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WAGES, UNEMPLOYMENT, AND THE UNDERGROUND ECONOMY

Ann-Sofie Kolm
Birthe Larsen

Department of Economics - Solbjerg Plads 3, C5 - DK-2000 Frederiksberg

Wages, Unemployment, and the Underground Economy*

Ann-Sofie Kolm† and Birthe Larsen‡

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Abstract

While examining the macroeconomic effects of increased government control of the informal sector, this paper develops a two-sector general equilibrium model featuring matching frictions and worker-firm wage bargaining. Workers search for jobs in both the formal and the informal sector. We analyse the impact of higher punishment rates and a higher audit rate on labour market performance. We find that a higher punishment rate reduces the size of the informal sector and reduces unemployment. A higher audit rate has an ambiguous impact on unemployment, and may actually increase the size of the underground economy.

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†Department of Economics, Uppsala University, Box 513, S-751 20 Uppsala, and Institute for Labour Market Policy Evaluation (IFAU), Box 513, S-751 20 Uppsala. Ph. +46 18 471 1131, Fax +46 18 471 1478. E-mail address: ann-sofie.kolm@nek.uu.se

‡Centre for Research in Social Integration and Marginalization (CIM) and Copenhagen Business School, Department of Economics, Solbjerg Plads 3, DK-2000 Copenhagen F. Ph. +45 38 15 25 44, E-mail address: bl.eco@cbs.dk

1 Introduction

Underground activities are unequally distributed across sectors within an economy. While analyzing comprehensive survey data for Denmark, Smith and Pedersen (1998) find that around 70 percent of the total hours performed in the informal sector is carried out within the service sector or construction sector. Hence, on a large scale, different goods are produced in the informal and in the formal sector. Although there may be different explanations for why this is the case, one reason may simply be that some types of activities and goods are easier to hide than others, and hence these goods are more likely to be produced in the informal sector. For example, cleaning jobs in private homes, gardening services, hairdressing, home repair activities, and other types of service jobs.

The purpose of this paper is to examine the macro economic effects of tax and punishment policies when different goods are being produced in the formal and the informal sector. In particular, we focus on how a revenue neutral change in the government controls of the informal sector affects labour market performance. Hence, if the government were to control the underground economy more severely, either through higher punishment fees or through a more frequent auditing of informal sector workers and/or firms, what would then happen to unemployment, sector allocation, and wages? For example, would higher punishment fees or a higher audit rate lead to a smaller informal sector? Furthermore, what happens to the number of unemployed workers?

In order to answer these questions, we develop a two-sector general equilibrium model featuring matching frictions and worker-firm wage bargains. The two sectors corresponds to the formal sector and the informal sector. Different goods are produced in the two sectors, and workers face job opportunities in both the formal sector and the informal sector.

We find that increased government control of the informal sector in terms of higher punishment fees (i) increases the size of the formal sector and reduce the size of the informal sector (ii) reduces real producer wages in both sectors, and (iii) reduces unemployment. Considering the impact of a higher audit rate is less clear cut. A higher audit rate has an ambiguous impact on unemployment and real producer wages, and may actually increase the size of the underground economy.

The principal contribution of the analysis in this paper is that we incorporate an imperfectly competitive labour market. This facilitates an analysis of how tax and punishment policies affect wage setting and unemployment. Previous research is mainly conducted within the public finance tradition.[1] In this literature wages are either assumed to be fixed or determined by market clearing, and by definition, such framework is unable to examine how involuntary unemployment is affected by tax and punishment policies. There have, however, been some recent studies of underground activity in models of involuntary unemployment; see Kolm and Larsen (2001, 2003), Cavalcanti (2002), Boeri and Garibaldi (2002) and Fugazza and Jacques (2003). The focus and modeling strategies are, however, very different in these papers. The studies by Kolm and Larsen (2001) and Fugazza and Jacques (2003) explores the consequences for unemployment when workers have moral considerations when deciding on informal sector work. With workers being heterogenous with respect to moral, only workers with low moral are willing to work in the informal sector. Kolm and Larsen (2003), has a different focus as it examines the potential for multiple equilibria when there are moral considerations and a social norm against tax evasion. The study by Cavalcanti (2002) also has a different focus as it explores how labour market policies affects unemploy-

[1] See Slemrod and Yitzhaki (2000) and Schneider and Eneste (2000) for two recent surveys of tax avoidence and tax evasion.

ment in a model with informal labour market opportunities. Firms in the informal sector differs from firms in the formal sector as they are assumed to face smaller job creation costs. The study by Boeri and Garibaldi (2002), considers control policies in a model of informal employment and involuntary unemployment. The modelling of the underground activity is, however, very different from the modelling in this paper. In their model, all jobs are started as legal jobs. Informal jobs come about as legal firms are hit by a bad productivity shock and face the option of becoming illegal.

This paper is organized as follows. Section 2 describes the model and the equilibrium variables are derived. In section 3, we examine how the equilibrium variables (tightness, relative prices, real producer wages, sector allocation, and unemployment) are affected by a fully financed change in the punishment fees and the audit rate. Finally, Section 4 concludes.

2 The Model[2]

The economy consists of two sectors, a formal sector and an informal sector. Different goods are produced in the two sectors. This captures the notion that certain types of goods and services are more likely to be produced in the informal sector than other types. For example, cleaning jobs in private homes, gardening services, hairdressing and other types of service jobs are more likely to be produced in the informal sector, whereas cars, televisions, radios etc. are less likely to be produced in the informal sector.[3]

[2] This model is along the line of Pissarides 2000, extended to a two sector version.

[3] If different goods are being produced, one could ask 'how come the workers and consumers are able to locate the informal firms whereas the tax authorities are not fully able to?' One answer is that the tax authorities cannot officially search in the same way as workers, and would need to use searching methods corresponding to each individual consumer. This is a time-consuming and expensive process, whereby only a fraction p of all informal firms and workers is detected. The tax authorities know what kind of firms

The government audits the economy. With probability p a worker-firm pair in the underground economy is detected and then has to pay a punishment fee and the match will be dissolved.[4]

2.1 Matching

Workers search for jobs in both the formal and the informal sector. We assume that only unemployed workers search for jobs. This is a simplification, i.e. we do not acknowledge that the connection to the labour market given by working in the formal sector, brings about job opportunities not available while unemployed. Workers accept job offers as long as the expected payoff exceeds their reservation wage.[5] We assume undirected search as in, for example, Albrecht and Vroman (2002). The matching function is given by:

$$X = v^{1-\eta} u^{\eta},$$

where u is unemployment and v is the total number of vacancies supplied by firms. The labour force is normalized to unity, whereby we interpret u as the unemployment rate and v as the vacancy rate. The number of vacancies supplied by the formal sector and the informal sector are $v^j, j = F, I$, and hence $v = v^F + v^I$. The worker's transition rates into the two sectors

to search for, but they do not know where to find them and their employees. The firms are not registered, they do not exist in any statistics, and officially their employees are unemployed.

[4] One can view the assumption of the match being dissolved in several ways. For example, once detected there may be a court process. This court process may take a long time. In case the legal system punishes one of the parties, say the employer, the employees may simply search for a new job. Alternatively, the match may be dissolved as the detected parties fear that the tax authority will return to the firm and workers with probability one if the match is continued.

[5] We focus on the non-trivial case where it is not optimal to reject job offers from one sector and wait for a job offer from the other sector in order to have an economy with both a formal and an informal sector. Moreover, we disregard from moral considerations; see Kolm and Larsen (2001) for a model where workers are heterogenous in terms of moral.

can be expressed as $\lambda^F = \beta\frac{X}{u} = \beta\theta^{1-\eta} = \beta\pi(\theta)$, and $\lambda^I = (1-\beta)\frac{X}{u} = (1-\beta)\theta^{1-\eta} = (1-\beta)\pi(\theta)$, where $\beta = \frac{v^F}{v}$ is the fraction of vacancies supplied in the formal sector and $\theta = v/u$ is overall labour market tightness. The term $\pi(\theta)$ can be interpreted as the probability of a worker getting any job offer, i.e., $\lambda^F + \lambda^I = \pi(\theta)$. The transition rates facing firms is equal across firms and given by $q = \frac{X}{v} = \theta^{-\eta}$. Furthermore, we define labour market tightness for the formal sector as $\theta^F = v^F/u$ and labour market tightness for the informal sector as $\theta^I = v^I/u$ where hence $\theta^F + \theta^I = \theta$.

2.2 Workers

Unemployed workers have the opportunity to apply for jobs in both the formal sector and the informal sector. Let λ^F and λ^I be interpreted as the probabilities per time unit of finding a job in the formal sector and in the informal sector, respectively. The present discounted value of unemployment, U, employment in the formal sector, E^F, and employment in the informal sector, E^I, are given in the following flow value equations:

$$
\begin{aligned}
rU &= \frac{R+\xi}{P} + \lambda^F(E^F - U) + \lambda^I(E^I - U), && (1)\\
rE^F &= \frac{R+\xi+w^F(1-t)}{P} + s(U - E^F), && (2)\\
rE^I &= \frac{R+\xi+w^I(1-p\delta)}{P} + (s+p)(U - E^I), && (3)
\end{aligned}
$$

where r is the exogenous discount rate, t gives the income tax rate, δ captures the proportion of the evaded wage a worker has to pay as a punishment fee if detected withholding the government taxes,[6] and p is the audit rate. s is the exogenous separation rate, R is a lump sum transfer received from

[6] It is of no importance for the results whether the punishment fee is imposed on evaded income or evaded taxes. This is not always the case in the previous literature of tax evasion, where the choice to base the fines of evasion on evaded income or evaded taxes may be of

the government, and ξ is profits received as dividends. Aggregate profits generated in the economy is distributed equally across the population.

The match is dissolved when detected which implies that the separation rate in the informal sector exceeds the formal sector separation rate.

The immediate income received in each state is expressed in real terms by division with the general price level, P. P is the cost-of-living index which is linear homogenous in the two goods prices, P^F and P^I, and derived from consumer preferences. This expression for real income could equivalently be interpreted as the instantaneous indirect utility function under certain conditions. In section 2.7 we discuss preferences in more detail.

The goods prices, and hence the general price level, is in equilibrium determined by market clearing and is taken as given by the individual firms and workers. It is hence of no importance for the results whether the flow value equations given in this section, and the next section, are given in terms of real income or in nominal income.

2.3 Firms

The marginal productivity of a worker is y.[7] Hiring costs are denoted $k^j, j = F, I$ and q is the firm's probability per time unit of finding a worker. Since the value functions for workers are expressed in real terms, we express the value functions for firms also in real terms.

Firms in the formal sector are characterized by the arbitrage equations:

$$rJ^F = \frac{P^F}{P}y - \frac{w^F(1+z)}{P} + s(V^F - J^F), \tag{4}$$

$$rV^F = q(J^F - V^F) - \frac{k^F}{P}, \tag{5}$$

significant importance for the results. See Yitzhaki (1974) who pointed out the importance of this critical assumption.

[7] There is no apriori reason to assume that one of the productivities should be greater than the other.

where J^F is the value of having a filled job in the formal sector, V^F is the value of an unfilled job in this sector, and the parameter z is the payroll tax rate.

Similarly, firms in the informal sector have J^I and V^I determined by:

$$rJ^I = \frac{P^I}{P}y - \frac{w^I(1+p\alpha)}{P} + (s+p)(V^I - J^I), \tag{6}$$

$$rV^I = q(J^I - V^I) - \frac{k^I}{P}, \tag{7}$$

where α is the proportion of the evaded wage the firm has to pay as a punishment fee for cheating the government on payroll taxes when supplying informal sector jobs.

2.4 Wages

In the wage bargains, the firm and the worker take the market clearing prices as given. Wages, $w^j, j = F, I$ solve first order conditions from the Nash Bargaining Solutions with the worker's bargaining power being equal to γ:

$$\frac{\gamma}{1-\gamma}\frac{1}{\phi^j}\left(J^j - V^j\right) = E^j - U, j = F, I, \tag{8}$$

where $\phi^F = \frac{1+z}{1-t}$ and $\phi^I = \frac{1+p\alpha}{1-p\delta}$ are the tax and punishment wedges.

By use of equations (1)-(7) in equations (8), and assuming free entry, $V^j = 0$, $j = F, I$, and symmetric conditions facing firms and workers within each sector, the relevant real producer wages are:

$$\omega^F = \frac{w^F(1+z)}{P^F} = \gamma y\left(1 + \rho\left(\theta^F + \frac{\theta^I}{\Delta}\right)\right), \tag{9}$$

$$\omega^I = \frac{w^I(1+p\alpha)}{P^I} = \gamma y\left(1 + \rho\Delta\left(\theta^F + \frac{\theta^I}{\Delta}\right)\right), \tag{10}$$

where

$$\Delta = \psi \frac{P^F}{P^I}, \tag{11}$$

and

$$\psi = \frac{\phi^I}{\phi^F} = \frac{1 + p\alpha}{1 - p\delta} / \frac{1 + z}{1 - t}, \tag{12}$$

where we have used that $k^j = \rho P^j y, j = F, I$. One interpretation of this specification of the vacancy cost is that the firm allocates its work force optimally between production and recruitment activities. The cost of hiring is proportional to its alternative cost, i.e., proportional to the value of the marginal product of labour. ψ is the punishment/tax wedge between the informal sector and the formal sector. We will simply refer to ψ as the wedge. It seems reasonable to focus on the case when $\psi < 1$, that is, when the government does not audit or punish the informal sector to the same extent as the formal sector is taxed. This is, however, of no importance for the results.

The wage rules in (9) and (10) capture the wage demands, i.e., the bargained wages for a given relative price and for given tightness. The relative price and sector tightness are clearly endogenous variables and will be determined in equilibrium. However, before proceeding to the determination of equilibrium, we can explore the consequences of a change in the tax and punishment rates, α, δ, z, and t, and the audit rate, p, on wage demands. For given equilibrium variables, an increase in the punishment rates or the audit rate for given tax rates, will reduce the formal sector wage demands, and increase the wage demands in the informal sector. The reason is that the value of employment has fallen in the informal sector relative to in the formal sector. Workers employed in the informal sector hence face a reduced value of employment relative to unemployment and will push for higher wages. The opposite holds for workers employed in the formal sector, which causes formal sector workers to moderate their wage demands. Analogous interpretation can be given for changes in the tax rates for a given punishment policy.

It also follows from (9) and (10) that proportional changes in the tax and punishment system, leaving the wedge unaffected, will have no impact on wage demands since these changes have no effect on the value of employment relative to the value of unemployment in each respective sector.

2.5 Labour market tightness

Labour market tightness for the formal and the informal sector is determined by equations (4),(5), (6) and (7) using the free entry condition and the wage equations (9) and (10) :

$$(r+s)\,\theta^{\eta} = \frac{(1-\gamma)}{\rho} - \gamma\left(\theta^{F} + \frac{\theta^{I}}{\Delta}\right), \tag{13}$$

$$(r+s+p)\,\theta^{\eta} = \frac{(1-\gamma)}{\rho} - \gamma\left(\Delta\theta^{F} + \theta^{I}\right). \tag{14}$$

Note, however, that equations (13) and (14) determine labour market tightness in the formal sector and the informal sector conditioned on the relative price, i.e., $\Delta = \psi\frac{P^F}{P^I}$. Thus we have two equations in the three unknowns, labour market tightness in the two sectors and the relative price, θ^F, θ^I, and $\frac{P^F}{P^I}$.

To close the system, we need to incorporate the product market. Before doing that it turns out to be useful to derive the employment rates.

2.6 Employment

Steady state employment and unemployment rates are derived by considering the flows into and out of employment and the labour force identity, $n^F +$

$n^I + u = 1$. The flow equations are given by $\lambda^F u = sn^F$ (formal sector) and $\lambda^I u = (s+p)\, n^I$ (informal sector).

Solving for the employment rates and the unemployment rate, we obtain:

$$n^I = \frac{\frac{\theta^I \theta^{-\eta}}{s+p}}{1 + \frac{\theta^F \theta^{-\eta}}{s} + \frac{\theta^I \theta^{-\eta}}{s+p}}, \tag{15}$$

$$n^F = \frac{\frac{\theta^F \theta^{-\eta}}{s}}{1 + \frac{\theta^F \theta^{-\eta}}{s} + \frac{\theta^I \theta^{-\eta}}{s+p}}, \tag{16}$$

$$u = \frac{1}{1 + \frac{\theta^F \theta^{-\eta}}{s} + \frac{\theta^I \theta^{-\eta}}{s+p}}, \tag{17}$$

$$u^o = u + n^I = \frac{1 + \frac{\theta^I \theta^{-\eta}}{s+p}}{1 + \frac{\theta^F \theta^{-\eta}}{s} + \frac{\theta^I \theta^{-\eta}}{s+p}}, \tag{18}$$

where u^o denotes official unemployment, that is, unemployment registered by the government. The relative sector size of employment is given by:

$$\frac{n^F}{n^I} = \frac{\theta^F}{\theta^I}\frac{s+p}{s}. \tag{19}$$

2.7 Product market equilibrium

Product markets clear in each period. We assume that individual preferences over the two goods are represented by a linear homogenous instantaneous utility function $\upsilon\left(C_i^F, C_i^I\right)$, where C^F is produced in the formal sector and C^I is produced in the informal sector.[8] Individuals choose the optimal mix of the two goods in each period by maximizing utility given their budget constraint. With a linear homogenous utility function, individual demand for each good is linear in the available income. Moreover, the indirect utility function is linear in income; i.e., $\upsilon\left(I_i, P^F, P^I\right)^* = I_i / P\left(P^F, P^I\right)$ where I_i is the individual's available income from the budget constraint, and $P\left(P^F, P^I\right)$

[8] U_F, $U_I > 0$, and U_{FF}, $U_{II} < 0$.

is the cost-of-living index. Note that the instantaneous real income measure used in the flow value equations (1)-(3) can also be interpreted as the instantaneous utility given workers are risk neutral and consume their full income in each period.

Let us now consider market clearing. Aggregating over individual demand in order to derive aggregate demand for the two goods is simply a matter of aggregating over individual income, as preferences are homothetic. Hence, we have that the aggregate demand for the two goods is given from the first-order condition for the individual consumer's optimal mix of commodities, i.e. $\frac{\upsilon_F\left(C^F,C^I\right)}{\upsilon_I\left(C^F,C^I\right)} = P^F/P^I$, in conjunction with the aggregate (economy wide) budget constraint. The relative price is obtained by equating demand and supply of commodities. The aggregate supplies of the two goods are given by production deducted vacancy costs. In the formal sector, we have $Y^F = yn^F - v^F \rho y = n^F y \left(1 - \rho\theta^\eta s\right)$. Similarly, in the informal sector, aggregate supply is $Y^I = yn^I - v^I \rho y = n^I y \left(1 - \rho\theta^\eta \left(s + p\right)\right)$.

Equalizing aggregate demand and aggregate supply leads to the following equation:

$$\frac{\upsilon_F\left(Y^F/Y^I, 1\right)}{\upsilon_I\left(Y^F/Y^I, 1\right)} = \frac{P^F}{P^I}. \tag{20}$$

For simplicity, we assume a Cobb Douglas Utility function $U = \left(C^F\right)^\sigma \left(C^I\right)^{1-\sigma}$. Using the Cobb Douglas assumption together with equation (19), we can rewrite equation (20) as:

$$\frac{\sigma}{1-\sigma}\frac{\theta^I}{\theta^F}\frac{\left(\frac{1}{s+p} - \rho\theta^\eta\right)}{\left(\frac{1}{s} - \rho\theta^\eta\right)} = \frac{P^F}{P^I}, \tag{21}$$

which is an equation in the three unknowns θ^F, θ^I, and $\frac{P^F}{P^I}$.

2.8 Equilibrium

Now we can characterize the equilibrium in the labour and goods markets with the equations (13), (14), and (21). We have:

$$
\begin{aligned}
(r+s)\left(\theta^F+\theta^I\right)^\eta &= \frac{(1-\gamma)}{\rho}-\gamma\left(\theta^F+\frac{\theta^I}{\Delta}\right), && (22)\\
(r+s+p)\left(\theta^F+\theta^I\right)^\eta &= \frac{(1-\gamma)}{\rho}-\gamma\Delta\left(\theta^F+\frac{\theta^I}{\Delta}\right), && (23)\\
\psi\frac{\sigma}{1-\sigma}\frac{\theta^I}{\theta^F}\frac{\frac{1}{s+p}-\rho\theta^\eta}{\frac{1}{s}-\rho\theta^\eta} &= \Delta, && (24)
\end{aligned}
$$

where we recall that $\Delta=\psi\frac{P^F}{P^I}$. Firms will enter into the two sectors as long as the expected vacancy costs are equal to the discounted profit. This is captured by equations (22) and (23). Equation (24) gives the relative price as a function of the relative supply derived from consumer preferences.

Because the separation rate for informal sector jobs is higher than the separation rate for formal sector jobs, it is more attractive for a firm to enter the formal sector since jobs on the average last a longer time in the formal sector. On the other hand if $\psi<1$, firms in the informal sector are expected to be punished less than firms in the formal sector are taxed, which makes it more attractive to enter the informal sector. However, whether it is more attractive to enter one or the other sector also depends on the prices consumers pay for the different goods produced. Or put differently, entry into one sector rather than the other sector because of the relative attractiveness of the tax/punishment system or the difference in the separation rates will be counteracted by adjustments in the relative price. Entry into the formal sector will increase the supply of formal goods and hence reduce the relative price P^F/P^I, which in turn reduces the relative attractiveness of entering the formal sector.

We only consider fully financed reforms. Hence, the government budget restriction is always satisfied and is given by:

$$n^F \omega^F \left(1 - \frac{1}{\phi^F}\right) + n^I \omega^I \frac{\psi}{\Delta}(1 - \frac{1}{\phi^I}) - c\left(p, n^I, n^F\right) = \frac{R}{P^F}, \tag{25}$$

where $c\left(p, n^F, n^I\right)$ is a function that captures that there is a cost associated with auditing.[9] The budget restriction is a function of the tax and punishment wedges, ϕ^F and ϕ^I, and the audit rate, p. Recall that the producer wages, employment rates, and Δ are functions of the wedge, $\psi = \frac{\phi^I}{\phi^F}$, and the audit rate p, where we note that p appears both in the wedge ψ and in the informal sector separation rate, $s + p$. The tax rates, t and z, and the punishment rates, δ and α, will not appear in the government budget restriction directly when all substitutions are done. This reflects that t and z are equivalent instruments, and so are δ and α. Hence it does not matter if we tax (punish) the firm side or the worker side. A change in δ and α is captured by a change in ϕ^I, and a change in z and t is captured by a change in ϕ^F.

From (25) it is clear that an increase in ϕ^F and ϕ^I that leaves ψ and p unaffected, will increase the government revenue. Hence for a given wedge, the government can choose t, z, δ and α so as to reap any level of revenue. This is very convenient and implies that we can investigate the impact of various reforms on the equilibrium variables, without explicitly incorporating the government budget restriction.

[9] In the literature on tax evasion it is commonly assumed that auditing is costly whereas punishment fees are costless. The auditing costs may depend on p and the number of producing firms in each sector. As will become clear below, any specification of the auditing costs that includes any of the real variables in the model or the auditing rate, p, or the wedge, ψ, will yield the same results.

3 Comparative statics

This section considers the impact of two reforms on tightness, relative prices, real producer wages, sector allocation and unemployment. The first reform involves a change in the punishment rates, α and/or δ, whereas the second reform involves a change in the audit rate p. Both reforms are fully financed and will be discussed in turn below.

Before considering the reforms we will engage in some substitution in order to reduce the equation system in (22)-(24) and to trace down some intuition. First, we eliminate $\left(\theta^F + \frac{\theta^I}{\Delta}\right)$ from (22) and (23) above. This yields

$$\Delta = \frac{\frac{(1-\gamma)}{\rho} - (r+s+p)\,\theta^{\eta} \quad > 0}{\frac{(1-\gamma)}{\rho} - (r+s)\,\theta^{\eta} \quad < 1}, \tag{26}$$

where $\Delta = \psi \frac{P^F}{P^I}$. Hence, the free entry conditions determine the relative price, $\frac{P^F}{P^I}$, conditional on total tightness θ. Changes in ψ will induce proportional adjustments in the relative price so that Δ is unaffected.

Equation (26) reflects the discussion in connection with equations (22)-(24), and verifies that it is ψ and the difference in the separation rates that are important for the entry and exit into the two sectors, and hence for the relative price. This is easily seen by considering the following two imaginary polar cases. If $\psi < 1$, and $p = 0$, we have $\Delta = 1$ and hence $\frac{P^F}{P^I} = \frac{1}{\psi} > 1$. That is, the informal sector is more attractive in the sense that informal firms are expected to be punished less than formal firms are taxed. Hence, firms keep entering the informal sector until the formal sector relative price has increased to such an extent that formal firms are fully compensated for the fact that $\psi < 1$. If on the other hand $\psi = 1$, and $p > 0$, we have that $\frac{P^F}{P^I} = \Delta < 1$. That is, the formal sector is more attractive in the sense that jobs on average last a longer time, and the entry of firms into the formal sector will reduce the relative price on formal goods below unity. However, with $\psi < 1$, and $p > 0$, we have $\frac{P^F}{P^I} = \frac{\Delta}{\psi}$ which can be either smaller or larger than unity reflecting the two counteracting incentives determining the relative attractiveness of the two sectors.

Moreover, we have that

$$\frac{\partial \Delta}{\partial \theta} = -\frac{\eta \theta^{\eta-1} (1-\gamma) p}{\rho \left(\frac{(1-\gamma)}{\rho} - (r+s) \left(\theta^F + \theta^I \right)^\eta \right)^2} < 0. \tag{27}$$

That is, the relative price $\frac{P^F}{P^I}$ falls with an increase in total tightness for a given ψ and for given separation rates. We know that $p > 0$ implies that the informal sector is relatively less attractive, since jobs last on average a shorter time in the informal sector. However, when θ is low, it is quite easy to fill a vacancy. The fact that jobs separate easier in the informal sector is hence not as important since, in case of separation, the open vacancy can quickly be filled again. A large θ will, for the same reasons, increase the importance of a long job duration. An increase in total tightness will reduce the attractiveness of the informal sector for given separation rates and ψ, which induces a reallocation of workers towards the formal sector with a reduction in the formal sector relative price, $\frac{P^F}{P^I}$, as a consequence.

By substituting the expression for Δ given by equation (26) into (22), we get

$$(r+s)\theta^\eta = \frac{(1-\gamma)}{\rho} - \gamma \left(\theta^F + \theta^I \left(\frac{\frac{(1-\gamma)}{\rho} - (r+s+p)\theta^\eta}{\frac{(1-\gamma)}{\rho} - (r+s)\theta^\eta} \right)^{-1} \right), \tag{28}$$

where $\theta = \theta^F + \theta^I$, which is one equation in the two unknowns θ^F, and θ^I. We hence have a relationship between sector tightness, and most convenient this relationship is independent of the relative punishment rate, ψ. The relative price will adjust so to make this relationship independent of ψ. It will, however, depend on p. Differentiating (28) with respect to sector tightness and we have

$$\frac{\partial \theta^F}{\partial \theta^I} = -\frac{(r+s)\,\eta\theta^{\eta-1} + \frac{\gamma}{\Delta}\left(1 - \frac{\theta^I}{\Delta}\frac{\partial \Delta}{\partial \theta}\right)}{(r+s)\,\eta\theta^{\eta-1} + \frac{\gamma}{\Delta}\left(\Delta - \frac{\theta^I}{\Delta}\frac{\partial \Delta}{\partial \theta}\right)} < -1 \tag{29}$$

where $\Delta < 1$. From (29), we have that informal tightness crowds out formal tightness, and vice versa. Moreover, a one unit increase in informal tightness will reduce formal tightness by more than one unit. This is a consequence of Δ being smaller than unity. Recall from (26) that $\Delta < 1$ follows because $p > 0$ prevents firms from entering the informal sector to some extent, and hence induces the relative price, $\frac{P^F}{P^I}$, to be lower than elsewise would have been the case. This price premium in the informal sector makes this sector relatively more attractive in the sense that informal firms face lower real producer wages, and workers in the informal sector face higher real consumer wages.[10]

To illustrate the intuition behind this sectorial trade-off, consider the following example: An exogenous reduction in informal sector tightness. This will induce wage moderation in the whole economy as the value attached to unemployment falls; the fall in informal tightness is dampened whereas formal tightness increases. This explains the negative sign in (29). The increase in formal sector tightness will, however, induce a wage push in the economy. The wage push following the increase in formal sector tightness will never dominate the wage moderation following the reduction in informal tightness. The reason is that the employment probability increases in the sector where the pay-off is the lowest (the formal sector) and falls in the

[10] >From (9) and (10) we have $\omega^F > \omega^I$, and $w^F(1-t) > w^I(1-p\delta)$.

sector where the pay-off is the highest (informal sector). That is, as workers, in the event of unemployment, now face an increased probability of finding themselves employed in the lower paying sector, the wage moderating effect will dominate. This implies that tightness in the formal sector increases by more than informal tightness falls.[11]

A shorter way of expressing why the wage response following a change in informal tightness is stronger than the wage response following an equally sized change in formal tightness is to say that the informal sector is given a larger weight in the wage bargains than is the formal sector. As the pay-off in the informal sector is higher, it follows that changes in job opportunities in this sector are also relatively more important for the wage bargains.

3.1 Changes in the punishment fee

This section is concerned with the impact on tightness, relative prices, real producer wages, sector allocation, and unemployment of a fully financed change in the punishment fee given by a change in α and/or δ. Tax rates z and/or t are adjusting so to keep the government budget in (25) balanced at all times. The audit rate p is taken as given throughout this reform. As is clear from (22)-(24), (15)-(17), (9) and (10), the equilibrium variables will only be affected by changes in α, δ, z, and t through the wedge, ψ. Hence, we can conduct comparative statics with respect to ψ without explicitly having to account for the government budget restriction since any government revenue can be reaped by appropriate changes in α, δ, z, and t at any given wedge.

[11] One may however argue that the informal sector is less attractive because the sectoral separation rate is higher in itself. However, the fact that the separation rate is higher in the informal sector has no impact on the wage bargains as it affects the worker and the firm equally. In addition, the direct effect of having for example, $\psi < 1$, has no effect on the wage bargains as this is counteracted by the relative price.

3.1.1 Labour market tightness and real producer wages

The effects on tightness, real producer wages, and the relative price are summarized in the following proposition.

Proposition 1 [12]*A fully financed increase in the punishment fee, δ or α, will increase tightness in the formal sector, θ^F, and reduce tightness in the informal sector, θ^I. Total tightness, θ, increases and real producer wages in the two sectors, ω^F and ω^I, fall. The relative price, $\frac{P^F}{P^I}$, falls.*

All propositions in the paper can be derived from the equilibrium equations. See the appendix for details.

An increase in ψ will make it relatively worse to be in the underground economy. Hence the value of employment in the informal sector falls relative to unemployment which increases informal sector wage demands. In the formal sector, on the other hand, wage demands fall because the value of formal employment has increased relative to the value of unemployment.

From the firms' perspective, these happenings tend to increase the producer costs in the informal sector whereas producer costs in the formal sector tend to fall. Consequently, exit from the informal sector and entry into the formal sector are initiated. That is, labour market tightness in the formal sector, θ^F, increases and labour market tightness in the informal sector, θ^I, falls.[13]

[12]For an intuitive interpretation, the propositions are expressed as if an increase in ϕ^I financed by adjustments in ϕ^F implies that $\psi = \phi^I/\phi^F$ increases. Other, although perhaps less plausible cases, are of course also incorporated. The propositions simply capture fully financed changes in the tax and punishment systems that affect the relative tax and punishment rates between the formal and informal economy.

[13]In fact, there is an additional effect reinforcing the reallocation process towards the formal sector. As a given reduction in θ^I induces θ^F to increase by more, total tightness increases. Thereby vacancy costs increase, which tends to reduce the supply of goods in both sectors. However, as the separation rate is higher in the informal sector, the fraction of total sector production that accounts for vacancy costs increases by more in the informal sector. This tends to further reinforce the reallocation process towards the formal sector.

The relative price is now affected. The reallocation of jobs towards the formal sector increases the production of formal sector goods relative to informal sector goods, which reduces the relative price, $\frac{P^F}{P^I}$. The relative price adjustments will eventually restore the equilibrium since the increase in the relative price eventually makes it profitable to produce informal goods and eliminates the profitability of producing formal goods.

Although the direct effect on real producer wages of an increase in ψ is that informal wages increase and formal wages fall, relative price adjustments fully counteracts these effects in equilibrium. The equilibrium effects on real producer wages is, in fact, entirely explained by changes in the reallocation of firms across sectors. Because $\Delta < 1$, the informal sector is given a larger weight in the wage bargains than is the formal sector. Hence the wage moderation following a reduction in θ^I is going to be larger than the wage push following an equally sized increase in θ^F. If, for the sake of the argument, total tightness where unaffected by this reallocation, also the time unit probability of finding any job would be the same, i.e., $\lambda^F + \lambda^I = \theta^{1-\eta}$. However, with total tightness being unaffected, the expected pay-off from finding any job must have fallen for an unemployed worker as the probability of finding a better paying job has fallen and the probability of finding lesser good paying job has increased. Hence the value of unemployment would in this case be lower, which calls for wage moderation. This wage moderation following the reallocation of jobs towards the formal sector is the driving force behind the reduction in the equilibrium real producer wages. Moreover, this wage moderation explains why total tightness must increase.[14]

[14] When total tightness increases it becomes relatively less attractive to enter the informal sector since the separation rate is higher in the informal sector than in the formal sector, which further reinforces the reallocation of firms towards the formal sector. This further reduces the relative price (that is, further reduces the relative price than was induced in order to fully counteract the direct effects on wage demands of a change in ψ).

3.1.2 Employment

We summarize the results on employment and unemployment in the following proposition:

Proposition 2 *A fully financed increase in the punishment fee, δ or α, will increase the employment rate in the formal sector, n^F, and reduce the employment rate in the informal sector, n^I. Both actual unemployment, u, and official unemployment, u^o, falls with the reform.*

It comes as no surprise that increased punishment fees relative to tax rates induce a reallocation of workers from the informal sector towards the formal sector. An increased wedge increases the transition rate into formal sector employment, whereas the opposite movements occur in the informal sector. Actual unemployment falls both because the overall transition rate into employment, $\lambda^F + \lambda^I = \theta^{1-\eta}$, increases and because the transition rate out of employment is lower in the formal sector. Official unemployment falls as both actual unemployment and informal sector employment fall.

3.2 Changes in the audit rate

This section is concerned with the impact on tightness, the relative price, real producer wages, sector allocation, and unemployment of a fully financed change in the audit rate p. The tax rates z and/or t and the punishment fees α and/or δ are adjusting so as to keep the government budget in (25) balanced at all times. As is clear from (22)-(24), (15)-(17), (9) and (10), the equilibrium variables will be affected by changes in p both through the wedge ψ, and through the informal sector separation rate, $s + p$. However,

This tends to increase real producer wages in the formal sector and reduce the informal sector real producer wages. However, this effect can never dominate the effects induced by the fact that the reallocation brings about stronger wage moderation than wage push; real producer wages fall in both sectors.

from the government budget restriction in (25) we know that there is always an appropriate adjustment in z and/or t and the punishment rates α and/or δ that will produce any level of government revenues for a given ψ. Hence to clarify how changes in p affect the equilibrium variables via the informal sector separation rate, $s+p$, this reform considers changes in p for a given ψ.

From the previous analysis we could conclude how changes in ψ affected the equilibrium variables and it is straight forward to extent the analysis below to incorporate that ψ is increased by an increase in p. The discussion in the introduction and in the conclusion summarizes the full effects of an increase in p.

3.2.1 Labour market tightness and real producer wages

The effect on tightness is summarized in the following proposition.

Proposition 3 *A fully financed increase in the audit rate, p (for a given ψ), will decrease total tightness, $\frac{\partial\theta}{\partial p}<0$.*

We first note that we cannot exclude that informal sector tightness actually increases with an increased audit rate. An increase in p reduces the profitability for firms to enter the informal sector by reducing the average length of a match, which works in the expected direction of reducing informal sector tightness, θ^I tends to fall. In addition, however, the relative price, $\frac{p^F}{p^I}$, is directly reduced by an increase in p since the outflow of informal sector workers increases for given tightness, and hence the production of informal goods fall. This relative price effect will increase the attractiveness for firms to enter the informal sector, tending to increase θ^I.

Since we cannot conclude whether θ^I falls or increases with p, we cannot conclude whether θ^F increases or falls with p. The direct negative effect on the relative price tends to reduce formal sector tightness by making the formal sector less attractive for firms to enter. However, if informal sector tightness falls, formal sector tightness tends to increase since the value of unemployment is reduced and hence wages are moderated in the formal sector. The overall impact on θ^F is ambiguous.

Total labour market tightness falls with an increase in the audit rate. This follows because the direct negative effect on the relative price reduces formal sector tightness. For example, if θ^I increases with the reform, we know from (29) that θ^F falls by more. Hence total tightness falls. The fact that $\frac{P^F}{P^I}$ falls as a direct effect of a higher p, will further reduce θ^F and total tightness. If, on the other hand, θ^I falls with the reform, formal sector tightness increases by more, and total tightness tends to increase. However, the fact that the relative price falls as a direct effect of an increase in p will reduce formal sector tightness, and hence the fall in θ^I is larger than the increase in θ^F; total tightness falls.[15]

In general, it cannot be determined whether the relative price increases or decreases with an increase in p. Considering for example equation (24), we can see that the direct effect of an increase in p makes the relative price fall. There is, however, a counteracting effect working through total tightness. As total tightness falls with an increase in p, the relative supply of informal goods falls due to that vacancy costs increases by more in the informal sector. In addition to that, relative tightness can move in either direction. However, if informal sector tightness falls, the relative price will fall. Hence, the effect working through vacancy costs can not dominate the effects working through p directly and through relative tightness.

[15] Due to that the relative price, $\frac{P^F}{P^I}$, falls as a direct effect of a higher p may even make θ^F fall. But again total tightness falls.

The effects on real producer wages is given by the following proposition:

Proposition 4 *A fully financed increase in the audit rate, p (for a given ψ), will increase the real producer wage in the formal sector, $\frac{\partial \omega^F}{\partial p} > 0$. The impact on the informal sector real producer wage is ambiguous.*

Firms enter into the two sectors until the expected vacancy costs are equal to the discounted profits. This implies that the expected time it takes to fill a vacancy is equal to the discounted profits relative to the per period vacancy cost. For reasons given above, we know that total tightness falls with an increase in p although we cannot determine how sector tightness and the relative prices are affected in general. If total tightness falls with the reform, and hence a vacancy is expected to be filled at a faster rate, discounted profits in the two sectors relative to per period vacancy costs has to fall as well. The reallocation of firms across the two sectors will assure that. In the formal sector this can only be achieved by an increase in the real producer wage. In the informal sector, however, an increase in p will reduce the expected profits since a match is expected to last a shorter number of periods. It is hence not necessarily the case that the real producer wage in the informal sector increases as a consequence of a higher p.

3.2.2 Employment

As we cannot exclude that relative tightness, $\frac{\theta^F}{\theta^I}$, actually falls following an increase in p, we cannot exclude that relative employment, $\frac{n^F}{n^I}$, falls with the reform. However, if relative tightness decreases with the reform, relative employment will fall as well. This follows both because the relative transition rate into formal employment falls, and because outflow of workers from the informal sector increases.

The impact on unemployment is, however, ambiguous. It is not possible to exclude the case that the unemployment rate falls with an increase in the audit rate. This is so although we know that total tightness falls, and hence the total transition rate into employment falls, and that the exit rate from the informal sector increases.[16] The reason is the reallocation effect. Consider, for example, that this reforms brings about increased inflow into formal sector employment. This may reduce the unemployment rate although the transition rate into the informal sector falls by more than the transition rate into formal sector increases. This follows because the formal sector separation rate is lower than the informal sector separation rate and hence for a given increase in the sector transition rate into employment, formal sector employment has to increase by more than informal sector employment in order to balance inflows with outflow in steady state. That is, employment in the formal sector is more sensitive to changes in its transition rate than are informal sector employment.

4 Conclusion

This paper developed a two-sector general equilibrium matching model with different goods produced in the formal sector and the informal sector. This enabled an analysis of how increased government control of the underground economy affects wage formation, sector allocation, and unemployment. This is something that to a large extent has been ignored in the previous literature where wages have been taken as either given or determined by market clearing.

Based on this framework, we have shown that increased government control of the underground economy in terms of higher punishment fees reduces the size of the underground economy, reduces real producer wages in the two sectors, and reduces actual and official unemployment.

[16] Recall that the total transition rate into employment is $\lambda^F + \lambda^I = \theta^{1-\eta}$.

The intuition behind these results is as follows. Increased punishment fees induce wage demands to increase in the informal sector and fall in the formal sector. As a consequence, firms find it profitable to exit the informal sector and enter the formal sector. In turn, this reallocation of production will reduce the formal sector relative price. The relative price adjustments will fully counteract the direct effects on real producer wages in equilibrium. Both real producer wages and unemployment instead fall due to the rather strong wage moderation that follows this reallocation process.

This strong wage moderation was essential and is here summarized. The reallocation of firms from the informal sector towards the formal sector will induce both increased and reduced wage demands. When firms enter the formal sector, wage demands increase since employment perspectives in the formal sector increase. Analogously, wage demands fall when firms exit the informal sector as employment perspectives in this sector fall. The wage moderation following a firm exiting the informal sector is, however, larger than the wage push following a firm entering the formal sector. The reason why wages are more responsive to changes in informal tightness is that the pay-off in the informal sector exceeds that of the formal sector. Hence, changes in job opportunities in the informal sector are relatively more important for the wage bargains as the pay-off in this sector is higher. This rather strong wage moderation explains why real producer wages and unemployment fall.

Considering the full effects on labour market performance of an increase in the audit rate produced less clear results. In addition to the effects previously described, there was a direct positive effect on the probability of a worker-firm match being separated. The outflow from informal sector employment into the unemployment pool hence increased. Furthermore, we found that the overall transition rate into employment fell caused by an increase in the informal sector separation rate. These effects tended to increase unemployment. Consequently, the overall impact of increased auditing on unemployment is ambiguous.

5 Appendix: Proof of propositions

5.1 Proof of Proposition 1

Differentiate equation (24) with respect to θ^I and ψ, taking into account that Δ is a function of θ^F and θ^I as given by (26) and that θ^F is a function of θ^I through equation (28). This yields

$$\frac{d\theta^I}{d\psi} = \frac{\theta^F}{\theta^I}\frac{\Delta}{\psi}\left(\Delta\frac{\partial\frac{\theta^F}{\theta^I}}{\partial\theta^I} + \left(\frac{\theta^F}{\theta^I}\frac{\partial\Delta}{\partial\theta} + \psi\frac{\sigma}{1-\sigma}\frac{\frac{1}{s}-\frac{1}{s+p}}{\left(\frac{1}{s}-\rho\theta^\eta\right)^2}\rho\eta\theta^{\eta-1}\right)\left(\frac{\partial\theta^F}{\partial\theta^I}+1\right)\right)^{-1}$$

where $\frac{\partial\left(\theta^F/\theta^I\right)}{\partial\theta^I} = \left(\frac{\partial\theta^F}{\partial\theta^I}\theta^I - \theta^F\right) / \left(\theta^I\right)^2 < 0$, and $\frac{\partial\theta^F}{\partial\theta^I}+1 < 0$ from (29). A sufficient condition for a negative sign is then:

$$\Delta\frac{\partial\frac{\theta^F}{\theta^I}}{\partial\theta^I} + \frac{\theta^F}{\theta^I}\frac{\partial\Delta}{\partial\theta}\left(\frac{\partial\theta^F}{\partial\theta^I}+1\right) < 0.$$

Substituting for $\left(\frac{\partial\theta^F}{\partial\theta^I}+1\right)$ and $\frac{\partial\frac{\theta^F}{\theta^I}}{\partial\theta^I}$ and simplifying we obtain:

$$-\Delta\frac{\theta^F}{\theta^I}\left(\left(r+s\right)\eta\theta^{\eta-1}+\gamma\right) - \left(r+s\right)\eta\theta^{\eta-1} - \frac{\gamma}{\Delta} + \left(\frac{\gamma}{\Delta}\theta^I + \theta^F\gamma\right)\frac{\partial\Delta}{\partial\theta} < 0.$$

Thus, $\frac{\partial\theta^I}{\partial\psi} < 0$. Thereby $\frac{\partial\theta^F}{\partial\theta^I}\frac{\partial\theta^I}{\partial\psi} > 0$ and $\frac{\partial\theta}{\partial\psi} = \left(\frac{\partial\theta^F}{\partial\theta^I}+1\right)\frac{\partial\theta^I}{\partial\psi} > 0$. Regarding the relative price we have $\frac{P^F}{P^I} = \frac{\Delta}{\psi}$ where $\frac{\partial\left(\frac{P^F}{P^I}\right)}{\partial\psi} = \left(\frac{\partial\Delta}{\partial\psi}\psi - \Delta\right)/\psi^2 < 0$ since $\frac{\partial\Delta}{\partial\psi} = \frac{\partial\Delta}{\partial\theta}\frac{\partial\theta}{\partial\psi} < 0$.

Differentiating equations (9) and (10) with respect to ψ yields:

$$\frac{\partial\omega^F}{\partial\psi} = \gamma\rho y\left(\frac{\partial\theta^F}{\partial\psi} + \frac{\partial\theta^I}{\partial\psi}\frac{1}{\Delta} - \frac{\theta^I}{\Delta^2}\frac{\partial\Delta}{\partial\theta}\frac{\partial\theta}{\partial\psi}\right), \tag{30}$$

$$\frac{\partial\omega^I}{\partial\psi} = \gamma\rho y\left(\frac{\partial\theta^F}{\partial\psi}\Delta + \theta^F\frac{\partial\Delta}{\partial\theta}\frac{\partial\theta}{\partial\psi} + \frac{\partial\theta^I}{\partial\psi}\right). \tag{31}$$

From (13) and (14), we have:

$$\theta^{\eta} = \frac{(1-\gamma)}{(r+s)\rho} - \gamma\frac{\left(\theta^F + \frac{\theta^I}{\Delta}\right)}{(r+s)}, \tag{32}$$

$$\theta^{\eta} = \frac{(1-\gamma)}{(r+s+p)\rho} - \gamma\frac{\Delta\left(\theta^F + \frac{\theta^I}{\Delta}\right)}{(r+s+p)}. \tag{33}$$

Differentiation brings out the following expressions

$$\eta\theta^{\eta-1}\frac{\partial\theta}{\partial\psi} = -\frac{\gamma}{(r+s)}\left(\frac{\partial\theta^F}{\partial\psi} + \frac{\partial\theta^I}{\partial\psi}\frac{1}{\Delta} - \frac{\theta^I}{\Delta^2}\frac{\partial\Delta}{\partial\theta}\frac{\partial\theta}{\partial\psi}\right), \tag{34}$$

$$\eta\theta^{\eta-1}\frac{\partial\theta}{\partial\psi} = -\frac{\gamma}{(r+s+p)}\left(\frac{\partial\theta^F}{\partial\psi}\Delta + \theta^F\frac{\partial\Delta}{\partial\theta}\frac{\partial\theta}{\partial\psi} + \frac{\partial\theta^I}{\partial\psi}\right), \tag{35}$$

where we know from the proof of proposition 1 that $\frac{\partial\theta}{\partial\psi} > 0$. This implies that $\frac{\partial\theta^F}{\partial\psi} + \frac{\partial\theta^I}{\partial\psi}\frac{1}{\Delta} - \frac{\theta^I}{\Delta^2}\frac{\partial\Delta}{\partial\theta}\frac{\partial\theta}{\partial\psi} < 0$ and $\frac{\partial\theta^F}{\partial\psi}\Delta + \theta^F\frac{\partial\Delta}{\partial\theta}\frac{\partial\theta}{\partial\psi} + \frac{\partial\theta^I}{\partial\psi} < 0$ have to hold. Hence we have that the equilibrium real producer wages have to fall in both sectors.

5.2 Proof of Proposition 2.

Differentiate equation (15) and (16) with respect to ψ gives:

$$\frac{\partial n^I}{\partial\psi} = \frac{\frac{\partial\left(\theta^I(\theta)^{-\eta}\right)}{\partial\psi}\left(1+\frac{\theta^F(\theta)^{-\eta}}{s}\right) - \frac{\theta^I(\theta)^{-\eta}}{s}\frac{\partial\left(\theta^F(\theta)^{-\eta}\right)}{\partial\psi}}{(s+p)\left(1+\frac{\theta^F(\theta)^{-\eta}}{s}+\frac{\theta^I(\theta)^{-\eta}}{s+p}\right)^2} < 0$$

$$\frac{\partial n^F}{\partial\psi} = \frac{\frac{\partial\left(\theta^F(\theta)^{-\eta}\right)}{\partial\psi}\left(1+\frac{\theta^I(\theta)^{-\eta}}{s+p}\right) - \frac{\theta^F(\theta)^{-\eta}}{s}\frac{\partial\left(\theta^I(\theta)^{-\eta}\right)}{\partial\psi}}{s\left(1+\frac{\theta^F(\theta)^{-\eta}}{s}+\frac{\theta^I(\theta)^{-\eta}}{s+p}\right)^2} > 0,$$

where we use that

$$\frac{\partial\left(\theta^I(\theta)^{-\eta}\right)}{\partial\psi} = \theta^{-\eta}\left(1-\eta\frac{\theta^I}{\theta}\left(\frac{\partial\theta^F}{\partial\theta^I}+1\right)\right)\frac{\partial\theta^I}{\partial\psi} < 0,$$

$$\frac{\partial\left(\theta^F(\theta)^{-\eta}\right)}{\partial\psi} = \theta^{-\eta}\left(\frac{\partial\theta^F}{\partial\theta^I}\left(1-\eta\frac{\theta^F}{\theta}\right) - \eta\frac{\theta^F}{\theta}\right)\frac{\partial\theta^I}{\partial\psi} > 0.$$

Furthermore, the unemployment rate is affected in the following way:

$$\frac{\partial u}{\partial \psi}=-\frac{\frac{s+p}{s}\frac{\partial\left(\theta^{F}(\theta)^{-\eta}\right)}{\partial\psi}+\frac{\partial\left(\theta^{I}(\theta)^{-\eta}\right)}{\partial\psi}}{\left(1+\frac{\theta^{F}(\theta)^{-\eta}}{s}+\frac{\theta^{I}(\theta)^{-\eta}}{s+p}\right)^{2}}\frac{1}{s+p}.$$

As $\frac{s+p}{s}>1$ and $\frac{\partial\theta^{I}}{\partial\psi}<0$ a sufficient condition for $\frac{\partial u}{\partial\psi}<0$ is that

$$\begin{aligned}\frac{\partial\theta^{F}}{\partial\theta^{I}}\left(1-\eta\frac{\theta^{F}}{\theta}\right)-\eta\frac{\theta^{F}}{\theta}+1-\eta\left(1-\frac{\theta^{F}}{\theta}\right)\left(\frac{\partial\theta^{F}}{\partial\theta^{I}}+1\right) &< 0\Leftrightarrow\\ (1-\eta)\left(\frac{\partial\theta^{F}}{\partial\theta^{I}}+1\right) &< 0,\end{aligned}$$

which is satisfied. Considering u^{o}, we have $\frac{\partial u^{o}}{\partial\psi}=\frac{\partial u}{\partial\psi}+\frac{\partial n^{I}}{\partial\psi}<0$ from this proof.

5.3 Proof of Proposition 3.

Differentiating the equilibrium system, equations (22) - (24) give:

$$\begin{aligned}\frac{d\theta^{F}}{dp} &= \frac{-\frac{\Phi\Delta+\gamma\frac{1}{\pi}H\theta^{I}}{\Delta}\theta^{\eta}+\gamma\psi\left(\theta^{F}\left(\Phi+\frac{\gamma}{\Delta}\right)+\frac{\Sigma+\gamma}{\Delta}\frac{\theta^{I}}{\Delta}\right)\frac{\sigma}{1-\sigma}\frac{\theta^{I}}{\theta^{F}}\frac{\frac{1}{(s+p)^{2}}}{\frac{1}{s}-\rho\theta^{\eta}}}{D},\\ \frac{d\theta^{I}}{dp} &= \frac{\frac{\Delta(\Phi+\gamma)+\gamma\frac{\theta^{I}}{\theta^{F}}+\gamma\frac{1}{\pi}H\theta^{I}}{\Delta}\theta^{\eta}-\gamma\psi\left(\theta^{F}\left(\Phi+\gamma\right)+\left(\frac{\Sigma}{\Delta}+\gamma\right)\frac{\theta^{I}}{\Delta}\right)\frac{\sigma}{1-\sigma}\frac{\theta^{I}}{\theta^{F}}\frac{\frac{1}{(s+p)^{2}}}{\frac{1}{s}-\rho\theta^{\eta}}}{D},\\ \frac{d\pi}{dp} &= \frac{\left(\frac{\Phi\pi+\gamma\pi}{\theta^{I}}+\frac{1}{\theta^{F}}\Phi\pi+\frac{1}{\theta^{F}}\gamma\frac{1}{\psi}+\frac{\gamma H}{\Delta}\left(\frac{1}{\Delta}-1\right)\right)\theta^{\eta}-\gamma\left(1-\Delta\right)\frac{\Sigma-\Phi\Delta}{\Delta}\frac{\sigma}{1-\sigma}\frac{\theta^{I}}{\theta^{F}}\frac{\frac{1}{(s+p)^{2}}}{\frac{1}{s}-\rho\theta^{\eta}}}{D},\end{aligned}$$

where

$$D=-\gamma\left(\left(\Phi+\gamma\right)\left(1+\frac{\theta^{F}}{\theta^{I}}\Delta\right)+\left(1+\frac{\theta^{F}}{\theta^{I}}\frac{1}{\Delta}\right)\left(\Sigma+\gamma\right)+\frac{\gamma}{\Delta}\psi H\theta\left(\frac{1}{\Delta}-1\right)\right)<0,$$

and

$$\Phi=\left(r+s\right)\eta\theta^{\eta-1},\Sigma=\left(r+s+p\right)\eta\theta^{\eta-1},H=\frac{\sigma}{1-\sigma}\frac{\theta^{I}}{\theta^{F}}\frac{\frac{1}{s}-\frac{1}{s+p}}{\left(\frac{1}{s}-\rho\theta^{\eta}\right)^{2}}\rho\eta\theta^{\eta-1}.$$

Adding the derivatives for labour market tightness we obtain:

$$\frac{d\theta}{dp}=\frac{1}{\Delta}\frac{\left(\Delta\gamma+\gamma\frac{\theta^{I}}{\theta^{F}}\right)\theta^{\eta}+\gamma\psi\left(\theta^{F}\gamma\left(1-\Delta\right)+\theta^{I}\gamma\left(\frac{1}{\Delta}-1\right)\right)\frac{\sigma}{1-\sigma}\frac{\theta^{I}}{\theta^{F}}\frac{\frac{1}{(s+p)^{2}}}{\frac{1}{s}-\rho\theta^{\eta}}}{D}<0.$$

5.4 Proof of Proposition 4.

Differentiating equations (9) and (10) with respect to p yields:

$$\begin{aligned}\frac{\partial\omega^F}{\partial p} &= \gamma\rho y\left(\frac{\partial\theta^F}{\partial p}+\frac{\partial\theta^I}{\partial p}\frac{1}{\Delta}-\frac{\theta^I}{\Delta^2}\frac{\partial\Delta}{\partial p}\right), \\ \frac{\partial\omega^I}{\partial p} &= \gamma\rho y\left(\frac{\partial\theta^F}{\partial p}\Delta+\frac{\partial\theta^I}{\partial p}+\theta^F\frac{\partial\Delta}{\partial p}\right).\end{aligned} \tag{36}$$

From (13) and (14), we have:

$$\begin{aligned}\theta^\eta &= \frac{(1-\gamma)}{(r+s)\,\rho}-\gamma\frac{\theta^F+\frac{\theta^I}{\Delta}}{(r+s)}, \\ \theta^\eta &= \frac{1}{(r+s+p)}\left(\frac{(1-\gamma)}{\rho}-\gamma\Delta\left(\theta^F+\frac{\theta^I}{\Delta}\right)\right).\end{aligned}$$

Differentiation brings out the following expression

$$\begin{aligned}\eta\theta^{\eta-1}\frac{\partial\theta}{\partial p} &= -\frac{\gamma}{(r+s)}\left(\frac{\partial\theta^F}{\partial p}+\frac{\partial\theta^I}{\partial p}\frac{1}{\Delta}-\frac{\theta^I}{\Delta^2}\frac{\partial\Delta}{\partial p}\right), \\ \eta\theta^{\eta-1}\frac{\partial\theta}{\partial p} &= -\frac{\gamma\Delta}{(r+s+p)}\left(\frac{\partial\theta^F}{\partial p}+\frac{\partial\theta^I}{\partial p}\frac{1}{\Delta}+\frac{\theta^F}{\Delta}\frac{\partial\Delta}{\partial p}\right)-\frac{\theta^\eta}{r+s+p},\end{aligned}$$

where we know from the proof of proposition (3) that $\frac{\partial\theta}{\partial p}<0$. This implies that $\frac{\partial\theta^F}{\partial p}+\frac{\partial\theta^I}{\partial p}\frac{1}{\Delta}-\frac{\theta^I}{\Delta^2}\frac{\partial\Delta}{\partial p}>0$.

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