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ADAPTIVE CONTRACTING: The Trial-and-Error Approach to Outsourcing

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Abstract: Adaptive contracting is defined as a strategy in which a principal experiments with the delegation of authority through leaving contracts incomplete. We highlight two potential benefits of an adaptive approach: First, the delegation of authority can be advantageous even if the agent acts opportunistically, since expected private benefits will be shared between the parties through price negotiation. Second, the principal extracts information from experimenting with delegation of authority and we identify a positive option value embodied in the principal's ability to extent or withdraw the delegated authority in future contracting periods.

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1 Introduction

Writing procurement contracts between a government institution and a private supplier is in general a complicated affair. The costs and benefits for the parties involved - including the public - have many components and a government institution normally has limited organizational resources and many political and economical objectives to satisfy.[1]

How detailed should such contracts be? How much time and effort should a public agency spend on analyzing all the contractual details before the contract is signed? What are the effects on the service provider and the service provided by not describing all possible details in the contract? When contracts are left incomplete, the agent will have the authority to decide on matters not specified in the contract. In this paper we provide a formal analysis of *adaptive contracting* defined as a repeated contracting approach, where the government experiments with the degree of contractual incompleteness. Such *trial-and-error* approach delegates authority to the contractor, who decides on all choices not specified in the contract.

Aghion and Tirole (1997) analyzes how the incentives to collect information as well as the congruence of preferences of the procurer and the contractor determines the optimal allocation of the decision authority in a setting where contracting is done once. They make the important distinction between real and formal authority, both of which can be allocated freely among the contracting parties.

We depart from their analysis by studying repeated contracting in a simple model of procurement where a government can experiment with how elaborate a contract shall be. We show that adaptive contracting can be optimal, because it entails an option value for the government. The contractor's response to the increased authority may increase or decrease the total value of the relationship in a way which is impossible to learn exactly without trying out this delegation in reality. However, in a repeated contracting

[1] James Q. Wilson (1989) provides a classical study of the workings of local and national government institutions.

setting, the government may use its superior contracting authority[2] to remove decision authority from the contractor again when contracts are renegotiated. Thus, the option value embodied in an adaptive contracting strategy arises from the government's ability to revise the contract later on: If the contractor uses his increased authority in a manner which increases the total value of the relationship, then he will keep the authority in future periods, thus in this case there will be a multi period benefit of transferring authority. On the other hand, if the contractor acts in a way which is decremental to the total value of the relationship, then the government can remove authority again, incurring only a limited - one period - cost.

The cost of delegating authority to the contractor arises from that this delegation increases his incentives to engage in opportunistic behavior. We show that this cost in general is moderated if the government can foresee it ex-ante, since expected private benefits to the contractor then will be reflected in the negotiated price.

Government agencies around the world frequently apply an adaptive strategy when making outsourcing contracts in practice (see e.g. Wilson (1995) and World Bank (1995)). An illustrative case is the outsourcing of local bus transportation in the Copenhagen region in Denmark.[3] The costs and benefits for the parties involved in a contract covering outsourcing of local bus operation are multidimensional including at least three central areas: First, there is the organization of the service provided, i.e. the routes to be serviced, the frequencies of buses, the number of stops, etc. etc. Secondly, there are a number of

[2]As mentioned in the text, Aghion and Tirole (1997) makes the important distinction between formal and real authority. In our model, formal authority is the right to decide which contract is offered to the agent. Formal authority cannot be delegated, it always belongs to the government. Real authority is the right to choose which action are implemented in situations where the contract is silent, we sometimes denote this decision authority.

[3]Private bus operation on local routes has a long tradition in Denmark where public and private operated bus routes have coexisted for more than fifty years. The first significant outsourcing of previous public operated local bus operation were implemented in the Copenhagen area in 1990 partly triggered by new EC rules requiring access for private operators in the market for local and national transportation. Today more than ten years after this first round of outsourcing, almost 80 pct. of the bus transportation is outsourced to private operators. The effect of this outsourcing has been increased competition, cost reductions and a change in market structure towards fewer and larger operators (see Konkurrencestyrelsen (1999) and Færdselsstyrelsen (2002) for an evaluation of the outsourcing of local bus transportation in Denmark and other countries).

cost related factors, e.g. the age and standard of the buses used, the quality and service provided to the passengers in a given bus, the environmental standard of the buses and the service. Finally, an important issue is the length of contracting and the conditions for terminating a contract, e.g. to which degree should long lasting investment costs be reimbursed if a contract is ended.

In 1990 the first contracting round were based on small and very incomplete contracts for a limited number of bus operations and a limited number of years. The authorities and the bus operators learned from these trial contracts and the experience obtained from giving the bus operators large authority has been the foundation for later contract revisions.[4] Essentially, the local authorities have used an adaptive contracting strategy in dealing with outsourcing of local bus transportation.

The trial and error approach to contracting may seem obvious in a setting with limited resources to write contracts. The public organization may lack manpower and be short of expert knowledge in contracting. In such a setting it is tempting to use the trial-and-error approach to save transaction cost on contracting relative to more comprehensive contracting approach (e.g. Williamson (1979) and (1985) and Wilson (1989)). However, our argument is different, we show that if certain production modes are non-verifiable ex-post, then the trial-and-error approach may be beneficial even in the absence of direct transaction costs, since this approach generates information about the agents' response to increased authority.

The existing theoretical contract literature has not focused on the option value of adaptive contracting. Aghion and Tirole (1997) discusses the possibility that a principal makes what they call "contingent delegation", i.e. makes clear that he may retain authority at a future date. They find that contingent delegation is intermediate in between delegation and no delegation in terms of initiative and loss of control for the principal. They do not consider the option value, which we focus upon.

[4] A visible indicator of this trial and error approach is the number of pages in the contracting material that the bus operators make their offer on. In the first round in 1990, the contract were appr. 30 pages long, where as the number of pages exceeded 300 in the 12. and latest round in 2002 (see HUR 1990-2002).

Part of the contracting literature has focused on optimal contracting in view of informational asymmetries, see e.g. Laffont and Tirole 1991 and 1993 for contributions related to outsourcing of public goods or services. This approach assumes that all relevant observable details are built into the contract.

Alternatively, the *incomplete* contracting literature assumes that contracts are necessarily incomplete (see Hart 1995). This approach takes as a premise that certain actions are observable but non-verifiable and therefore cannot be built into a contract. Hence, the gabs in existing contracts reflect issues which are essentially non-contractible. Hart, Shleifer and Vishny (1997) shows that non-verifiability in contracting can have important consequences for outsourcing of public services.[5] Our paper relates to this literature, by assuming that certain modes of producing the service cannot be sufficiently described ex-ante and, therefore, cannot be verified ex-post. We extend this literature by showing that given the external restrictions on contracting, it may be optimal to deliberately leave gaps in contracts to explore the outcome of delegating authority.

One strand of the contracting literature has explicitly focused on the impact of re negotiating multi period contracts. Dewatripont (1989) shows that renegotiation can be costly if it blocks the contracting parties ability to commit to contracts which are ex-ante efficient but ex-post inefficient in settings with asymmetric information. Another cost identified with renegotiation is the *ratchet effect* which is present when information is revealed gradually over time (see e.g. Laffont and Tirole (1988) and Hart and Tirole (1988)). Our contribution in relation to this literature is to identify the option value embodied in using short term contracts that allows for renegotiation.

This paper relates to a large organizational literature analyzing adaptive behavior in organizational design (see for instance the case study by Van de Ven and Polley (1992) and the special issue of *Organizational Design* Vol.2, No.1, (1990)). In economics, the focus has been on adaptive behavior in economic decision making. One approach to

[5] In Bennedsen and Schultz (2002) we extend Hart, Shleifer and Vishny (1997)'s incomplete contracting approach to outsourcing of public service and analyze the interaction between a government's incentive to outsource and the degree of competition in the private market for the good or service.

this has been to view optimal rational behavior as the outcome of a converging series of trial and errors as pointed out by e.g. Smith (1982) and Lucas (1986).[6] In addition, our analysis relates to two other literatures: First it contributes to the literature on outsourcing and privatization. The welfare effects of outsourcing is well documented both theoretically (see Laffont and Tirole (1991), Shapiro and Willig (1993), Shleifer and Vishny (1994), Schmidt (1996) and Bennedsen (1999) for different theoretical models of the welfare consequences of privatization and Shleifer (1998) for a general survey) and empirically (see Vickers and Yarrow (1988), La Porta *et.al* (1997) and World Bank (1995) and (1997) for empirical studies of privatization and outsourcing and - again - Konkurrencestyrelsen (1999) and Færdselsstyrelsen (2002) for some Danish experience in outsourcing in general and outsourcing of local bus transportation in particular). Second, since an important aspect of our analysis is the optimal allocation of authority, our paper also contributes to a large organizational literature on this issue builded upon the seminal contribution in sociology by Max Weber.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 analyzes the model under the assumption that there are no inter-period contract renegotiation. Section 4 analyzes the optimal use of adaptive contracting when contract can be renegotiated. Finally, Section 5 concludes.

2 The Model:

We analyze a simple two period model of adaptive contracting in outsourcing of public services such as cleaning, renovation, local bus and train operation etc. There are two players, a service buying government, G, and a private service provider, S. The price for the service is p. We assume that G decides on contract length and contract type, but that p is determined through bargaining, such that the parties split the generated surplus

[6]Lucas states that "Technically, I think of economics as studying decision rules that are steady states of some adaptive process, decision rules that are found to work over a range of situations and hence are no longer revised appreciably as more experience accumulates," (Lucas 1986, p.402).

within each period.

We assume that the service provider can organize the service production in a number of ways. Two of these are standard modes of organizing the production of this particular service and these can be described ex-ante and verified ex-post in a court. These two standard production modes give raise to respectively a low and a high quality service. Hence, G, can write a complete contract specifying one of these two standard modes of organizing the service and such a contract would be verifiable to a court. Notice, we assume that production modes can be verifiable to a court but not the quality level as such. However if a verifiable production mode has been used and it is known from prior experience which quality level is associated with this particular production mode, it is as if the court can verify the quality level directly. We will, thus, occasionally write the verifiable high and low quality contract as a shortcut for a contract specifying the verifiable standard production modes, generating a high or low quality service. When either of these two standard contracts are written, we will say that the government keeps authority, since it then decides the precise production method.

The agents' payoffs in such a complete contract is given by:

$$U^g = \begin{cases} B - p & \text{if a low quality standard production organization is chosen,} \\ B + \Delta B - p & \text{if a high quality standard production organization is chosen,} \end{cases}$$
$$U^s = \begin{cases} p - C & \text{if a low quality standard production organization is chosen,} \\ p - C - \Delta C & \text{if a high quality standard production organization is chosen.} \end{cases}$$

The basic - standard - quality level yields a benefit, $B \geq 0$, to the government and the high - standard - quality level yields $B+\Delta B$, where $\Delta B > 0$. The service provider incurs a cost of $C \geq 0$ if he provides the basic quality level, and incurs an additional cost, $\Delta C > 0$, by providing the extra quality level.

The government can also refrain from specifying the production mode precisely. In this case *authority* - in the form of decision rights with respect to the production mode -

is transfered from G to S. S can choose one of the two standard production organization ways, or he can choose from a - possible infinite - number of unverifiable ways to organize the production. To be specific, we assume the existence of $n \geq 2$ other unverifiable modes of producing the service.

The first of these we denote the (expected) best alternative production mode and it has ex-ante unknown pay-off consequences for both G and S. This (expected) best alternative (which we henceforth denote the alternative production mode) can be to test the newest undocumented technology, some new materials or new ways of organizing the labor input. Since, the alternative technology is unproven and learned through implementation, it is impossible to describe this alternative approach ex-ante or to verify it precisely ex-post. We assume that the pay-off consequences can only be learned through implementation in practice. In short, the alternative non-verifiable production mode gives rise to a third non-contractible, quality level. The associated payoffs are:

$$\begin{aligned} U^g &= B - p + R_g \\ U^s &= p - C + R_s \\ \text{where } R_g &= \alpha\mu + r_g \text{ with } r_g \sim UD[-\bar{r}_g, \bar{r}_g] \\ \text{and } R_s &= \mu + r_s \text{ with } r_s \sim UD[-\bar{r}_s, \bar{r}_s]. \end{aligned}$$

If the alternative production mode is chosen, the service provider receives a private net benefit of $R_s = \mu + r_s$ where r_s is uniformly distributed on a non-degenerate interval $[-\bar{r}_s, \bar{r}_s]$. The term $\mu \geq 0$ measures the expected benefit to the service provider of this alternative service quality. To make the analysis below interesting, we assume that there is sufficient uncertainty, so that S's preferred quality level depends on the realization of the private benefit, i.e. we assume that $\bar{r}_s > \mu$.

The alternative approach gives G an additional utility - in excess of the standard benefit B - equal to $R_g = \alpha\mu + r_g$ where $r_g \sim UD[-\bar{r}_g, \bar{r}_g]$ and $\alpha \in [-1, 1]$. The parameter α measures the alignment of G's and S's *expected* utility with respect to the alternative service organization. The model is thus general enough to capture the cases of independent preferences (α is zero), positively aligned preferences in expectation (α is positive) or negatively aligned preferences in expectation (α is negative).[7] We assume that R_g and R_s are observable but non-verifiable ex-post and that if the parties negotiate in a future period, they cannot write a complete contract specifying that the alternative production mode should be used, even in the case where it has been used in the first period.[8]

In addition to the alternative production mode, S has $n-1$ other non-verifiable ways to provide the service. The expected surplus from these other production modes are smaller than the expected surplus of the (best) alternative. In a more realistic setting, these other production modes would be variations of the alternative mode, implying that pay-off would be random and only learned through experimentation. However, to simplify the analysis, we assume that these have known pay-off consequences and the minimum cost of delivering the service using any of these other unverifiable production modes is exactly C. Furthermore, we assume that the mode that delivers the service at cost C also yields a benefit B for G. These other production modes play no part in the analysis below, however, they secure that G cannot de facto make the alternative contract verifiable to a court by specifying in a contract that neither of the two verifiable standard modes must be used. Hence, if authority is allocated to S through opting for an incomplete contract, S will de facto choose between the alternative production mode or a production mode - verifiable or unverifiable - that generates a pay-off identically to the low quality service.[9]

[7] The combined restriction that $|\alpha| \leq 1$ and $\mu \geq 0$ implies that the alternative organization method in a one period version of the model always provides in expectation at least as much surplus as the basic quality service. This assumption reduces the number of cases below; however, it does not change the fundamental results of the paper and it is straightforward - but notationally cumbersome - to analyze cases where either $\mu < 0$ and/or $|\alpha| > 1$.

[8] Alternatively, we could have assumed that it is possible to contract on the alternative quality level if it has been used in a previous period. As we briefly discuss in Section 4 below, this would increase the incentives to use adaptive contracting and thus strengthen the key insight from our analysis.

[9] None of the general insight of the paper is driven by this assumption, however, to conduct the analysis wee need to know which production modes the agent prefer. Changing the assumption that the best other production mode yields B to the government at a cost C for the service provider would not change the results, however it would affect some formulas below.

The timing of the model is as follows: First G decides on the length of the contract (one or two periods) and on whether to choose a complete contract specifying the (standard) quality level or to delegate authority through leaving the contract incomplete. If the contract is complete, the service provider then delivers the service using the desired production mode. If the contract is incomplete, the service provider then decides on which production mode to apply. If he chooses the alternative production method, Nature reveals the payoffs associated with this production method for both parties and S delivers the service.

To simplify matters further we make the following assumption:

Assumption 1.

$$\Delta H \equiv \Delta B - \Delta C \geq (1 + \alpha)\mu.$$

Assumption 1 states that the extra social welfare (over the welfare of low quality contract) generated from a high quality contract is higher than the expected extra social welfare from choosing the alternative production mode. Since $(1 + \alpha)\,\mu \geq 0$, Assumption 1 also implies that the high quality contract with verified contract conditions is better than the low quality contract. The assumption thus strengthen our focus on delegation of authority through adaptive contracting, since in the absence of any dynamic authority issues, G would always write a complete contract specifying a verifiable production mode that generates the high quality service.

Finally, we assume that if contracts are complete, the service provider has no other incentives or possibilities to investigate the alternative production mode, and therefore will neither of the parties learn the true values of r_s and r_g. The utility consequences of the alternatives we have in mind can thus not be figured out in a laboratory or in the development department of the service provider, i.e. they can only be learned through implementation. Since the complete contract provides a detailed specification of how the service is produced, experimenting is impossible.[10]

[10] As an example consider the demand consequences of changing the route or schedule in a bus-line. It cannot be precisely estimated without actually implementing the changes.

Our focus in the next section is on contracting in a repeated service provision setting. Hence, we assume that each period is as described above. No new relevant information is revealed to the parties in the period before the service provider delivers the service. Hence, they have no incentive to renegotiate the contract within the period. Of course they may have an incentive to negotiate a new contract after the passing of period 1 where they possibly have learned the utility consequences of the alternative production mode.[11]

3 Delegating authority in the absence of inter-period contract revision.

It is useful to begin the analysis by assuming that contracts cannot be revised between the two periods. This assumption implies that G chooses ex-ante between proposing a complete contract for two periods or delegating authority through proposing an incomplete contract for two periods. The restrictive contracting setting highlights one important feature of delegating authority: It increases the agent's incentive to engage in opportunistic behavior, however, we show that this may be beneficial to all contracting parties if it can be foreseen ex-ante.

The Bayesian Nash equilibrium outcome of the restricted outsourcing game is characterized in Proposition (1):

Proposition 1. *Let,*

$$\overline{\Delta H} \equiv \frac{1}{2}(1+\alpha)\mu + \frac{\bar{r}_s + \mu}{4\bar{r}_s}((\frac{1}{2}+\alpha)\mu + \frac{1}{2}\bar{r}_s) \tag{1}$$

The equilibrium of the contracting game implies that

a) If $\Delta H \geq \overline{\Delta H}$*, the government chooses a complete contract specifying a high quality level of the provided service.*

b) If $\Delta H \leq \overline{\Delta H}$*, the government leaves the contract incomplete. The service provider*

[11] Also notice, that there is no private information in the model. The utility consequences of the different modes of production are common knowledge (if known to somebody). Hence there are no incentives for the service provider to act strategically in order to manipulate beliefs, i.e. *signaling* is not an issue in our model.

chooses the alternative production mode in period 1. In period 2, the service provider chooses the alternative production mode if R_s is non-negative and the basic quality service otherwise.

Proof: Since prices are set such that the expected total surplus are distributed equally among G and S, G offers the contract that generates the highest expected total surplus.

First, we compare expected total surplus, ETS , from a contract specifying high quality in both periods with a contract specifying high quality in the first period and leaving authority to S in the second:

$$\begin{aligned} ETS_{hh} &= 2(B - C + \Delta H), \qquad (2) \\ ETS_{ha} &= 2B - 2C + \Delta H + E\{R_g + R_s\} = 2B - 2C + \Delta H + (1+\alpha)\mu, \end{aligned}$$

where $E\{R_s+R_g\}$ denotes the expected value of R_s+R_g. By assumption $\Delta H > (1+\alpha)\mu \geq 0$, hence, $ETS_{hh} > ETS_{ha}$. The agents' expected pay-offs from a two period high quality contract are $U^g_{hh} = U^s_{hh} = B - C + \Delta H$, respectively.

Second, if G instead leaves authority to S, S never picks the delivers the high quality service, since it will not be rewarded. Remember, that delivering a low quality service is always an option, since this can be done in an unverifiable way. Hence, S chooses between providing low quality service in both periods, low quality in the first period and the alternative production mode in the second period or the alternative production mode in both periods. The expected total surplus generated from the first two options are,

$$\begin{aligned} ETS_{ll} &= 2(B - C), \\ ETS_{la} &= 2(B - C) + E\{R_s + R_g\} = 2(B - C) + (1+\alpha)\mu \geq ETS_{ll}. \end{aligned}$$

Comparing with equation (2) it is seen that both of these strategies are dominated by writing a high quality contract in both periods.

Third, if S picks the alternative service organization in period 1, he learns r_s and picks the alternative organization again in period 2 if R_s is non-negative and the low quality service otherwise. Notice, since r_s is uniformly distributed, the probability that $R_s \geq 0$ is $\frac{\bar{r}_s+\mu}{2\bar{r}_s}$. The expected total surplus in this case is,

$$\begin{aligned} ETS_{a\cdot} &= 2(B-C) + E\{R_g + R_s\} + \frac{\bar{r}_s+\mu}{2\bar{r}_s} E\{R_s + R_g | R_s \geq 0\} \\ &= 2(B-C) + (1+\alpha)\mu + \frac{\bar{r}_s+\mu}{2\bar{r}_s}((\frac{1}{2}+\alpha)\mu + \frac{1}{2}\bar{r}_s). \end{aligned}$$

Through price negotiation, S receives half this value, implying that S chooses the alternative production method whenever the contract allocates authority to her at date 1. Hence, G chooses a two period complete contract if and only if,

$$U^g_{hh} = B - C + \Delta H \geq B - C + \frac{1}{2}(1+\alpha)\mu + \frac{\bar{r}_s+\mu}{4\bar{r}_s}((\frac{1}{2}+\alpha)\mu + \frac{1}{2}\bar{r}_s) = U^g_{a\cdot},$$

which is equivalent to

$$\Delta H \geq \frac{1}{2}(1+\alpha)\mu + \frac{\bar{r}_s+\mu}{4\bar{r}_s}((\frac{1}{2}+\alpha)\mu + \frac{1}{2}\bar{r}_s) \equiv \overline{\Delta H}.$$

□

Proposition 1 provides the first main insight from the present analysis: delegating authority increases the expected private benefit of the service provider and this is in both parties' interest as long as it increases the expected total welfare from the relationship. To see this, it is helpful to begin with the case were $\mu = 0$, i.e. were there is no expected benefit or loss from the alternative provision mode to either party relative to producing the basic service quality. In this case the threshold value, $\overline{\Delta H}$, reduces to:

$$\overline{\Delta H} \equiv \frac{1}{8}\bar{r}_s. \tag{3}$$

This condition says that, for the government to delegate authority, it must be the case that the (per period) net benefit of writing a complete - high quality - contract does not exceed half of S's expected gain from having authority over the production decision. The expected private gain for S arises from the option value in period two whenever he chooses the alternative production mode in period one: By assumption, there is no expected gain from using the alternative production mode in period one; however, by experimenting,

S learns the real value of R_s. This provides S with the option to choose the alternative production mode in period two iff $R_s \geq 0$ and choose a production mode that generates the the basic quality service otherwise. The expected value of this option is the expected value of R_s, conditioned on R_s being positive, times the probability that R_s is indeed positive. In total this equals $\frac{1}{2} \cdot \frac{1}{2}\bar{r}_s$. When the uncertainty is large, the option value is large. The government receives half of this expected value through the price negotiations ex ante. Hence, the Proposition makes clear, that the more uncertainty there is for the service provider about the utility consequences of the alternative production mode, the more likely it is that the government offers an incomplete contract which leaves authority to the service provider.

The additional terms in equation (1) reflect the caveats that arise when $\mu > 0$. The larger is α, the more aligned are the preferences in expected terms; the higher is μ, the higher is the expected benefit to the service provider from choosing the alternative production mode. Straightforward differentiation gives that $\frac{\partial \overline{\Delta H}}{\partial \alpha}$ and $\frac{\partial \overline{\Delta H}}{\partial \mu}$ are both positive. Hence the higher α and μ are, the more likely it is that the government chooses to delegate authority through leaving gaps in the contract.

Notice that the condition given in Proposition 1, does *not* depend on the uncertainty around the impact of the alternative production mode on the utility of the government, as measured by $\bar{r}_g$. Since the government cannot use the information about its private benefit to revise the contract before period 2, from an ex-ante perspective there is no option value for the government.[12]

How, then, does the government's willingness to delegate authority depend on the amount of uncertainty about the service provider's private gain? The derivative $\frac{\partial \overline{\Delta H}}{\partial \bar{r}_s}$ is positive, if $\alpha < -\frac{1}{2}$. If $\alpha > -\frac{1}{2}$, the derivative is positive iff $\bar{r}_s > \sqrt{(1 + 2\alpha)}\mu$. In these cases, larger uncertainty about the service providers payoff increases the likelihood that the government prefers delegation. The intuition here is more involved, many effects are at

[12]Clearly, introducing risk aversion into the model would imply an independent channel through which uncertainty would affect the government's decision to delegate authority.

play, when $\bar{r}_s$ increases. Still of course, the option is more valuable for the service provider, and the government gets part of this in the initial negotiations. However, the condition determining when the service provider chooses one and the other form for provision is also affected. The probability that R_s is positive equals $\frac{\bar{r}_s+\mu}{2\bar{r}_s}$. This probability decreases in $\bar{r}_s$ and this creates an offsetting effect.

These comparative static results are summarized in the following Corollary

Corollary 1. *a) Increasing uncertainty about the government's payoff from the alternative production mode does not affect the government's incentive to delegate authority.*

b) Assume $\mu = 0$: Increasing uncertainty about the service provider's payoff from the alternative production mode increases the government's incentive to delegate authority through leaving contract incomplete.

c) If $\mu > 0$, the government is more likely to prefer an incomplete contract the higher is μ and α. The effect of an increase in $\bar{r}_s$ is ambiguous. If $\alpha < -\frac{1}{2}$ or $\bar{r}_s > \sqrt{(1+2\alpha)}\mu$, then an increase in $\bar{r}_s$ tends to make delegation preferable.

Even though we do not allow for a complete trial-and-error approach before next section, Proposition 1 - and in particular Equation (3) - clarifies that our analysis does not hinge on the presence of (specific) transactions cost. What drives the result is the ability to learn from delegating authority through leaving contracts incomplete and the associated option value linked to this learning feature.[13]

4 Adaptive contracting.

We now proceed to analyze the complete version of our two period contracting model. We keep all the assumptions from the previous section, except that we allow for inter-period contract revision of the following kind. The government can choose between a two period long-term contract ex-ante or a one-period short term contract. In the latter case a new

[13] Obviously, if transaction cost is defined more general as any contractual constraint that makes costless first-best contracting infeasible, then these "general" transactions costs drive the results in the present analysis.

contract will be negotiated before period 2 starts. This framework therefore opens the possibility of adaptive contracting.

There are many possible contracting strategies from which the government and the service provider can choose. However, Lemma 1 significantly reduces the relevant contracting strategies.

Lemma 1. *In equilibrium G offers one of two contracts:*
a) A complete long-term contract specifying a high quality standard production mode in both periods;
b)An adaptive contract consisting of a short-term incomplete contract, which is replaced in period 2 with either another short-term incomplete contract or a short-term complete contract specifying high quality production mode.

Proof:

We prove the Lemma backwards. If a short term contract specifying either low quality (LQ) or high quality (HQ) standard production modes was chosen in period 1, nothing is learned about r_s and r_g, and the optimal contract in the second period specifies HQ, due to assumption 1. It thus follows that the incomplete contract, IC, can only be chosen in the second period if it was chosen in the first period.

It is never optimal to specify LQ under any circumstances in any period, again due to Assumption 1, recalling that $(1+\alpha)\,\mu \geq 0$. If IC is chosen in the first period, the government learns the realizations of the stochastic variables r_s and r_g. If it only writes a short time contract, it has the option to specify HQ in the second period, should it wish to, and as we show below this may indeed be valuable. It will therefore not wish to choose IC for both periods already in period 1.

If it specifies HQ already in the first period, it may as well specify it for the second period also, as it will be the choice anyway in the second period. This proves the Lemma.□

The Lemma states that there are two relevant contracts to analyze. Either the extra value of having high quality is so high that the government prefers this outcome ex-ante,[14] or it is better to leave authority to the service provider, giving him incentives to pick the alternative organization of the service provision.

If the long-term high quality contract is chosen, the expected surplus is given in Equation (2).

We now analyze the parties' expected surplus from using the adaptive contracting strategy. When the contract is incomplete, and S has authority in period 1, he chooses the alternative production mode, since it gives (weakly) more expected utility than choosing the low quality service, which again yields higher benefit for S than choosing high quality. In period 2, there are three options: The government stipulates a high quality contract effectively removing the authority of the service provider; S keeps authority but has learned that R_s is negative, implying that he chooses the low quality; and, finally that S keeps authority and has learned that R_s is positive, so he chooses the alternative production mode again. Since the last case happens with a positive probability, S is strictly better off in expected terms choosing the alternative production mode in period 1. Notice, that since the alternative production mode and the related payoffs are non-verifiable by assumption, it is not possible to write a second period contact specifying that this mode *should* be chosen. Effectively, the only way to implement this mode in the second period is to write an incomplete contract, and the service provider therefore still has authority over whether this mode should be used. Accordingly, it will only be used if his private utility from using it, R_s, is positive. Below we briefly discuss the case, where the alternative production mode is contractible in the second period if and only if it has been used in the first period.

After S chooses the alternative production mode in period 1, G and S learn the realizations of the private benefits. Both S and G have the option to avoid the alternative quality level again in period 2. S opts for the alternative production mode in the second

[14] Since none of the parties learn anything about their potential private benefits, it is obvious that this long-term contract could be replaced by two short-term contracts. However a small positive negotiating cost would make it strictly more beneficial to both parties to negotiate a long-term contract.

period if and only if $R_s \geq 0$, whereas, as we will show below, G only delegates authority if it is in the parties' joint interests, i.e. if $R_s \geq 0$ and $R_g + R_s \geq \Delta H$. We are therefore interested in the ex ante probability of the event $R_g + R_s \geq \Delta H$ given it is known that $R_s \geq 0$.

As $R_s + R_g = (1+\alpha)\mu + r_s + r_g$, and $R_s = \mu + r_s$, the probability that $R_s + R_g > \Delta H$ conditioned on $R_s \geq 0$ is equal to the probability that $r_s + r_g > \Delta H - (1+\alpha)\mu$ conditioned on $r_s \geq -\mu$. Let

$$T \equiv \Delta H - (1+\alpha)\mu.$$

T expresses the expected additional surplus from the high quality service to the alternative service quality. Let $\Phi(T)$ denote the probability that $r_s + r_g < T$ given $r_s \geq -\mu$.[15]. The distribution function Φ is fully characterized in the Appendix. Lemma 2 gives Φ for the parameter values, we will focus on below. The proof is in the Appendix.

Lemma 2. *If* $-\bar{r}_g + \bar{r}_s \leq T \leq \bar{r}_g - \mu$, *then*

$$\Phi(T) = \frac{2\bar{r}_g - \bar{r}_s + 2T + \mu}{4\bar{r}_g}.$$

From Lemma 1 we know that there are only two possible equilibrium outcomes. Recall that the parties split the surplus in the negotiations. Hence, in order to determine which contract type the government prefers, it suffices to determine whether the long-term complete high quality or the adaptive contract yields the highest total surplus.

Consider first the long-term complete high quality contract. From Equation (2) we notice that the additional total surplus relative to a low quality contract over the two periods equals:

$$2\Delta H = 2(\Delta B - \Delta C). \tag{4}$$

Next, if G offers the adaptive contract, the expected additional surplus from period 1 is $(1+\alpha)\mu$. If $R_s \geq 0$ and $R_s + R_g \geq \Delta H$ then the second period contract will be left

[15] Hence, the probability that $T > r_s + r_g$ given $r_s \geq \mu$, then equals $1 - \Phi(T)$.

incomplete. Let this event be denoted IC_2. The probability that $R_s \geq 0$ is $\frac{\bar{r}_s+\bar{\mu}}{2\bar{r}_s}$, while the probability that $R_s + R_g \geq \Delta H$ equals $1 - \Phi(\Delta H - (1+\alpha)\mu) = 1 - \Phi(T)$. Hence, the incomplete contract is chosen again with probability $\left(\frac{\bar{r}_s+\mu}{2\bar{r}_s}\right)(1-\Phi(T))$.

Let the expected additional surplus conditioned on the event IC_2 be denoted $E\{R_s + R_g|IC_2\}$. The expected surplus from the adaptive contracting strategy is:

$$(1+\alpha)\mu + \left(\frac{\bar{r}_s+\mu}{2\bar{r}_s}\right)(1-\Phi(T))E\{R_s + R_g|IC_2\}+ \\ \left(1-\left(\frac{\bar{r}_s+\mu}{2\bar{r}_s}\right)(1-\Phi(T))\right)\Delta H. \tag{5}$$

The incomplete contract will be chosen in the first period if the surplus in (5) exceeds the surplus in (4). Simplifying a bit proves the following,

Proposition 2. *The government delegates authority through offering an incomplete contract to the service provider if and only if,*

$$E\{r_s + r_g|IC_2\} \geq \frac{T + \left(\frac{\bar{r}_s+\mu}{2\bar{r}_s}\right)(1-\Phi(T)\Delta H}{\left(\frac{\bar{r}_s+\mu}{2\bar{r}_s}\right)(1-\Phi(T))}. \tag{6}$$

Equation (6) provides the condition for when the adaptive - trial-and-error - approach to outsourcing is optimal. With probability $\left(\frac{\bar{r}_s+\mu}{2\bar{r}_s}\right)(1-\Phi(T))$ the service provider will be allowed to keep the authority in the second period which generates an expected surplus of $E\{R_s + R_g|IC_2\}$ to the government. In this case the government observes that the joint private benefit is higher than the potential gain from using the high quality complete contract. The opportunity cost of delegating authority is the net benefit given up in the first period. In the second period, when private benefits from the alternative production is known, adaptive contracting is costless for the government. The second period contract will only be incomplete, if the government gains from it.

It is interesting to know how the incentives to use adaptive contracting is affected by the degree of uncertainty about the agents' private benefit. Unfortunately, the formulas become rather lengthy and there are a number of cases to consider. We now derive a clear result in the important case where $\mu = 0$, i.e. where there is no (one period) expected surplus from the alternative process relative to the basic service provision. In this case, $T = \Delta H$, and condition (6) reduces to

$$E\{r_s + r_g | IC_2\} \geq \frac{(2 + (1 - \Phi(\Delta H))\, \Delta H}{1 - \Phi(\Delta H)}. \tag{7}$$

As is clear from the full expression for Φ given in the Appendix, there are quite a number of sub cases to consider. Rather than going tediously through all possible sub cases, we will focus first on the case, where $\bar{r}_g \geq \bar{r}_s$ and the high quality contract is not overwhelmingly attractive in the sense that $\Delta H < \bar{r}_g$. Then Φ is given in Lemma 2.

Proposition 3. *Consider the case where the expected utility from choosing the alternative production mode is zero for both parties, i.e. $\mu = 0$, where the uncertainty for the government is at least as large as for the service provider, $\bar{r}_g \geq \bar{r}_s$, and where the high quality contract is not overwhelmingly attractive, $\Delta H < \bar{r}_g$. In this case, the government offers a short term incomplete contract to the service provider if and only if*

$$\Delta H \leq 5\bar{r}_g + \frac{1}{2}\bar{r}_s - 2\sqrt{6\bar{r}_g^2 + \bar{r}_g\bar{r}_s}. \tag{8}$$

This is more likely to be fulfilled, the smaller is the expected gain from the high quality contract, ΔH, and the larger is the uncertainty about the utility consequences of the alternative production mode for either of the parties, i.e. the larger is $\bar{r}_s$ and $\bar{r}_g$.

Proof:

We first evaluate the left hand side of the inequality (7).

$$\begin{aligned} E\{r_s + r_g | IC_2\} &= E\{r_s + r_g | r_s \geq 0 \text{ and } r_g + r_s \geq \Delta H\} \\ &= \int_0^{\bar{r}_s} \frac{1}{\bar{r}_s} \left(\int_{\Delta H - r_s}^{\bar{r}_g} \frac{r_s + r_g}{\bar{r}_g - (\Delta H - r_s)} dr_g \right) dr_s. \end{aligned}$$

This is true since, the conditional density, given $r_s \geq 0$, at a particular r_s equals $\frac{1}{\bar{r}_s}$. With this r_s, r_g has to be larger than $\Delta H - r_s$ for $r_g + r_s \geq \Delta H$ to hold. Since r_g is uniformly distributed, the conditional density at an r_g fulfilling $r_g \geq \Delta H - r_s$ is $\frac{1}{\bar{r}_g - (\Delta H - r_s)}$. We sum r_s and r_g, multiply with the relevant densities and integrate over the relevant ranges. Integrating yields

$$E\{r_s + r_g | IC_2\} = \frac{1}{2}\Delta H + \frac{1}{2}\bar{r}_g + \frac{1}{4}\bar{r}_s.$$

In the range of variables, we consider here Φ is given in Lemma 2. Inserting into the right hand side of (7) , and manipulating a bit gives

$$\frac{(2 + (1 - \Phi(\Delta H))\,\Delta H}{1 - \Phi(\Delta H)} = \frac{(10\bar{r}_g + \bar{r}_s - 2\Delta H)\,\Delta H}{2\bar{r}_g + \bar{r}_s - 2\Delta H}.$$

Therefore condition (7) becomes

$$\frac{1}{2}\Delta H + \frac{1}{2}\bar{r}_g + \frac{1}{4}\bar{r}_s \geq \frac{(10\bar{r}_g + \bar{r}_s - 2\Delta H)\,\Delta H}{2\bar{r}_g + \bar{r}_s - 2\Delta H}.$$

Solving for ΔH yields that in the relevant range (recall that we assume that $\Delta H \leq \bar{r}_g$), this is equivalent to

$$\Delta H \leq 5\bar{r}_g + \frac{1}{2}\bar{r}_s - 2\sqrt{6\bar{r}_g^2 + \bar{r}_g\bar{r}_s},$$

which is condition (8) of the Proposition. Finally, differentiating the right hand side of (8) yields

$$\frac{\partial RHS}{\partial \bar{r}_s} = \frac{1}{2}\frac{\sqrt{\bar{r}_g\,(6\bar{r}_g + \bar{r}_s)} - 2\bar{r}_g}{\sqrt{\bar{r}_g\,(6\bar{r}_g + \bar{r}_s)}} > 0$$

as $r_g, r_s > 0$. Similarly

$$\frac{\partial RHS}{\partial \bar{r}_g} = \frac{5\sqrt{\bar{r}_g\,(6\bar{r}_g + \bar{r}_s)} - 12\bar{r}_g - \bar{r}_s}{\sqrt{\bar{r}_g\,(6\bar{r}_g + \bar{r}_s)}}$$

and therefore we get that

$$\frac{\partial RHS}{\partial \bar{r}_g} > 0 \text{ iff } 6\bar{r}_g^2 + \bar{r}_g\bar{r}_s - \bar{r}_s^2 > 0$$

which is fulfilled under our assumption that $\bar{r}_g \geq \bar{r}_s$.

$\square$

Under adaptive contracting the alternative production mode is only implemented in period 2 when it is privately beneficial for the service provider and jointly beneficial for both parties. Contrary to the case with no inter-period contracting, the government is now also insured against the downside realization of the uncertainty. The renegotiation in the current setting gives both parties insurance, and hence the option value is extended to both parties. This implies that more uncertainty, whether it concerns the government's utility or the service provider's utility, strengthen the incentives to use the trial-and-error approach. This is a qualitatively different result from the ones we obtained in the previous section.

When $\mu \neq 0$, the formulas becomes substantially more involved. We therefore restrict ourselves to consider a single case where $\mu > 0$, and where the uncertainty is the same for both parties, so that $\bar{r}_s = \bar{r}_g = \bar{r}$. Again, we assume that the high quality contract is not very attractive, so that $T \leq \bar{r} - \mu$, which is equivalent to $\Delta H \leq \bar{r} + \alpha\mu$. Then the relevant part of $\Phi(T)$ is given in Lemma 2. Let

$$H(\mu, \alpha, \bar{r}) \equiv \frac{1}{2(\mu + \bar{r})}(11\bar{r}^2 + 3\bar{r}\mu -$$
$$\sqrt{112\bar{r}^4 + 16(1-2\alpha)\bar{r}^3\mu + (4\alpha^2 - 20\alpha - 23)\bar{r}^2\mu^2 + (24\alpha + 18 + 8\alpha^2)\bar{r}\mu^3 + (12\alpha + 9 + 4\alpha^2)\mu^4}).$$

Following the steps of the previous proof, inserting for Φ in condition (6) and solving the resulting inequality for ΔH yields the following result.

Proposition 4. *Consider the case where the expected utility from the alternative production mode is positive for the service provider, i.e. $\mu > 0$, the uncertainty is the same for both parties, $\bar{r}_g = \bar{r}_s = \bar{r}$, and the high quality contract is not overwhelmingly attractive, $\Delta H < \bar{r} + \alpha\mu$. In this case, the government offers a short term incomplete contract in the first period to the service provider if and only if*

$$\Delta H \leq H(\mu, \alpha, \bar{r}).$$

Unsurprisingly, a low expected value of the high quality complete contract tends to make the incomplete contract relatively more attractive. The expression for $H(\mu, \alpha, \bar{r})$ is unfortunately not particularly informative. In order to gain further insight, Figure 1 below presents three dimensional plot of H, for the case $\mu = 1$.

**** FIGURE 1 TO BE PLACED HERE****

Figure 1 reveals that increasing uncertainty makes the incomplete contract relatively more attractive, as was the case when $\mu = 0$. The effect of α is slightly more complicated. Straightforward differentiation gives that $\frac{\partial H}{\partial \alpha}$ is positive for $\alpha < \frac{1}{2}\frac{8r^2-3\mu^2-3r\mu}{\mu(\mu+r)}$ and negative otherwise. For $\mu = 1$, this imply that $\frac{\partial H}{\partial \alpha}$ is positive for all $\alpha \in [-1, 1]$ provided, $\bar{r} \geq \frac{5}{16} + \frac{1}{16}\sqrt{185} \approx: 1.16$. Hence, except for low degrees of uncertainty, we have the result, that increasing the degree of alignment of preferences makes delegation more attractive as would appear natural. For small degrees of uncertainty, the result may, however, be the opposite, as is also clear from figure 1.

Before we end this section it is worth discussing the assumption that the alternative production mode is unverifiable in period two, even if it has been applied in period one. Let us instead assume that if the alternative production mode has been used in period one, then it is feasible to write a contract specifying this production mode in period 2. This changes the condition for choosing the alternative quality in period 2 from the joint event of $R_s \geq 0$ and $R_s + R_g \geq 0$ to only $R_s + R_g \geq 0$. Obviously this would increase the government's incentive to choose adaptive contracting. Hence, in all the cases where the joint surplus is positive, but where the service provider's private utility is lower than what he obtains from delivering a low quality service, we observed more delegation in the second period under this alternative assumption. We conclude, that the alternative assumption strengthen our analysis since it makes adaptive contracting more valuable.

5 Conclusion

Adaptive contracting is widely used around the world, in particular it is a standard approach when government agencies write outsourcing contracts with private firms. Many observers and participants argue that the trial and error approach is the only feasible way to write a contract for a bureaucratic public organization with its many limitations due to time-, organizational- and various political constraints. The main insight of the present analysis has been to show that the trial-and-error approach can be an optimal form of contracting even without such constraints.

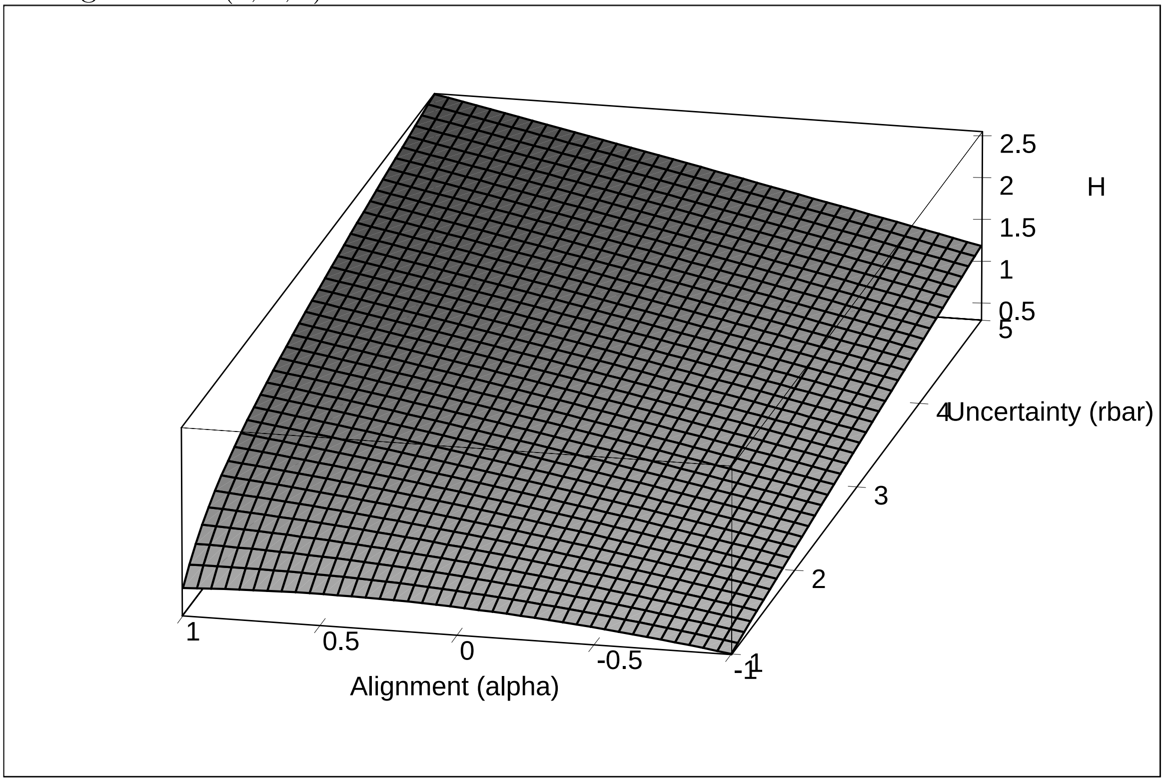
Our theoretical analysis highlighted the connection between adaptive contracting and the optimal allocation of authority in form of decision rights about matters not specified in the contract. The general principle resulting from our analysis is that authority should be allocated to the party that uses it for most surplus generation independently on how this surplus is distributed ex-post. The trial-and-error approach allocates authority initially to the service provider who uses this authority opportunistically. This is beneficial for both parties as long as the service provider's choices increase the total value generated in the relationship. The ex-post distribution of this rent is not important because it will be corrected through the negotiated service price.

Adaptive contracting enables a government to learn about the organizational reactions to delegating authority. In the contract renegotiation phase, this knowledge becomes valuable since the government always has the option of elaborate on contracts in places where it has been revealed that the agent reacts to having discretion in a way which is decremental to maximizing the total surplus from the relationship.

Further work has to be undertaken to obtain a comprehensive knowledge about which organizational settings that favor the adaptive relative to a comprehensive contracting approach. As shown in the previous sections, the more uncertainty about the consequences of the delegation of authority, the more incentive the parties have to apply adaptive contracting, since it increases the option value associated with experimenting in addition to increasing the cost of writing complete contracts. This observation is particular interesting in a multi-dimensional contract where the quality of the service may have many parameters each of which it is possible to delegate authority over.

Our analysis were restricted to a two period contracting setting. In reality many outsourcing decisions covers many periods. For instance, in the Danish local bus transportation example, there has presently been twelve rounds of outsourcing. We conjecture that extending our model to a multi-period setting only increases the incentives to use adaptive contracting, since the embedded option value increases.

Figure 1: $H(1, \alpha, \bar{r})$.



Appendix: Complete characterization of $\Phi(.)$.

Lemma 2. *The distribution function* $\Phi(.)$ *is given by:*

- *If* $-\bar{r}_g + \bar{r}_s \leq \bar{r}_g + \mu$ *then,*

$$\Phi(T) = \begin{cases} \frac{1}{4}\frac{T^2+2T\bar{r}_g+\bar{r}_g^2+2T\mu+\mu^2+2\bar{r}_g\mu}{(\bar{r}_s+\mu)\bar{r}_g} & \text{if } -\bar{r}_g-\mu \leq T \leq -\bar{r}_g+\bar{r}_s, \\ -\frac{1}{4}\frac{\bar{r}_s-2\bar{r}_g-2T-\mu}{\bar{r}_g} & \text{if } -\bar{r}_g+\bar{r}_s \leq T \leq \bar{r}_g-\mu, \\ -\frac{1}{4}\frac{-2T\bar{r}_s+\bar{r}_s^2-2\bar{r}_g\bar{r}_s+T^2+2T\bar{r}_g-3\bar{r}_g^2}{(\bar{r}_s+\mu)\bar{r}_g}+\frac{T-\bar{r}_g+\mu}{\bar{r}_s+\mu} & \text{if } \bar{r}_g-\mu < T < \bar{r}_g+\bar{r}_s. \end{cases}$$

- *If* $-\bar{r}_g + \bar{r}_s > \bar{r}_g + \mu$ *then,*

$$\Phi(T) = \begin{cases} \frac{1}{4}\frac{T^2+2T\bar{r}_g+\bar{r}_g^2+2T\mu+\mu^2+2\bar{r}_g\mu}{\bar{r}_g\bar{r}_s} & \text{if } -\bar{r}_g-\mu \leq T \leq \bar{r}_g-\mu, \\ \frac{\bar{r}_g}{\bar{r}_s+\mu}+\frac{T-\bar{r}_g+\mu}{\bar{r}_s+\mu} & \text{if } \bar{r}_g-\mu \leq T \leq -\bar{r}_g+\bar{r}_s, \\ -\frac{1}{4}\frac{-2T\bar{r}_s+\bar{r}_s^2-2\bar{r}_g\bar{r}_s+T^2+2T\bar{r}_g-3\bar{r}_g^2}{(\bar{r}_s+\mu)\bar{r}_g}+\frac{T-\bar{r}_g+\mu}{\bar{r}_s+\mu} & \text{if } -\bar{r}_g+\bar{r}_s < T < \bar{r}_g+\bar{r}_s. \end{cases}$$

Proof. We are interested in the probability that the sum $R_g + R_s < \Delta H$, where $\Delta H = \Delta B - \Delta C - d$, given that $R_s \geq 0$.

This problem can be written

$$\begin{aligned} &\text{Prob}\left(\mu + r_s + \alpha\mu + r_g < \Delta H \;\mid \mu + r_s > 0\right) \\ &= \text{Prob}\left(r_s + r_g < \Delta H - (1+\alpha)\mu \;\mid r_s > -\mu\right) \\ &= \text{Prob}\left(r_s + r_g < T \;\mid r_s > -\mu\right) \text{ since} T \equiv \Delta H - (1+\alpha)\mu. \end{aligned}$$

Notice that given $r_s \geq -\mu$, the conditional density at any r_s is $\frac{1}{\bar{r}_s+\mu}$.

Recall that $\Phi(T)$ denotes the probability that $r_g + r_s \leq T$ given $r_s \geq -\mu$. Then $\Phi(T)$ equals the integral over positive r_s, given the conditional density of r_s, multiplied by the probability that $r_g \leq T - r_s$ at this particular r_s. We therefore have,

$$\Phi(T) = \int_{-\mu}^{\bar{r}_s} \frac{1}{\bar{r}_s+\mu} G(T - r_s)\, dr_s,$$

where $G(\cdot)$ denotes the cdf of r_g.

Remember

$$G(r_g) = \begin{cases} 0 \text{ for } r_g < -\bar{r}_g, \\ \frac{r_g-(-\bar{r}_g)}{2\bar{r}_g} \text{ for } -\bar{r}_g \leq r_g \leq \bar{r}_g, \\ 1 \text{ for } \bar{r}_g < r_g. \end{cases} \tag{9}$$

This feature implies that Φ will be pasted together from three different parts. The support of Φ is $[-\bar{r}_g-\mu, \bar{r}_g+\bar{r}_s]$.

We need to distinguish between two cases according to whether

$$-\bar{r}_g+\bar{r}_s \lesseqgtr \bar{r}_g-\mu. \tag{10}$$

First we consider the case

$$-\bar{r}_g+\bar{r}_s < \bar{r}_g-\mu. \tag{11}$$

Notice that if

$$\bar{r}_g > \bar{r}_s,$$

then we for sure have that (11) is fulfilled.

In the present case, the real line looks like this

$$\text{----} -\bar{r}_g-\mu\text{--}-\bar{r}_g+\bar{r}_s\text{-------------}\bar{r}_g-\mu\text{---}\bar{r}_g+\bar{r}_s.$$

Let us consider $G(T-r_s)$.

When $-\bar{r}_g+\bar{r}_s < \bar{r}_g-\mu$, then the interval $T \in [-\bar{r}_g+\bar{r}_s, \bar{r}_g-\mu]$, is non-empty. For T in this interval, we have that $T-r_s \in [-\bar{r}_g, \bar{r}_g]$ for all realizations of r_s in $[-\mu, \bar{r}_s]$. In this case the relevant formula for $G(T-r_s)$ is the second line in (9) above - this explains the second line in (12) below.

When $T \in [-\bar{r}_g-\mu, -\bar{r}_g+\bar{r}_s]$, then high realizations of r_s will imply that $T-r_s < -\bar{r}_g$, in which case the first line in (9) is relevant. For a given $T \in [-\bar{r}_g-\mu, -\bar{r}_g+\bar{r}_s]$ the probability that $T-r_s < -\bar{r}_g$ equals

$$\text{Prob}\,(\ r_s > T+\bar{r}_g) = \frac{\bar{r}_s-(T+\bar{r}_g)}{\bar{r}_s+\mu}.$$

On the other hand the second line in (9) is relevant for small realizations of r_s, i.e. for all realizations $r_s < T+\bar{r}_g$. This explains the first line in (12) below.

Then consider high T, $T \in [\bar{r}_g-\mu, \bar{r}_g+\bar{r}_s]$. Low realizations of r_s imply that $T-r_s > \bar{r}_g$, in which case the relevant part of G is given by the third line in (9). This is true for $r_s < T - \bar{r}_g$. The probability of such an r_s is

$$\text{Prob}\,(r_s < T - \bar{r}_g) = \frac{T - \bar{r}_g + \mu}{\bar{r}_s + \mu}.$$

For high realizations of r_s ($r_s > T - \bar{r}_g$), the relevant part of G is the second line in (9). This explains the third line in (12).

We therefore have that the cdf of the sum is

$$\Phi(T) = \begin{cases} \int_{-\mu}^{T+\bar{r}_g} \frac{1}{\bar{r}_s+\mu} \frac{T-r_s+\bar{r}_g}{2\bar{r}_g} dr_s \; + 0\frac{\bar{r}_s-(T+\bar{r}_g)}{\bar{r}_s+\mu} \text{ for } -\bar{r}_g - \mu \leq \; T \leq -\bar{r}_g + \bar{r}_s, \\ \int_{-\mu}^{\bar{r}_s} \frac{1}{\bar{r}_s+\mu} \frac{T-r_s+\bar{r}_g}{2\bar{r}_g} dr_s \text{ for } -\bar{r}_g + \bar{r}_s \leq T \leq \; \bar{r}_g - \mu, \\ \int_{T-\bar{r}_g}^{\bar{r}_s} \frac{1}{\bar{r}_s+\mu} \frac{T-r_s+\bar{r}_g}{2\bar{r}_g} dr_s \; + 1\frac{T-\bar{r}_g+\mu}{\bar{r}_s+\mu} \text{for } \bar{r}_g - \mu < T < \bar{r}_g + \bar{r}_s. \end{cases} \tag{12}$$

Performing the integrations yields,

$$\Phi(T) = \begin{cases} \frac{1}{4}\frac{T^2+2T\bar{r}_g+\bar{r}_g^2+2T\mu+\mu^2+2\bar{r}_g\mu}{(\bar{r}_s+\mu)\bar{r}_g} \text{ for } -\bar{r}_g - \mu \leq \; T \leq -\bar{r}_g + \bar{r}_s, \\ -\frac{1}{4}\frac{\bar{r}_s-2\bar{r}_g-2T-\mu}{\bar{r}_g} \text{ for } -\bar{r}_g + \bar{r}_s \leq T \leq \; \bar{r}_g - \mu, \\ -\frac{1}{4}\frac{-2T\bar{r}_s+\bar{r}_s^2-2\bar{r}_g\bar{r}_s+T^2+2T\bar{r}_g-3\bar{r}_g^2}{(\bar{r}_s+\mu)\bar{r}_g} + \frac{T-\bar{r}_g+\mu}{\bar{r}_s+\mu} \text{for } \bar{r}_g - \mu < T < \bar{r}_g + \bar{r}_s. \end{cases}$$

The density function is

$$\phi(T) = \begin{cases} \frac{1}{2}\frac{T+\bar{r}_g+\mu}{(\bar{r}_s+\mu)\bar{r}_g} \text{ for } -\bar{r}_g - \mu \leq \; T \leq -\bar{r}_g + \bar{r}_s, \\ \frac{1}{2\bar{r}_g} \text{ for } -\bar{r}_g + \bar{r}_s \leq T \leq \; \bar{r}_g - \mu, \\ -\frac{1}{2}\frac{-\bar{r}_s+T-\bar{r}_g}{(\bar{r}_s+\mu)\bar{r}_g} \text{for } \bar{r}_g - \mu < T < \bar{r}_g + \bar{r}_s. \end{cases}$$

Now consider the case where

$$-\bar{r}_g + \bar{r}_s > \bar{r}_g - \mu \tag{13}$$

Now the real line looks like this

____ $-\bar{r}_g - \mu$__$\bar{r}_g - \mu$____________ $-\bar{r}_g + \bar{r}_s$____$\bar{r}_g + \bar{r}_s$.

As above we have that , when $T \in [-\bar{r}_g - \mu, -\bar{r}_g + \bar{r}_s]$, then high realizations of r_s will imply that $T - r_s < -\bar{r}_g$, in which case the first line in (9) is relevant. For a given $T \in [-\bar{r}_g - \mu, -\bar{r}_g + \bar{r}_s]$ the probability that $T - r_s < -\bar{r}_g$ equals

$$\text{Prob}\,(\; r_s > T + \bar{r}_g) = \frac{\bar{r}_s - (T + \bar{r}_g)}{\bar{r}_s + \mu}.$$

On the other hand when $T \in [\bar{r}_g - \mu, \bar{r}_g + \bar{r}_s]$, then low realizations of r_s imply that $T - r_s > \bar{r}_g$, in which case the relevant part of G is given by the third line in (9). This is true for $r_s < T - \bar{r}_g$. The probability of such an r_s is

$$\text{Prob}\left(r_s < T - \bar{r}_g\right) = \frac{T - \bar{r}_g + \mu}{\bar{r}_s + \mu}.$$

For the rest of the realizations of r_s, i.e. for $r_s \in [T - \bar{r}_g, T + \bar{r}_g]$, then $T - r_s \in [-\bar{r}_g, \bar{r}_g]$. Hence for $T \in [-\bar{r}_g - \mu, -\bar{r}_g + \bar{r}_s]$, the cdf is given by the second line in (14) below.

For small $T \in [-\bar{r}_g - \mu, \bar{r}_g - \mu]$, only the probability that high realizations of T imply that $T - r_s < -\bar{r}_g$ is relevant. This explains the first line in (14) below.

Finally for high $T \in [-\bar{r}_g + \bar{r}_s, -\bar{r}_g + \bar{r}_s]$ only the probability that low realizations of T imply that $T - r_s > \bar{r}_g$ is relevant. This explains the third line in (14)

$$\Phi\left(T\right) = \begin{cases} \int_{-\mu}^{T+\bar{r}_g} \frac{1}{\bar{r}_s} \frac{T - r_s + \bar{r}_g}{2\bar{r}_g} dr_s \ + 0 \frac{\bar{r}_s - (T + \bar{r}_g)}{\bar{r}_s + \mu} \text{ for } -\bar{r}_g - \mu \leq \ T \leq \bar{r}_g - \mu, \\ \int_{T-\bar{r}_g}^{T+\bar{r}_g} \frac{1}{\bar{r}_s + \mu} \frac{T - r_s + \bar{r}_g}{2\bar{r}_g} dr_s + 0 \frac{\bar{r}_s - (T + \bar{r}_g)}{\bar{r}_s + \mu} + 1 \frac{T - \bar{r}_g + \mu}{\bar{r}_s + \mu} \text{ for } \bar{r}_g - \mu \leq T \leq \ -\bar{r}_g + \bar{r}_s, \\ \int_{T-\bar{r}_g}^{\bar{r}_s} \frac{1}{\bar{r}_s + \mu} \frac{T - r_s + \bar{r}_g}{2\bar{r}_g} dr_s \ + 1 \frac{T - \bar{r}_g + \mu}{\bar{r}_s + \mu} \text{for } -\bar{r}_g + \bar{r}_s < T < \bar{r}_g + \bar{r}_s. \end{cases} \tag{14}$$

Integrating yields,

$$\Phi\left(T\right) = \begin{cases} \frac{1}{4} \frac{T^2 + 2T\bar{r}_g + \bar{r}_g^2 + 2T\mu + \mu^2 + 2\bar{r}_g\mu}{\bar{r}_g\bar{r}_s} \text{ for } -\bar{r}_g - \mu \leq \ T \leq \bar{r}_g - \mu, \\ \frac{\bar{r}_g}{\bar{r}_s + \mu} + \frac{T - \bar{r}_g + \mu}{\bar{r}_s + \mu} \text{ for } \bar{r}_g - \mu \leq T \leq \ -\bar{r}_g + \bar{r}_s, \\ -\frac{1}{4} \frac{-2T\bar{r}_s + \bar{r}_s^2 - 2\bar{r}_g\bar{r}_s + T^2 + 2T\bar{r}_g - 3\bar{r}_g^2}{(\bar{r}_s + \mu)\bar{r}_g} + \frac{T - \bar{r}_g + \mu}{\bar{r}_s + \mu} \text{for } -\bar{r}_g + \bar{r}_s < T < \bar{r}_g + \bar{r}_s. \end{cases}$$

The density becomes

$$\phi\left(T\right) = \begin{cases} \frac{1}{2} \frac{T + \bar{r}_g + \mu}{\bar{r}_g\bar{r}_s} \text{ for } -\bar{r}_g - \mu \leq \ T \leq \bar{r}_g - \mu, \\ \frac{1}{\bar{r}_s + \mu} \text{ for } \bar{r}_g - \mu \leq T \leq \ -\bar{r}_g + \bar{r}_s, \\ -\frac{1}{2} \frac{-\bar{r}_s + T - \bar{r}_g}{(\bar{r}_s + \mu)\bar{r}_g} \text{for } -\bar{r}_g + \bar{r}_s < T < \bar{r}_g + \bar{r}_s. \end{cases}$$

□

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