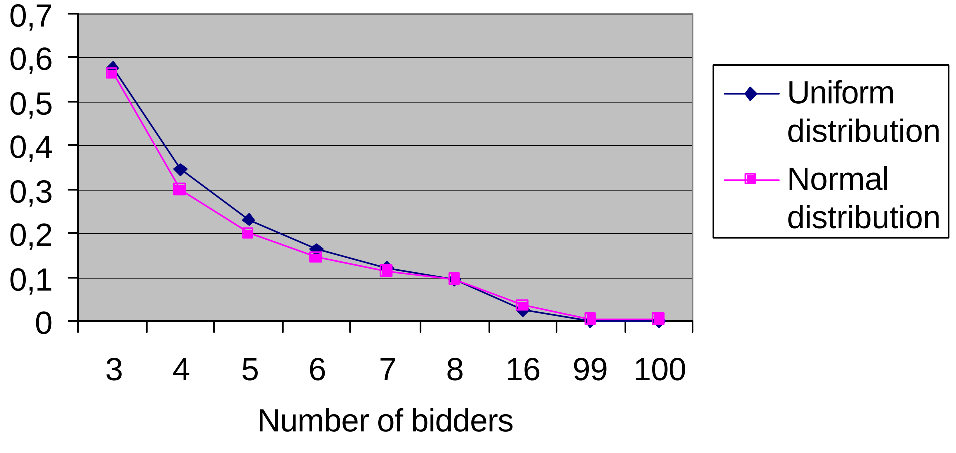
Figure 1. Redistribution relative to std. deviation both with mean 0.5 and std. deviation 0.2887



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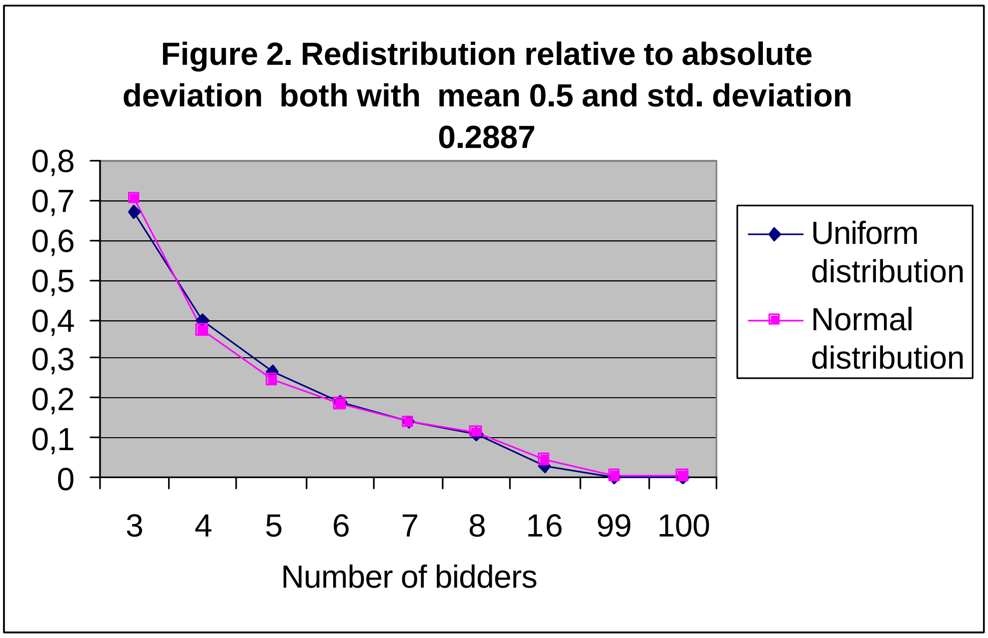

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