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MORAL COSTS, THE INFORMAL SECTOR AND UNEMPLOYMENT

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Moral costs, the informal sector and unemployment*

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Abstract

While examining the macroeconomic effects of increased government control of the informal sector, this paper develops a two-sector general equilibrium model featuring matching frictions and worker-firm wage bargaining. The same good is produced in the formal and in the informal sector. Moral considerations are determinant for whether the worker search for jobs in the formal or in the informal sector. We analyse the impact of higher punishment fees and a higher audit rate on wages, sector division, unemployment and welfare.

1 Introduction

Some goods are produced in both the formal and in the informal sector. From the worker's perspective, a given worker therefore faces a decision of whether to perform his or her activities in the formal or the informal sector. When making that decision, the worker compares wages, employment perspectives and taxes/expected punishment in the two sectors. Considering this sectorial choice, one question emerges: why do not all workers of a given skill type apply for jobs in either the formal sector or the informal sector? Why do both sectors exist?

One prominent explanation is that workers differ in terms of morality. Entering the informal sector is associated with moral costs and some workers have a high moral preventing them from entering the informal sector. Other workers have a lower moral and gladly enter the informal sector. The division into a formal and an informal sector is then such that the informal sector is constituted by relatively low morality workers and the formal sector consists of relatively high

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morality workers. However, for all workers we have that employment perspectives, wages, taxes, detection probabilities and punishment fees are important for the decision of entering into the formal or the informal sector.

The purpose of this paper is to examine how a larger control of the informal sector, affects the size of the informal sector, wages, unemployment and welfare.

To that end, we develop a two-sector general equilibrium model featuring matching frictions, heterogenous workers in terms of different morality, worker-firm wage bargains and endogenous sector division.

Recently, there has been a large amount of economic writings on tax avoidance and tax evasion.[1] Early theoretical contributions of tax evasion are provided by Allingham and Sandmo (1972) and Srinivasan (1973), where underreporting of income is modeled as a decision made under uncertainty. Since these early contributions, a number of papers have enhanced the basic model of individual behaviour by, for example, incorporating endogenous labour supply decisions.[2] Several theoretical papers have also recognized that the opportunities for tax evasion differ across sectors.[3] Risk aversion is usually the determinant of the sector division. Although there has been a recent explosion of the literature on tax evasion and tax avoidance, the research is mainly carried out within the public finance tradition. In this literature wages are either assumed to be fixed or determined by market clearing.[4]

The main novelty of this paper is that we incorporate an imperfectly competitive labour market. This facilitates an analysis of how punishment policies affect wage setting and unemployment. Previous literature on tax evasion has either assumed that wages are fixed or determined by market clearing, which obvious is an inadequate framework to use when analyzing how tax evasion opportunities affect wage setting and unemployment.

The paper is organised as follows. In Section 2 the model is described and Section 3 derives the effects of a higher audit rate and higher punishment fees on labour market tightness, wages, unemployment and sector division. Section 4 gives a welfare analysis and the last section concludes.

2 The Model[5]

The economy consists of two sectors producing a homogenous good; a formal sector and an informal sector.

We assume that workers have some moral considerations, being an important factor for whether the worker applies for a job in the formal or the informal sector. There is a distribution of moral values, m, in the economy, $m \in [0,1]$.

[1] See Slemrod and Yitzhaki (2000) and Schneider and Eneste (2000) for two recent surveys of tax avoidence and tax evasion.

[2] See for example Andersen (1977) and Sandmo (1981) for early contributions of endogenous labour supply and underreporting of income.

[3] See for example Watson (1985), Pestieu and Possen (1991), and Jung et al. (1994).

[4] An exception is Chang and Lai (1996) who examines the relationship between underreporting of income and total tax revenues by taking into account the efficiency wage hypothesis.

[5] The model is along the line of Pissarides 1990.

For simplicity we assume the distribution to be uniform. Moral costs of applying for a job in the informal sector, is denoted $c(m)$. If m is large the moral costs are large, $c'(m) > 0$.

2.1 Matching

We assume that only unemployed workers search for jobs. This is a simplification, i.e., we do not acknowledge that the connection to the labour market given by working in the formal sector, brings about job opportunities not available while unemployed. Workers accept job offers as long as the expected payoff exceeds their reservation wage.

The matching functions for the formal and informal sectors are given by

$$X^j = \left(v^j\right)^{1-\eta} \left(u^j\right)^{\eta}, j = F, I,$$

where $u^j, j = F, I$ are the unemployment rates for the formal and the informal sector, respectively. The unemployment rate in each sector is defined as the number of unemployed workers in the specific sector relatively to the labour force in the sector. The total labour force is normalized to unity, which is divided into the two sectors. The vacancy rates, i.e., the number of vacancies supplied in each sector relative to the sector labour force, are $v^j, j = F, I$. The worker's and firm's transition rates can be expressed as $\lambda^j = X^j/u^j = \left(\theta^j\right)^{1-\eta}, j = F, I$ and $q^j = X^j/v^j = \left(\theta^j\right)^{-\eta}, j = F, I$. Here $\theta^j = \frac{v^j}{u^j}, j = F, I$ are the labour market tightness for the two sectors.

2.2 The formal and the informal sector

Let λ^F and λ^I be interpreted as the probabilities of finding a job in the formal sector and the informal sector, respectively. The present discounted values of unemployment as a worker in the two sectors, U^F and U^I, are then given by:

$$\begin{aligned} rU^F &= \lambda^F(E^F - U^F) && (1) \\ rU^I &= \lambda^I(E^I - U^I) - c\,(m)\,, && (2) \end{aligned}$$

where E^F and E^I are the present discounted values of employment in the formal sector and the informal sector, respectively. r is the exogenous discount rate. For simplicity, we assume that unemployment benefits are equal to zero. The present values of employment are determined by the equations:

$$\begin{aligned} rE^F &= w^F\,(1-t) + s(U^F - E^F), && (3) \\ rE^I &= w^I\,(1-p\delta) - c\,(m) + (s+p)(U^I - E^I). && (4) \end{aligned}$$

Here $w^j, j = F, I$ denotes wages in the formal sector and the informal sector and s is the exogenous separation rate. The parameter t is the proportional income tax rate, p is the rate at which a worker is detected by the government

working in the informal sector, and δ is the proportion of the evaded income the worker has to pay as a punishment fee if detected. The informal sector worker faces the separation rate $s + p$ as the worker may be separated from his or her job due to an exogenous market separation or due to detection.

The moral cost is paid by all workers in the informal sector irrespective of whether they are employed or unemployed as the decision of applying for a job in this sector involves the moral considerations.

Firms in the formal sector are characterized by the arbitrage equations:

$$\begin{aligned} rJ^F &= y - w^F(1+z) + s(V^F - J^F), & (5)\\ rV^F &= q^F(J^F - V^F) - k, & (6) \end{aligned}$$

where J^F is the value of having a filled job in the formal sector, V^F is the value of an unfilled job in this sector, and the parameter z is the payroll tax rate. The marginal productivity of manual workers is y. Hiring costs are denoted k.

Similarly, firms in the informal sector have J^I and V^I determined by:

$$\begin{aligned} rJ^I &= y - w^I(1+p\alpha) + (s+p)(V^I - J^I), & (7)\\ rV^I &= q^I(J^I - V^I) - k, & (8) \end{aligned}$$

where α is the proportion of the evaded wage the firm has to pay as a punishment fee for cheating the government on payroll taxes when supplying informal sector jobs.

2.2.1 Wage determination

When the worker and firm meet they bargain over wages. Wages, $w^j, j = F, I$ solve first order conditions from the Nash Bargaining Solutions with the worker's bargaining power being equal to γ:

$$\begin{aligned} \frac{\gamma}{1-\gamma}\frac{1}{\phi^F}\left(J^F - V^F\right) &= E^F - U^F, & (9)\\ \frac{\gamma}{1-\gamma}\frac{1}{\phi^I}\left(J^I - V^I\right) &= E^I - U^I, & (10) \end{aligned}$$

where $\phi^F = \frac{1+z}{1-t}$ and $\phi^I = \frac{1+p\alpha}{1-p\delta}$ are the tax- and punishment wedges.

By use of equation (1)-(8) in equations (9) and (10), and assuming free entry, $V^j = 0, j = F, I$, and symmetric conditions facing firms and workers within each sector, producer wages are derived to be:

$$\begin{aligned} \omega^F &= w^F(1+z) = \gamma\left(y + \theta^F k\right), & (11)\\ \omega^I &= w^I(1+p\alpha) = \gamma\left(y + \theta^I k\right). & (12) \end{aligned}$$

We note that the tax rates and punishment fees have no impact on the producer wages in the two sectors.[6]

2.2.2 Labour Market Tightness

Labour market tightness for the formal sector and the informal sector is determined by equation (5), (6),(7), and (8) using the free entry condition and the wage equations (11) and (12):

$$\frac{k\left(r+s\right)}{q^F} = \left(1-\gamma\right)y-\gamma\theta^F k, \tag{13}$$

$$\frac{k\left(r+s+p\right)}{q^I} = \left(1-\gamma\right)y-\gamma\theta^I k. \tag{14}$$

Changes in the tax and punishment rates, (t, z, δ, α) do not have any impact on tightness in the two sectors. However changes in the audit rate, p, will have an impact on tightness in the informal sector. An increase in p reduces the average length of an informal sector job, and it becomes less profitable to enter the informal sector; θ^I falls. Note that this is a fully financed change in p since changes in the tax rates (t, z) have no impact on tightness and hence these parameters can always be altered in order to balance the government budget.

Note that labour market tightness is higher in the formal than in the informal sector, $\theta^F > \theta^I$, since $s + p > s$ which makes the producer wages higher in the formal sector relative to in the informal sector, that is, $\omega^F > \omega^I$.

In accordance with empirical evidence we have that the consumer wages are higher for informal sector workers than for formal sector workers (Pedersen and Smith (1998)). By assuming $w^F\left(1-t\right) < w^I\left(1-p\delta\right) < w^I$, we require that

$$\psi\gamma\left(y+\theta^F k\right) < \gamma\left(y+\theta^I k\right), \tag{15}$$

and hence the necessary condition that $\psi < 1$ must hold, where

$$\psi = \frac{\phi^I}{\phi^F} = \frac{1+p\alpha}{1-p\delta}\Big/\frac{1+z}{1-t}, \tag{16}$$

is the wedge between the informal sector and the formal sector. We will simply refer to ψ as the wedge.

2.2.3 Unemployment

Steady state employment- and unemployment rates are derived by considering the flows into and out of employment. The equations determining the unemployment rates u^F and u^I are given by:

$$\lambda^F u^F\left(1-\hat{m}\right) = s\left(1-u^F\right)\left(1-\hat{m}\right),$$

$$\lambda^I u^I \hat{m} = \left(s+p\right)\left(1-u^I\right)\hat{m},$$

[6] The fact that taxes are fully borne by labor is a standard result in many models of equilibrium unemployment considering the standard assumptions made; see for example Pissarides (1998).

where $\hat{m}$ and $1-\hat{m}$ defines the labour forces in the informal- and formal sector. The unemployment rates can then be solved from the flow equations as:

$$u^F = \frac{s}{s+\lambda^F}, \tag{17}$$

$$u^I = \frac{s+p}{s+p+\lambda^I}. \tag{18}$$

The total number of unemployed workers are given by the following expression:

$$U_{TOT} = u^F(1-\hat{m}) + u^I\hat{m}. \tag{19}$$

2.3 Sector Division

Workers enter the unemployment pool and choose whether to apply for formal jobs or informal jobs. In making the choice they compare the value of being in the formal sector and the informal sector. That is, they compare rU^F with rU^I. The marginal worker is just indifferent between entering the unemployment pool in the formal sector and in the informal sector. The following equation solves the moral for the marginal worker, $\hat{m}$;

$$rU^F = rU^I. \tag{20}$$

Substituting from equations (1) and (2) into (20) we obtain:

$$\lambda^I\left(E^I - U^I\right) - \lambda^F\left(E^F - U^F\right) = c(\hat{m}), \tag{21}$$

Workers with low moral, $m \leqq \hat{m}$, choose to work in the informal sector whereas workers with high moral, $m > \hat{m}$, choose to work in the formal sector. Hence $\hat{m}$ and $1-\hat{m}$ resolve the labour forces in the informal- and formal sector.

The condition in (21) may be rewritten in two alternative ways. If we substitute for the employment gains by using equations (1), (2), (3) and (4) we have:

$$\frac{\lambda^I w^I(1-p\delta)}{r+s+p+\lambda^I} - \frac{\lambda^F w^F(1-t)}{r+s+\lambda^F} = c(\hat{m}), \tag{22}$$

where $\mu^F = \frac{\lambda^F}{r+s+\lambda^F}$ and $\mu^I = \frac{\lambda^I}{r+s+p+\lambda^I}$ are the weights associated with the payoffs in the formal- and informal sector, respectively. When $r \to 0$, the value attached to payoffs in the informal sector is the employment rate, n^I, which is the expected proportion of time spent as employed in the informal sector. Hence, higher n^I or higher consumer wages in the informal sector, $w^I(1-p\delta)$, tends to increase the number of workers moving into the informal sector; $\hat{m}$ increases. Similarly, a higher employment rate in the formal sector, n^F and a higher formal consumer wage, $w^F(1-t)$, tend to reduce the number of workers

in the informal sector; $\hat{m}$ decreases. Including the discount rate, $r > 0$, the future is less important than the presence and unemployed workers value future payoffs from employment slightly lower than the employment rates.

Alternatively, we can substitute using the first order condition for wages, free entry ($V^j = 0$) and that $J^j = \frac{k}{q^j}$ giving

$$k\frac{\gamma}{1-\gamma}\frac{1}{\phi^p}\left(\theta^F - \psi\theta^F\right) = c\left(\hat{m}\right), \tag{23}$$

We observe that for the informal sector to exist, that is for $\hat{m}$ to be positive, we need that labour market tightness in the informal sector relatively to labour market tightness in the formal sector is larger than the wedge, $\frac{\theta^I}{\theta^F} > \psi$. Put differently, the wage premium for workers employed in the informal sector has to be large enough to counteract that the expected time spent in unemployment is lower in the formal sector.

Consider the specific matching function where $\eta = \frac{1}{2}$. In this case the condition reduces to:

$$\left(\frac{-\left(r+s+p\right)+\sqrt{\left(r+s+p\right)^2+4\gamma\frac{(1-\gamma)y}{k}}}{-\left(r+s\right)+\sqrt{\left(r+s\right)^2+4\gamma\frac{(1-\gamma)y}{k}}}\right)^2 > \psi.$$

The smaller p is, the smaller is the difference between the two labour market tightness, and hence the more likely it is that the condition is satisfied. The more p increases, the less attractive the informal sector becomes as employment opportunities are diminished and wages are reduced in the informal sector. The left hand side therefore decreases in p. The right hand side, on the other hand, increases in p. Hence, there exists a value for the audit rate, p^*, for which for $p \in (0, p^*)$ the informal sector exists and for which $p \in (p^*, 1)$ the informal sector does not exist. Empirically the informal sector exists, wherefore we concentrate on the former range.

Furthermore, for a given distribution of morals, the form of the cost function, $c(m)$, will be determinate for the size of the informal sector relative to the formal sector. We assume that the cost function is such that $c\left(1\right) = \infty$ to guarantee the existence of a formal sector. Ceteris paribus, both a high audit rate and high moral costs cause a small informal sector.

3 Comparative Statics

This section is concerned with the impact of the punishment system on tightness, equilibrium producer wages, employment- and unemployment rates, the number of unemployed workers, and the division of workers into the formal and informal sector. We consider both how an increase in the audit rate, p, and an increase in the punishment fees, α and δ, affects the equilibrium variables. We only consider fully financed changes in the punishment system. Hence changes in p,

α, and δ is always followed by adjustments in the tax rates t and z in order to balance the government budget. The government budget restriction is given by:

$$R = w^F (t+z)(1-\hat{m})\left(1-u^F\right) + \hat{m}w^I (p\delta + p\alpha)\left(1-u^I\right), \tag{24}$$

which may be rewritten as

$$R = \omega^F \left(1-\frac{1}{\phi^F}\right)(1-\hat{m})\left(1-u^F\right) + \hat{m}\omega^I \left(1-\frac{1}{\phi^I}\right)\left(1-u^I\right). \tag{25}$$

3.1 Labour Market Tightness

This section considers tax financed changes in the punishment system on labour market tightness. Since adjustments in the tax rates, z and t, have no impact on tightness, we can study the impact on tightness of a fully financed increase in the audit rate, p, and the punishment rates, α and δ, without explicitly incorporating the government budget restriction. The effects on tightness is summarized in the following proposition.

Proposition 1 *A fully financed increase in the audit rate, p, will have no impact on tightness in the formal sector, θ^F, and reduce tightness in the informal sector, θ^I. Both labour market tightness, θ^F and θ^I, are unaffected by fully financed changes in the punishment rates, δ, or α.*

Proof. From equation (13) it is apparent that tightness in the formal sector is neither affected by changes in the tax rates nor by changes in the punishment rates, or the audit rate, $\frac{\partial\theta^F}{\partial p} = \frac{\partial\theta^F}{\partial x} = 0, x = t, z, \alpha, \delta$. Differentiating equation (14) with respect to p, δ, α, t and z gives:

$$\begin{aligned} \frac{\partial\theta^I}{\partial p} &= -\frac{\frac{1}{q^I}}{\gamma - \frac{(r+s+p)}{(q^I)^2}\frac{\partial q^I}{\partial\theta^I}} = -\frac{\frac{1}{q^I}}{\gamma + \eta\frac{r+s+p}{q^I\theta^I}} < 0, \\ \frac{\partial\theta^I}{\partial x} &= 0, x = t, z, \alpha, \delta. \end{aligned} \tag{26}$$

■

When the audit rate increases, the expected duration of a match in the informal sector decreases. It is therefore less profitable for a firm to open informal sector vacancies, whereby the number of vacancies supplied relatively to the number of unemployed informal sector workers decreases.

3.2 Wages

This section considers tax financed changes in the punishment system on real producer wages (c.f. equations (11)-(13)). Since adjustments in the tax rates, z and t, have no impact on real producer wages, we can study the impact of a fully financed increase in the audit rate, p, and the punishment rates, α and δ, without

explicitly incorporating the government budget restriction. We summarize the effects of tax financed changes in the punishment system on producer wages in the following proposition:

Proposition 2 *A fully financed increase in the audit rate, p, will have no impact on formal sector producer wages, ω^F, and reduce the producer wage in the informal sector, ω^I. Both formal and informal sector producer wages, ω^F and ω^I, are unaffected by a fully financed increase in the punishment rates, δ and α.*

Proof. Differentiating equation (11) with respect to p and $x = t, z, \alpha, \delta$ give, $\frac{\partial \omega^F}{\partial p} = \frac{\partial \omega^F}{\partial x} = 0, x = t, z, \alpha, \delta$. Differentiating equation (12), considering that tightness is affected according to (14), with respect to p and $x = t, z, \alpha, \delta$ yield:

$$\begin{aligned} \frac{\partial \omega^I}{\partial p} &= \gamma k \frac{\partial \theta^I}{\partial p} < 0, \\ \frac{\partial \omega^I}{\partial x} &= \gamma k \frac{\partial \theta^I}{\partial x} = 0, x = t, z, \alpha, \delta. \end{aligned}$$

■

When the audit rate increases, labour market tightness decreases, whereby lower hiring costs are induced. Hence producer wages decrease.

3.3 Unemployment Rates

This section considers tax financed changes in the punishment system on labour sector unemployment rates. Since adjustments in the tax rates, z and t, have no impact on the unemployment rates, we can study the impact of a fully financed increase in the audit rate, p, and the punishment rate, α and δ, without explicitly incorporating the government budget restriction. We summarize the results in the following proposition:

Proposition 3 *A fully financed increase in the audit rate, p, will have no impact on the unemployment rate in the formal sector, u^F, and increase the informal sector unemployment rate, u^I. Neither the formal nor the informal sector unemployment rates, u^F and u^I, are affected by a fully financed increase in the punishment rates, δ and α.*

Proof. Differentiating equation (17) with respect to p and $x = t, z, \alpha, \delta$ yield, $\frac{\partial u^F}{\partial p} = \frac{\partial u^F}{\partial x} = 0, x = t, z, \alpha, \delta$. Differentiating the unemployment equation, equation (18,), with respect to p and $x = t, z, \alpha, \delta$, we have:

$$\begin{aligned} \frac{\partial u^I}{\partial p} &= \frac{\lambda_m^I - (s+p)\frac{\partial \lambda_m^I}{\partial p}}{\left(s + p + \lambda_m^I\right)^2} > 0, \qquad (27) \\ \frac{\partial u^I}{\partial x} &= 0, \ x = t, z, \alpha, \delta. \end{aligned}$$

■

Informal sector unemployment increases for two reasons. A higher audit rate increases the separation rate in the informal sector which directly raises the informal sector unemployment rate, but also reduces labour market tightness and thereby reduces the unemployed worker's transition rate into the informal sector employment. Both effects increase the informal sector unemployment rate.

3.4 Sector Division

This section is concerned with how the division of labour across the two sectors is affected by a tax financed change in the punishment system. We observe from (23) that $\hat{m}$ is affected by p, α, δ, z, and t. Therefore, in order to consider the effects of a fully financed change in the punishment system on sector division, we have to account for repercussions on $\hat{m}$ following adjustments in the tax rates; the government budget restriction has to be incorporated explicitly. Let us first consider the impact on $\hat{m}$ of a change in the audit rate, punishment rates and tax rates separately. Differentiating equation (23) with respect to p and $\hat{m}$ yield:

$$\frac{\partial \hat{m}}{\partial p}|_{t,z,\alpha,\delta} = \frac{1}{c'(\hat{m})}\frac{k}{\phi^I}\frac{\gamma}{1-\gamma}\left(-\left(\frac{\theta^I}{\phi^I}-\frac{\psi\theta^F}{\phi^I}\right)\frac{\partial \phi^I}{\partial p}+\frac{\partial \theta^I}{\partial p}-\frac{\partial \psi}{\partial p}\theta^F\right)<0. \tag{28}$$

A higher audit rate reduces employment perspectives and consumer wages in the informal sector implying that a smaller number of workers will allocate towards the informal sector; $\hat{m}$ falls.

Differentiating equation (23) with respect to $x_1 = \alpha, \delta$ and $\hat{m}$ we obtain:

$$\begin{aligned}\frac{\partial \hat{m}}{\partial x_1}|_{t,z,p} &= \frac{\partial \hat{m}}{\partial \phi^I}|_{\phi^F}\frac{\partial \phi^I}{\partial x_1}|_{p,t,z} \\ &= \frac{\partial \phi^I}{\partial x_1}\frac{k}{c'(\hat{m})}\frac{-\gamma}{1-\gamma}\left(\frac{\theta^I-\psi\theta^F}{\left(\phi^I\right)^2}+\frac{\partial \psi}{\partial \phi^I}\frac{\theta^F}{\phi^I}\right)<0, x_1=\alpha,\delta. \end{aligned} \tag{29}$$

Hence, both an increase in p, α or δ, reduces the informal sector as the informal sector becomes relatively less attractive.

Differentiating equation (23) with respect to $x_2 = t, z$ we obtain:

$$\begin{aligned}\frac{\partial \hat{m}}{\partial x_2}|_{\alpha,\delta,p} &= \frac{\partial \hat{m}}{\partial \phi^F}|_{\phi^I}\frac{\partial \phi^F}{\partial x_2}|_{\alpha,\delta,p} \\ &= -\frac{k}{c'(\hat{m})}\frac{\gamma}{1-\gamma}\frac{1}{\phi^I}\frac{\partial \psi}{\partial \phi^F}\frac{\partial \phi^F}{\partial x_2}\theta^F>0,\ x_2=t,z. \end{aligned} \tag{30}$$

Increases in the tax rates, t and z, leads to a larger informal sector, $\frac{d\hat{m}}{d\phi^F}|_{\phi^I} > 0$, as the formal sector becomes relatively less attractive.

In order to consider fully financed increases in the punishment system given by increased audition, p, and increased punishment rates, α and or δ, we need to consider the impact of the audit-, punishment-, and tax rates on the government revenue. Changes in the punishment system and the tax system affect the government revenue in a number of ways (details are given in the Appendix). Considering that we are located on the positively sloped side of the Laffer curve, in the sense that dynamic adjustments in equilibrium wages, employment rates and labour force are not dominating the direct effects, government revenue increases with increased audit-, punishment-, and tax rates. An increase in the audit rate, p, or the punishment rates, α and δ, then calls for reductions in the tax rates in order to maintain a balanced budget. We can hence rewrite the government budget restriction in (25) as $\phi^F = h(p, \alpha, \delta)$, where $\frac{\partial \phi^F}{\partial p} < 0$, and $\frac{\partial \phi^F}{\partial x_1} < 0$, $x_1 = \alpha, \delta$. That is an increase in α, δ, or p will induce a government surplus which calls for a reduction in ϕ^F, which is captured by a reduction in either t or z, or both. We can summarize the effects on sector division in the following proposition:

Proposition 4 *A fully financed increase in the audit rate or the punishment rates, p, α, and δ, will reallocate workers from the informal sector towards the formal sector considered that we are located on the positively sloped side of the Laffer curve.*

Proof. The impact on the sector division of labor stemming from an increase in the audit rate, p, and the punishment rates, α and δ, are given by:

$$\frac{\partial \hat{m}}{\partial p} = \frac{\partial \hat{m}}{\partial p}|_{t,z,\alpha,\delta} + \frac{\partial \hat{m}}{\partial \phi^F}|_{\phi^I} \frac{\partial \phi^F}{\partial p} < 0, \tag{31}$$

$$\frac{\partial \hat{m}}{\partial x_1} = \frac{\partial \hat{m}}{\partial x_1}|_{t,z,p} + \frac{\partial \hat{m}}{\partial \phi^F}|_{\phi^I} \frac{\partial \phi^F}{\partial x_1} < 0, \tag{32}$$

where $\frac{\partial \phi^F}{\partial p} < 0$, and $\frac{\partial \phi^F}{\partial x_1} < 0$ according to the budget restriction in (25) if we are located on the positively sloped side of the Laffer curve. $\frac{\partial \hat{m}}{\partial \phi^F}|_{\phi^I} > 0$ from (30) and $\frac{\partial \hat{m}}{\partial p}|_{t,z,\alpha,\delta} < 0$ from (29) and $\frac{\partial \hat{m}}{\partial x_1}|_{t,z,p} < 0$ from (30). ■

3.5 Total Unemployment

This section is concerned with how the number of unemployed workers is affected by fully financed changes in the punishment system. As is clear from section 2.2.3, the total number of unemployed workers depends on the division of labour across sectors. We can summarize the results on total unemployment in the following proposition:

Proposition 5 *A fully financed increase in the audit rate, p, has an ambiguous impact on total unemployment, U^{TOT}, whereas a fully financed increase in the punishment rates, α and δ, reduces total unemployment, U^{TOT}, considered that we are located on the positively sloped side of the Laffer curve.*

Proof. Differentiating equation (19) with respect to U^{TOT}, and the policy parameters p, α, and δ and considering the government budget restriction gives:

$$\frac{\partial U_{TOT}}{\partial p} = \frac{\partial \hat{m}}{\partial p}\left(u^I - u^F\right) + \hat{m}\frac{\partial u^I}{\partial p}, \tag{33}$$

$$\frac{\partial U_{TOT}}{\partial x_1} = \frac{\partial \hat{m}}{\partial x_1}\left(u^I - u^F\right) < 0, \; x_1 = \alpha, \delta \tag{34}$$

where $\frac{\partial \hat{m}}{\partial p} < 0$, $\frac{\partial \hat{m}}{\partial x_1} < 0$ from the proof of proposition (4) and $\frac{\partial u^I}{\partial p} < 0$ from (27). ■

A higher audit rate p induces a reallocation of workers towards the formal sector. This tends to reduce total unemployment as the formal sector unemployment rate is smaller than the informal sector unemployment rate. However, the unemployment rate in the informal sector increases since the shorter duration of jobs discourages firms to enter the informal sector; this tends to raise total unemployment. The overall effect on unemployment of an increase in the audit rate is hence ambiguous. An increase in the punishment rates, α and δ, will however unambiguously reduce total unemployment since only the reallocation effect is at work.

4 Welfare

This section is concerned with welfare analyses of punishment systems. To that end, we make use of a utilitarian welfare function, which is obtained by adding all individuals' and firms' steady state flow values of welfare. The social welfare function is written as:

$$W = \tilde{W}^F\left(1 - \hat{m}\right) + \int_0^{\hat{m}} \tilde{W}^I dm,$$

where $\tilde{W}^F = u^F r U^F + n^F r E^F + n^F r J_m^F + v^F r V^F$, and $\tilde{W}^I = u^I r U^I + n^I r E^I + n^I r J^I + v^I r V^I$. We assume that firms are owned by "reinters" who do not work. By making use of the asset equations for workers and firms in the two sectors, imposing the flow equilibrium conditions as well as the government budget restriction in (25), and considering the case of no discounting, i.e., $r \to 0$, we can write the welfare function as:

$$W = W^F\left(1 - \hat{m}\right) + \int_0^{\hat{m}} W^I dm, \tag{35}$$

where

$$W^F = y\left(1-u^F\right) - u^F\theta^F k, \tag{36}$$
$$W^I = y\left(1-u^I\right) - u^I\theta^I k - c(m). \tag{37}$$

With the assumption of risk neutral individuals, we ignore distributional issues and hence wages will not feature in the welfare function. The government budget is balanced at all times.

4.1 Welfare effects of the punishment system

Before considering the impact of the punishment system on welfare, we can conclude that welfare depends on both the welfare generated in each sector and the allocation of workers across the two sectors. Let us first consider how changes in the allocation of workers across sectors affect welfare. We can conclude the following:

$$\frac{\partial W}{\partial \hat{m}} = W^I\left(\hat{m}\right) - W^F = 0.$$

There are no welfare effects from changes in sectorial division. The reason is that both workers and firms are unaffected by these labour movements. Workers that change their sectorial status are indifferent between the two states since the moral costs equals the expected gain of informal sector work for these workers. Firms, in addition, make zero profits in the long run.

Hence, considering how the punishment system affects welfare, we can disregard from the reallocation effect. What is important is how welfare associated with the two sectors are affected by the punishment system. A closer look at the welfare measures associated with each sector reveals that changes in α, δ, z, and t will have no impact on W^F and W^I. Furthermore, we have that changes in the audit rate p have no impact on W^F but will influence W^I. Therefore, we can study the impact on overall welfare of a fully financed increase in the audit rate, p, and the punishment rates, α and δ, without explicitly incorporating the government budget restriction since adjustments in the tax rates, z and t, have no impact on welfare. We summarize the results in the following proposition.

Proposition 6 *A fully financed increase in the audit rate, p, will reduce welfare whereas higher punishment rates, α and δ, will have no impact on welfare.*

Proof. As is clear from equations (36) and (37) which are independent of α, δ, z, and t, and the fact that sector reallocation of workers will have no impact on overall welfare, tax financed changes in, α and δ, will have no impact on welfare. Differentiating (35), (36) and (37) with respect to p gives:

$$\frac{\partial W}{\partial p} = \hat{m}\frac{\partial W^I}{\partial p} = -\hat{m}\left(\left(y+\theta^I k\right)\frac{\partial u^I}{\partial p} + u^I\frac{\partial \theta^I}{\partial p}\right).$$

Substituting for $\frac{\partial u^I}{\partial p}$ and $\frac{\partial \theta^I}{\partial p}$ from (26) and (27) brings us:

$$\frac{\partial W}{\partial p} = -\left(\lambda^I \left(y + \theta^I k \right) \frac{1 - (s+p)(1-\eta) \left(\theta^I \right)^{-1} \frac{\partial \theta^I}{\partial p}}{\left(s + p + \lambda^I \right)^2} + u^I \frac{\partial \theta^I}{\partial p} k \right)$$

Then using equation and (2) and reduce to obtain:

$$\frac{\partial W}{\partial p} = -(s+p) \left(\left(y + \theta^I k \right) \left(\frac{\lambda^I}{s+p} - (1-\eta) \frac{\lambda^I}{\theta^I} \frac{\partial \theta^I}{\partial p} \right) + \frac{\partial \theta^I}{\partial p} k \left(s + p + \lambda^I \right) \right).$$

Reducing further using (26) gives when $r = 0$:

$$\frac{\partial W}{\partial p} = -\left(\left(y + \theta^I k \right) \lambda^I \gamma + \frac{s+p}{q^I} \left(y q^I - (s+p) k \right) \right).$$

By use of the labour market tightness equation for the informal sector we observe that the last term is always positive and hence $\frac{\partial W}{\partial p} < 0$. ■

A higher audit rate leads to lower welfare as welfare for the informal sector is reduced. Welfare for the informal sector is decreasing with a higher audit rate, as the increase in the unemployment rate is dominating the increased welfare due to lower hiring costs.

From a welfare economic perspective, the informal sector should not be audited as welfare is reduced for informal sector workers without affecting welfare for formal sector workers. The induced reallocation of workers towards the formal sector has no impact on economic welfare.

5 Conclusion

We have shown that increased government control of the underground economy in terms of more frequent auditing will reduce the size of the underground economy. The reason is that employment perspectives and consumer wages in the formal sector are reduced. More frequent auditing reduces the profitability for firms to open vacancies in the underground economy since the average time length of a job is reduced. Hence, less firms will open informal vacancies and employment perspectives in the informal sector falls.

We have also shown that increased auditing reduces welfare but may not necessarily reduce overall unemployment. The ambiguous impact on overall unemployment stems from the reduced employment perspectives in the informal sector being counteracted by the fact that workers reallocate towards the formal sector where the unemployment rate is lower. Welfare is reduced since employment perspectives in the informal sector are reduced and the reallocation of workers leaves welfare unaffected.

Considering increased punishment fees, we concluded that the higher punishment fees reduce the size of the underground economy by reducing the consumer

wages for informal sector workers. Moreover, higher punishment fees will have no impact on welfare but will reduce overall unemployment. However, an agenda for future work could be to include moral considerations concerning the size of the informal sector or consumption of the informal sector good, which could modify the welfare results.

References

[1] Andersen Per, 1977, Scandinavian Journal of Economics 79, 375-83.

[2] Allingham, Michael G. and Sandmo, Agnar, 1972, Income tax evasion: A theoretical analysis, Journal of Public Economics 1, 323-338.

[3] Chang , Wen-Ya, and Lai, Ching-Chong, 1996, Public Finance Quarterly 24, 163-172.

[4] Jung, Young H., Snow Arthur, Trandel, Gregory A., 1994, Tax evasion and the size of the underground economy. Journal Of Public Economics 54, 391-413.

[5] Pissarides, C.A., 1998, The impact of Employment Tax Cuts on Unemployment and Wages: The Role of Unemployment and Tax Structure, European Economic Review 42, 155-183.

[6] Pissarides, C.A., 1990: Equilibrium Unemployment Theory, Blackwell, Oxford.

[7] Pissarides, C.A., 1986, Unemployment and vacancies in Britain, Economic Policy 3, 499-559.

[8] Pestieau, Pierre, and Possen, Uri M., 1991, Tax Evasion and occupational choice, Journal of Public Economics 45, 107-125.

[9] Sandmo, Agnar, 1981, Income Tax Evasion, Labour supply, and the equity-efficiency trade-off, Journal of Public Economics 16, 265-88.

[10] Schneider, Friedrich, and Dominik Eneste, 2000, Shadow Economies: Size,Causes, and Consequences, Journal of Economic Literature, 38(1), 77-114.

[11] Srinivasan,T.N.,1973, Tax Evasion, A Model, Journal of Public Economics 2, 339-346.

[12] Pedersen, S. and Nina Smith, 1998: Sort arbejde og sort løn i Danmark.

[13] Slemrod, Joel, and Shlomo Yitzhaki, 2000, Tax Avoidence, Evasion, and Administration, Forthcoming in Handbook of Public Economics, Working Paper, NBER:7473.

[14] Watson, Harry, 1985, Tax evasion and labour markets, Journal of Public Economics 27, 231-246.

6 Appendix

6.1 Impact on revenue

Differentiating the government budget constraint in (25) with respect to p and $x = t, z, \delta, \alpha$ gives the following expressions:

$$\begin{aligned}\frac{\partial R}{\partial p} &= \frac{\partial \hat{m}}{\partial p}\left(\omega^I\left(1-\frac{1}{\phi^p}\right)n^I - \omega^F\left(1-\frac{1}{\phi^t}\right)n^F\right) \\ &+\hat{m}\frac{\partial \omega^I}{\partial p}\left(1-\frac{1}{\phi^p}\right)n^I \\ &+\hat{m}\omega^I\left(1-\frac{1}{\phi^p}\right)\frac{\partial n^I}{\partial p} \\ &+\hat{m}\omega^I\frac{1}{\left(\phi^p\right)^2}n^I\frac{\partial \phi^p}{\partial p},\end{aligned} \tag{38}$$

$$\begin{aligned}\frac{\partial R}{\partial x_1} &= \frac{\partial \hat{m}}{\partial x_1}\left(\omega^I\left(1-\frac{1}{\phi^p}\right)n^I - \omega^F\left(1-\frac{1}{\phi^t}\right)n^F\right) \\ &+\hat{m}\omega^F\frac{1}{\left(\phi^p\right)^2}\left(1-u^I\right)\frac{\partial \phi^p}{\partial x_1},\ x_1 = \delta, \alpha.\end{aligned} \tag{39}$$

$$\begin{aligned}\frac{\partial R}{\partial x_2} &= \frac{\partial \hat{m}}{\partial x_2}\left(\omega^I\left(1-\frac{1}{\phi^p}\right)n^I - \omega^F\left(1-\frac{1}{\phi^t}\right)n^F\right) \\ &+\omega^F\frac{1}{\left(\phi^t\right)^2}\left(1-\hat{m}\right)\left(1-u^F\right)\frac{\partial \phi^t}{\partial x_2},\ x_2 = t, z,\end{aligned} \tag{40}$$

where we can divide the influences on the government revenue into four categories characterized by each row in the two equations. The *first* row in each equation captures how revenues are altered by the change in the number of workers choosing the informal sector. The *second* row in equation (38) captures how revenues are influenced by changes in the equilibrium producer wage for informal sector workers. The *third* row in equation (38) gives the impact on revenues due to employment changes for informal sector workers. Finally, the *fourth* row in equation (38) and the second row in equations (39) and (40) gives the direct effect.

Consider first how revenues change with an increase in audit rate, p (cf. equation (38). From the discussion in section 3.4 we concluded that the number of informal sector workers decreases with a higher audit rate. Revenues then increase to the extent that formal sector workers pay more taxes than informal sector workers pay in punishment fees. The second row encapsulates that a higher audit rate implies lower informal sector producer wages, the government revenues fall. Since the employment rate for informal sector workers decreases, the government revenues fall. Finally, the direct effect will always increase revenues.

Since the dynamic effects move in different directions it is difficult to determine whether they reinforce or weaken the direct effect. However, we assume that we are located on the upward sloping part of the Laffer curve and hence the dynamics effects will never dominate the direct effect.

Analogous reasoning can be conducted for equation (39). We derived that the number of informal sector workers decreases with higher punishment fees. Revenues then increase to the extent that formal sector workers pay more taxes than informal sector workers pay in punishment fees. The direct effect will always increase revenues.

Finally, from equation (40) we observe that the reallocation effect between the two sectors tends to decrease revenues if formal sector workers pay more taxes than informal sector workers pay in punishment fees, whereas the direct effect increases revenues.