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**VALUATION, LEVERAGE AND THE COST
OF CAPITAL IN THE CASE OF DEPREE-
CIABLE ASSETS: REVISITED.**

Diderik Lund

Department of Economics - Copenhagen Business School
Solbjerg Plads 3, DK-2000 Frederiksberg

Valuation, leverage and the cost of capital in the case of depreciable assets: Revisited

Diderik Lund

Department of Economics, University of Oslo

P.O.Box 1095, Blindern, NO-0317 Oslo, Norway

Phone +47 22855129

Fax +47 22855035

e-mail: diderik.lund@econ.uio.no

web page: <http://folk.uio.no/dilund>

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Abstract

Levy and Arditti (1973) introduced depreciable assets into the Modigliani and Miller (1958) model, and analyzed the implications for the cost of capital. Assuming that the firm reinvests indefinitely to maintain a constant expected cash flow, they found that depreciation increases the cost of capital before and after tax. Most of their assumptions are maintained. However, commitment to perpetual reinvestment is in most cases not a reasonable assumption. Without it, depreciation decreases the cost of capital before and after tax. The effect of depreciation is less in absolute value than in Levy and Arditti, but not insignificant.

KEYWORDS: Cost of capital, depreciation, corporate taxes

JEL CLASSIFICATION NUMBERS: G31, H25

Levy and Arditti (1973) (LA73 hereafter) extended the Modigliani and Miller (1958, 1963) (MM58, MM63) analysis by introducing depreciation. In spite of the long time which has passed, one particular assumption in the analysis deserves attention today. One main result concerns the effect of depreciation on the required expected rate of return on investment. While both LA73 and MM assumed that the firm commits to a constant expected cash flow forever, a more realistic assumption leads to a reversal of the sign of this effect from positive to negative. The magnitude of the effect can still be substantial.

There are strong indications that most companies do their project valuations with methods which are deficient in one or more ways, at least as seen from a theoretical viewpoint. Even some terms which are fairly easy to predict and/or adjust for, so that practice could be brought more in line with theory, are often neglected. A leading textbook by Brealey and Myers (2003) suggests that “In practical capital budgeting, a single discount rate is usually applied to all future cash flows” (p. 239). The empirical investigation by Graham and Harvey (2001) confirms this. Brealey and Myers (2003) also state that “Depreciation tax shields contribute to project cash flow, but they are not valued separately; they are just folded into project cash flows along with dozens, or hundreds, of other specific inflows and

outflows” (p. 546). This paper, as well as LA73, shows the inadequacy of such a practice. It is demonstrated that one feasible method is to adjust the cost of capital for depreciation tax shields.

A company often operates, or contemplates to operate, under various tax rates and/or various tax depreciation schedules. This could be due to operation in different countries, or in sectors (within a country) which are taxed differently (e.g., natural resource extraction), or simply due to using assets with different tax depreciation. According to the view of LA73, maintained in this paper, the companies should not apply the same discount rates (neither before nor after corporate taxes) in these different situations. The same view is relevant for analyses of tax reforms or hypothetical tax systems. Lund (2002a) describes how some oil companies based their arguments against a tax reform on a fixed discount rate, inconsistent with theory. The topic is highly relevant today, and the purpose of this paper is to suggest an alternative model with opposite results from those in LA73.

LA73 assumed economic depreciation which results in an investment outlay being required each period in order to maintain the perpetual revenue stream, constant in expected terms. The depreciation is tax deductible, and for simplicity, tax depreciation equals economic depreciation. They also assumed that both the replacement outlay and the tax value of the deduction are known with certainty for all future periods. The result is that depreciation increases the required expected rate of return, both before and after taxes.

Bradford (1975) suggested that the tax value of the depreciation deductions might be risk free, but that the replacement outlays would be risky. This resulted in a completely different conclusion, namely that depreciation decreases the required expected rate of return.

The present paper suggests that the results in both papers are misleading. The basic situation in which the concept of a required expected rate of return applies, is an investment with no commitment to perpetual reinvestment. Tax depreciation deductions are still applicable, however.

Results will be presented in the form of required expected rates of return both before and after corporate taxes. This is in line with MM63 and LA73. While the economics literature concentrates on the distortions, which show up in the before-tax required rates

of return, it is also interesting to know what would be the right discount rates for companies to use after taxes.

Like the cited sources, the present paper does not rely on a particular model of the capital market, such as the CAPM. The results rely on value additivity only. Two related papers, Lund (2002b) and Lund (2003), give the implications of these results in a CAPM framework. The exact relation between this paper and Lund (2002b, 2003) is explained in Lund (2003). Briefly, the differences are as follows. Lund (2002b, 2003) are less general since they assume a CAPM type model, and since the company's operating cash flow is always given as price times quantity, PQ , of which Q is chosen by the company while P has an exogenous probability distribution (under competitive conditions). On the other hand, results are arrived at there without specifying a particular profile for economic or tax depreciation. In addition, the model in Lund (2002b, 2003) allows for a variety of corporate and personal tax systems, and for uncertainty over future tax positions of the companies, i.e., the tax value of future deductions being risky.

I The model with perpetual reinvestment

The model in this section follows LA73 and Bradford (1975) as closely as possible. The model portrays a firm with the same expected cash flow, element by element, in every period after the initial investment. The firm has debt, which is not repaid, but which requires a yearly interest payment. The expected after-tax cash flow before payment of interest, but including the resulting value of the tax deduction for interest, is

$$\bar{X}^t = (1 - t)\bar{C} - \bar{K}^* + tK + tR, \quad (1)$$

where C is annual before-tax operating cash flow, t is the corporate tax rate, K^* is replacement investment outlay, K is the (risk free) tax depreciation deduction, and R is (risk free) interest payment. The bar over variables denotes expected values. Inflation is neglected (or the tax system is inflation adjusted). The possible difference between K and K^* was introduced by Bradford (1975).

Both papers sort the cash flow elements in two categories, one risk free and one risky. The two papers differ over the riskiness of the replacement outlay, with expected value \bar{K}^* .

The value of the levered firm according to LA73 (their equation (4)) is

$$V_\ell = \frac{(1-t)\bar{C}}{\rho} - \frac{\bar{K}^*}{r} + \frac{tK}{r} + \frac{tR}{r} \quad (2)$$

(but with $K^* = K$), while Bradford (1975) uses

$$V_\ell = \frac{(1-t)\bar{C}}{\rho} - \frac{\bar{K}^*}{\rho} + \frac{tK}{r} + \frac{tR}{r}, \quad (3)$$

for the same value (his equation (9)). Here, ρ is the risk-adjusted discount rate applicable to the operating cash flow, while r is the market interest rate, applicable as a discount rate for the risk free cash flow elements. Both rates are assumed to be constant over time. By value additivity we can assume that the same discount rate is applicable to C and $(1-t)C$, as long as t is constant. Not only does Bradford assume that economic depreciation may have a different expected value than tax depreciation, but also that it has a different risk, namely the same risk as the operating cash flow.

It should be noted that both papers assume that the firm pays taxes every period, so that there is no risk connected to the tax payments, not even to their timing. This is obviously a simplifying assumption, cf. footnote 5 of MM63. Even if the operating revenue is risky, the assumption may not be too unrealistic at the margin if the firm has other projects which are weakly (or negatively) correlated, or more profitable, or both. But of course, every year some firms are out of tax position, carry-forward reduces present values, and some deductions are lost completely. Lund (2003) considers uncertainty about the firm's tax position.

What are the effects of depreciation on the required expected return on investment? If we compare with the expression without depreciation,

$$V_\ell = \frac{(1-t)\bar{C}}{\rho} + \frac{tR}{r}, \quad (4)$$

the consequences of the LA73 assumptions become clear. Assuming $K^* = K$, they subtract the positive amount $(1-t)K/r$ from the value in (4), or equivalently, they subtract $(1-t)K$ from each year's cash flow. This is similar to an increased operating leverage, and thus raises the risk of the net cash flow as long as the risk of K is less than the net risk of the other cash flow elements.

In general the sign of a cash flow element's contribution to the systematic risk of the net, overall cash flow of a company depends on the element's own systematic risk and whether it is an inflow or an outflow. When it is assumed that the risk of the elements falls in only two categories, distinguished by r and ρ , we are left with only four cases, and qualitative results (the signs of effects) follow easily. The model of the next section gives more detailed results, since we are also interested in the magnitude of effects, cf. table 1. These, of course, depend on relative magnitudes of the cash flow elements as well.

The assumptions of Bradford (1975) lead to a less clear-cut result than those of LA73. Comparing with (4) one finds that the effect of introducing depreciable assets has an undetermined sign. But Bradford shows that two assumptions are sufficient to determine a negative sign: $\bar{K}^* = K$ together with $\rho > r$. The latter means a positive risk premium, or a "positive beta" in the CAPM jargon. The former reintroduces one of the LA73 assumptions, so that the objection made by Bradford boils down to the riskiness of the replacement outlay. By continuity, since $\bar{K}^* = K$ gives a negative sign, then \bar{K}^* close to K also does.

We are left with two opposing views on the effect of introducing depreciation in the MM model. Levy and Arditti (1975) recognize that the alternative assumptions of Bradford lead to the opposite conclusion. But they disagree with Bradford's view that the replacement investment outlays are exactly as risky as the operating cash flow. Thus they end up with an even less clear-cut conclusion, since it is clear that an intermediate discount rate leaves the sign undetermined.

II An alternative to perpetual reinvestment

The objection to the perpetual reinvestment model is that it is unrealistic and thus misleading. It is very rare that an investment decision involves a commitment to perpetual reinvestment. As will be shown, this objection has important implications for the conclusion. The standard, basic use of the concept "required expected rate of return" is for an investment decision made once. There may be some commitment to investment outlays

over some limited period, but the present value of these is normally included as part of the initial investment.

Since 1975 the literature on real options has emerged (see, e.g., Brennan and Schwartz (1985) or McDonald and Siegel (1986)), as well as the literature on the analogy between nonlinear taxes and financial options (e.g., Ball and Bowers (1982) or Green and Talmor (1985)). We can distinguish between three different types of projects: (i) A project with commitment to (re)investment only in a limited number of periods. (ii) A project with investment in the first period and commitment to (re)investment in all future periods. (iii) A project with investment in the first period plus options to (re)invest in some or all future periods. Type (ii) is hardly seen in practice. The third type, however, may be the most realistic and has been the focus of recent research. Nevertheless a leading book on real options states that, “This does not mean that static (passive) NPV should be scrapped; rather, it should be seen as a necessary input to an options-based expanded-NPV approach to capital budgeting,” Trigeorgis (1996, p. 152). By “NPV” Trigeorgis means net present value with a risk-adjusted discount rate. This means that the treatment of the most basic, first type of projects is still of interest.

The objection to perpetual reinvestment does not imply a criticism of the original MM model. Modigliani and Miller were consistent in ignoring depreciation and assuming a perpetual cash flow stream with a constant expected value. They also mentioned the possible extensions, depreciation, non-perpetual cash flows, and non-perpetual debt, cf. footnotes 9 and 16 in MM63. It is the introduction of depreciable assets which raises the question of reinvestment. For the purpose of characterizing the required expected rate of return, it is more appropriate to leave reinvestment aside.

Dropping the commitment to perpetual reinvestment leaves a simplified model. The term K^* disappears from the cash flow expressions and from the value expressions. For many types of assets there will still be tax depreciation, however. But it will — for realistic tax systems — decrease over time, since the tax depreciation is now related to the initial investment only, not to the reinvestment. If assets depreciate, and there is no commitment to reinvestment, the cash flow is unlikely to have a constant expectation.

In order to understand the differences between the three different models, MM63, LA73, and the suggested model with exponential decline and no reinvestment, consider the following general model which encompasses them all. The time period is denoted by τ . The assumptions are:

1. The firm invests an amount $I > 0$ in period $\tau = 0$. In each subsequent period, τ , the amount reinvested is μ times the capital in the previous period $\tau - 1$. Capital also depreciates at a rate ξ , so that capital declines at a net rate $(\xi - \mu) \in [0, 1)$.
2. The firm borrows a fraction $(1 - \eta) \in [0, 1)$ of the gross investment in each period, so that η is the equity share. Repayment of the debt follows depreciation, so that the total debt is always the same fraction $1 - \eta$ of total capital.
3. The firm is always in tax position, and it always pays its debt service.
4. From period $\tau = 1$ onwards the four elements of the expected cash flow — the expected operating cash flow, the reinvestment, the tax value of the depreciation deduction, and the debt service — all decline at the same exponential rate, $(\xi - \mu) \in [0, 1)$.
5. In period $\tau = 1$ the before-tax operating cash flow is C , the reinvestment is μI , the tax depreciation deduction is $K = \xi I$, and the interest payment is $R = r(1 - \eta)I$.
6. The operating cash flow is the only source of risk of the cash flow stream. To find its market value at $\tau = 0$, its expected values, $\bar{C}(1 - \xi + \mu)^{\tau-1} > 0$, can be discounted at the constant risk-adjusted rate ρ .
7. The non-risky elements of the cash flow stream can be discounted at the risk free interest rate r , with $0 < r < \rho$.
8. A corporate income tax at a constant rate $t \in [0, 1)$ applies, with the tax base including the operating cash flow minus tax depreciation and minus interest payments.

The MM63 assumption is $0 = \mu = \xi$, LA73 is obtained with $0 < \mu = \xi$, while the new model suggested in the present paper has $0 = \mu < \xi$. The original articles (and the

subsequent literature) had a strong focus on the effect of debt on the cost of capital. Thus debt is included here as well, although it has no particular significance for the results, as long as debt financing is less than one hundred percent. The case of the unlevered firm is obtained by letting $\eta = 1$.

The value as of $\tau = 0$ of the after-tax cash flows of the levered firm from $\tau = 1$ to infinity is

$$\begin{aligned} V &= \sum_{\tau=1}^{\infty} \left[\frac{(1-t)\bar{C}(1-\xi+\mu)^{\tau-1}}{(1+\rho)^{\tau}} + \frac{(t(K+R)-\mu I)(1-\xi+\mu)^{\tau-1}}{(1+r)^{\tau}} \right] \\ &= \frac{(1-t)\bar{C}}{\rho+\xi-\mu} + \frac{t(K+R)-\mu I}{r+\xi-\mu}, \end{aligned} \quad (5)$$

where the debt service itself is left out, while the value of the tax deduction for interest is included. This is thus the value of the cash flow to equity and debt taken together.

The criterion for accepting an (additional) investment is that

$$\frac{dV}{dI} = \frac{1-t}{\rho+\xi-\mu} \frac{d\bar{C}}{dI} + \frac{t(\xi+r(1-\eta))-\mu}{\xi+r-\mu} \geq 1, \quad (6)$$

since the left-out debt and debt service (implied by the tax deduction for the interest payment) has a net value of zero.

In line with MM63 and LA73 the criterion (6) will be expressed as requirements for (i) the expected rate of return before tax, and for three different expected rates of return after tax, both (ii) the weighted average cost of capital (WACC), (iii) the expected rate of return to debt and equity together, and (iv) that to equity only. In each of these four cases we first need to identify what that expected rate of return is for the kind of cash flow profile under consideration, without specifying that the project should be marginal. Then we can reformulate the requirement given by (6) in terms of a requirement for that particular rate. The underlying requirement is the same each time, but expressed as requirements for different rates (based on different cash flow definitions), which could be calculated for any given project.

The before-tax expected rate of return is that rate ρ_b which solves

$$\sum_{\tau=1}^{\infty} \frac{(d\bar{C}-\mu dI)(1-\xi+\mu)^{\tau-1}}{(1+\rho_b)^{\tau}} = dI. \quad (7)$$

The solution is $\rho_b = d\bar{C}/dI - \xi$, independent of μ .

The requirement given by (6) can now be reformulated. The resulting minimum value of ρ_b will be denoted ρ_b^* . Inequality (6) is equivalent to

$$\rho_b = \frac{d\bar{C}}{dI} - \xi \geq (1 - w)\rho \left(1 + \frac{t\eta}{1 - t}\right) + wr \left(1 + \frac{t\eta}{1 - t}\right) \equiv \rho_b^*, \quad (8)$$

where

$$w = \frac{\frac{\xi t\eta}{1 - t(1 - \eta)} - \mu}{r + \xi - \mu}. \quad (9)$$

The expression $1 + t\eta/(1 - t)$ is recognized from MM63, and is found in equation (10) below. This reflects the basic distortion in the before-tax rate of return, caused by the tax system. If ρ and r are equal, the expression equals the relative tax wedge, ρ_b^*/ρ . The required rate is thus a weighted average of two adjusted versions of ρ and r , respectively, both adjusted to reflect the tax wedge.

The weight w is not always between zero and unity. In the model with depreciation, but without reinvestment, with $0 = \mu < \xi$, we have $w \in [0, 1)$, so that the required rate of return is between $\rho[1 + t\eta/(1 - t)]$ and $r[1 + t\eta/(1 - t)]$. But in the LA73 model, with $0 < \mu = \xi$, the weight is negative, so that the required rate exceeds $\rho[1 + t\eta/(1 - t)]$. This suggests (which will be even more clear below) that dropping reinvestment from the general model turns the LA73 result around: The required expected rate of return is then decreasing in the depreciation rate, not increasing.

The weight itself can be interpreted as the present discounted value of an exponentially declining perpetual stream, thus the denominator $r + \xi - \mu$. The numerator is perhaps easiest to interpret in the case with $\eta = 1$, when the expression is simplified to $\xi t - \mu$. The first term is the tax value of the depreciation allowance, while the second is the outlay for reinvestment, both as fractions of capital. The weight on the risk free component in (8) is thus the present value of the risk free cash flow stream. This can be exactly zero if (assuming $\eta = 1$) reinvestment is equal (in absolute value) to the tax value of depreciation, $\mu = \xi t$. If $\mu/\xi < t$, then the weight is positive, while $\mu/\xi > t$ makes it negative.

Leverage makes this more complicated since reinvestment carries an additional tax benefit of debt. This concludes the interpretation of the before-tax required expected rates of return.

Setting $0 = \mu = \xi$ gives

$$\rho_b^* \rightarrow \rho \frac{1-t(1-\eta)}{1-t} \equiv \rho \left(1 + \frac{t\eta}{1-t}\right) \equiv A, \quad (10)$$

which is the expression given in MM63, equation (7), p. 440. In case there is no debt, the relative tax wedge A/ρ will equal $1/(1-t)$. But interest deductibility implies that the wedge is reduced towards unity as borrowing increases. Setting $0 < \mu = \xi$ gives the expression in LA73, their equation (10),

$$B \equiv \rho \frac{1-t(1-\eta)}{1-t} + \xi \frac{\rho-r}{r}. \quad (11)$$

Setting $0 = \mu < \xi$ gives the expression suggested here, for the case with no reinvestment,

$$\rho_b^* = \rho \frac{1-t \left(1 - \eta \frac{r}{\xi+r}\right)}{1-t} + \frac{r\xi t\eta}{(r+\xi)(1-t)}. \quad (12)$$

For this last model, the net effect of depreciation is given by

$$\frac{\partial \rho_b^*}{\partial \xi} = \frac{tr\eta(r-\rho)}{(1-t)(\xi+r)^2}. \quad (13)$$

This is negative when $r < \rho$. Thus we have obtained an effect with the opposite sign from that in LA73, who find that a higher rate of depreciation increases the before-tax required expected rate of return. The main reason for the difference is the omission here of the yearly investment outlay. In LA73 this has the same effect as operating leverage. Removing it means lower risk in the net cash flow.

They illustrate the importance of their result by a table, their table 1 on p. 692, which shows the ratio of this B to the MM63 expression, denoted A in (10) above. They use the values $t = 0.5$, $r = 0.05$, $\eta = 0.5$ throughout. The similar ratio of ρ_b^* (from (12)) to A is shown in the right-hand half of table I. The left-hand half reproduces parts of the table in LA73 for comparison.

TABLE

The table clearly illustrates the strong differences in results. The very high required rates of return implied by the LA73 assumptions are replaced by rates which are below the MM63 values, but which do not differ as much. The difference is not insignificant, however. For a realistic case of $\rho = 0.12$ and $\xi = 0.2$, the numbers are $\rho_b^* = 0.152$ and $A = 0.18$. A lower tax rate of $t = 0.34$, not shown in the table, gives $\rho_b^* = 0.136$ and $A = 0.151$. A higher equity share, η , would lower the ratio, i.e., increase the difference between the rates.

I HERE.

Next we consider the after-tax required expected rates of return. For the sake of readability, we skip the general model and consider only the case with no reinvestment. We then compare the results with those of MM63 and LA73. It should be noted that they will here be expressed as marginal rates of return, while formally, both MM63 and LA73 express them as average rates of return. They are really also requirements on marginal rates of return, however, since the tax system is linear, and the average rates they derive, determine projects with exactly zero net value. This follows since in MM63, eq. (11.c), and in LA73, eq. (7), the perpetual rates of return are expressed as \bar{X}^t/V_ℓ , not as some arbitrary \bar{X}^t/I .

Set now $\mu = 0$. The WACC is based on an after-tax cash flow definition which excludes the tax benefit of debt. It is that rate ρ_w which solves

$$\sum_{\tau=1}^{\infty} \frac{[d\bar{C}(1-t) + tdK](1-\xi)^{\tau-1}}{(1+\rho_w)^\tau} = dI. \quad (14)$$

The solution is $\rho_w = (1-t)d\bar{C}/dI - (1-t)\xi = (1-t)\rho_b$.

The last of these equations is the reason why we present the WACC first of the after-tax requirements: There is a very simple relationship between ρ_w and ρ_b . This relative tax wedge, ρ_b/ρ_w , only depends on the tax rate, as it is equal to $1/(1-t)$.

Since the tax advantage of debt is not included in the numerator in (14), it will instead (for a given marginal project, i.e., given $d\bar{C}$ and $dK (= \xi dI)$) affect the denominator. The requirement given by (6) can be reformulated. The resulting minimum value of ρ_w will be denoted ρ_w^* . Inequality (6) is equivalent to

$$\rho_w = (1-t)\frac{d\bar{C}}{dI} + t\xi - \xi \geq \rho(1-t) + \frac{(\xi + \rho)tr\eta}{\xi + r} \equiv \rho_w^*. \quad (15)$$

As a control, we observe that if $\xi \rightarrow 0$, then

$$\rho_w^* \rightarrow \rho[1 - t(1 - \eta)] \quad (16)$$

which is the value given in MM63, equation (8), p. 442. Compared with their result, the last term (containing η) is reduced when $\xi > 0$, since by assumption, $\rho > r$. From (13) and $\rho_w = (1-t)\rho_b$, it is clear that $\partial\rho_w^*/\partial\xi < 0$. In this model, contrary to the model of LA73, the after-tax cost of capital is decreasing in the depreciation rate. This is true when it is

defined as the WACC, and next we show that it also holds for other concepts of after-tax costs of capital.

The after-tax expected rate of return to equity and debt is based on an after-tax cash flow definition which includes the tax benefit of debt. It is that rate ρ_a which solves

$$\sum_{\tau=1}^{\infty} \frac{[d\bar{C}(1-t) + t(dK + dR)](1-\xi)^{\tau-1}}{(1+\rho_a)^\tau} = dI. \quad (17)$$

The solution is $\rho_a = (1-t)d\bar{C}/dI + t(\xi + r(1-\eta)) - \xi$.

The requirement given by (6) can again be reformulated. The resulting minimum value of ρ_a will be denoted ρ_a^* . Inequality (6) is equivalent to

$$\rho_a = (1-t)\frac{d\bar{C}}{dI} + t(\xi + r(1-\eta)) - \xi \geq \rho - (\rho - r)t\frac{\xi + (1-\eta)r}{\xi + r} \equiv \rho_a^*. \quad (18)$$

As a control, we observe that if $\xi \rightarrow 0$, then

$$\rho_a^* \rightarrow \rho - (\rho - r)t(1-\eta), \quad (19)$$

which is the value given in MM63, equation (11.c), p. 439. Compared with their result, there are two opposing new effects when $\xi > 0$ (assuming also $\eta < 1$). First their expression is increased by multiplying $1 - \eta$ with the fraction $r/(\xi + r)$. But then another term is subtracted.

It turns out that the second effect is the greater in absolute value. The net result of depreciation is given by

$$\frac{\partial \rho_a^*}{\partial \xi} = \frac{tr\eta(r - \rho)}{(\xi + r)^2}. \quad (20)$$

This is negative when $r < \rho$. Thus we have obtained an effect with the opposite sign from that in LA73, who find that a higher rate of depreciation increases the after-tax required expected rate of return to equity and debt, their expression (7), p. 690.

Next we consider the expected rate of return to equity alone.

The after-tax expected rate of return to equity is that rate ρ_e which solves

$$\sum_{\tau=1}^{\infty} \frac{[d\bar{C}(1-t) + t(dK + dR) - (\xi + r)dD](1-\xi)^{\tau-1}}{(1+\rho_e)^\tau} = \eta dI. \quad (21)$$

The solution is $\rho_e = [(1-t)d\bar{C}/dI + t(\xi + r(1-\eta)) - (\xi + r)(1-\eta)]/\eta - \xi$.

The requirement given by (6) can again be reformulated. The resulting minimum value of ρ_e will be denoted ρ_e^* . Inequality (6) is equivalent to

$$\rho_e = \frac{(1-t)\frac{d\bar{C}}{dt} + t(\xi + r(1-\eta)) - (\xi + r)(1-\eta)}{\eta} - \xi \geq \rho + (\rho - r)(1-t)\frac{1-\eta}{\eta} - t\xi\frac{\rho - r}{\xi + r} \equiv \rho_e^*. \quad (22)$$

As a control, we observe that if $\xi \rightarrow 0$, then

$$\rho_e^* \rightarrow \rho + (\rho - r)(1-t)\frac{1-\eta}{\eta}, \quad (23)$$

which is the value given in MM63, equation (12.c), p. 439. Compared with their result, an extra term is subtracted when $\xi > 0$.

The net result of depreciation is given by

$$\frac{\partial \rho_e^*}{\partial \xi} = \frac{tr(r - \rho)}{(\xi + r)^2}. \quad (24)$$

This is negative when $r < \rho$. Thus we have obtained an effect with the opposite sign from that in LA73, who find that a higher rate of depreciation increases the after-tax required expected rate of return to equity, their expression (9), p. 690.

From (22) we also find the familiar WACC formula,

$$\rho_w^* = \eta\rho_e^* + (1-\eta)r(1-t). \quad (25)$$

This verifies that our definitions of ρ_w and ρ_e are the conventional ones.

III Discussion

The results show that in the model presented, the effect of tax depreciation is quite different from what LA73 suggested. The difference is due to a different description of the decision for which the required expected rate of return will be used. There is no objection here to using the results in LA73 for a situation in which commitment to perpetual reinvestment is realistic. However, mistakes will be made if the required rate suggested by them is applied to a decision without such a commitment. Too many projects will be rejected, since the requirement found in the present paper is lower. Their required rate should only be applied if (i) cash flows are specified with perpetual reinvestment, and (ii) the

commitment to perpetual reinvestment is the only choice, or can somehow be shown to be the best choice.

If one first considers an investment project like in this paper, and finds that the expected rate of return is satisfactory, and one then realizes that there are *options* to reinvest, this can only make the project more attractive. Options to reinvest is something quite different from commitment to reinvest.

The results may have important implications for the use of market data in capital budgeting. If a potential investment project has an operating cash flow with the same risk as some existing firm, it is often recommended that the market's required rate of return for that firm be used to find the cost of capital applicable in the investment decision. The procedure of unlevering the discount rate (or the beta in a CAPM setting) is well known. This is reflected in the difference between ρ and ρ_e^* as it appears in (22) above.

The results show that another adjustment may be necessary if the new project uses different kinds of assets with different depreciation rates. This idea was implied also in LA73, but their assumptions are not applicable for most projects.

An adjustment is also needed if the new project uses assets with the same depreciation rate, but is subject to a different tax rate, e.g., in a different year, country, or sector. While it is well known that taxes affect the cost of capital in a levered firm, we show that this also happens for an unlevered firm when assets are depreciable. These are partial equilibrium effects, assuming ρ and r are unaffected by a change in t . From the inequalities (12) and (18) above, we find:

$$\frac{\partial \rho_b^*}{\partial t} = \frac{(\xi + \rho)r\eta}{(\xi + r)(1 - t)^2} > 0, \quad (26)$$

and

$$\frac{\partial \rho_a^*}{\partial t} = -\frac{[\xi + r(1 - \eta)](\rho - r)}{\xi + r} < 0. \quad (27)$$

That the first of these is positive, is not surprising. More will be said about the tax wedge below. That the second is negative, is often ascribed to leverage. But here we observe that even with $\eta = 1$, i.e., no leverage, the negative effect remains as long as $\xi > 0$. Again this can be explained as analogous to operating leverage, but this time with a negative sign.

The effect of depreciation on the expected rate of return after corporate taxes is something quite different from the effect before corporate taxes. The requirement after corporate taxes is taken directly from the capital markets. The reason why this requirement depends on the tax depreciation schedule, is that the depreciation rate alters the risk characteristic of the after-tax cash flows. Adding a risk free cash flow each period (namely, the tax value of the depreciation deduction) has the opposite effect of borrowing. In the CAPM jargon it decreases the systematic risk of the cash flow.

The requirement before tax, however, shows how much the tax system distorts the investment decision as compared with a no-tax situation. This was also found to be negatively affected by the depreciation rate. We can now find the effect of the depreciation rate on the absolute tax wedge, $\rho_b^* - \rho_a^*$. From (13) and (20) we get

$$\frac{\partial(\rho_b^* - \rho_a^*)}{\partial\xi} = \frac{t^2 r \eta (r - \rho)}{1 - t} < 0. \quad (28)$$

The absolute tax wedge is decreasing in ξ , as the before-tax requirement decreases faster than the after-tax requirement.

Like in LA73 and Bradford (1975), the analysis so far does not show the separate effect of regulating the tax depreciation schedule. It is based on the simplifying assumption that this is always equal to economic depreciation. While this is not always realistic, it was done so in order to obtain simple analytical results. However, for the before-tax required rate ρ_b^* , the separate effect of tax depreciation is not difficult to derive if we are willing to assume that debt is repaid at the same exponential rate as tax depreciation. This yields a tractable solution. In Assumption 5, $K = \xi I$ should be replaced by $K = \nu I$. Assumption 4 should be replaced with:

- 4a From period $\tau = 1$ onwards the two risk free cash flow elements — the tax value of the depreciation deduction, and the debt service — both decline at the same exponential rate, $\nu \in [0, 1)$. This rate is thus both the rate of tax depreciation and the rate of repayment of the (remaining) debt. The expected operating cash flow, however, declines at the rate $\xi \in [0, 1)$.

Of course, if $\xi > \nu$, the assumption that the firm is always in position to pay debt service and taxes relies even more heavily on income from other sources than the marginal project considered here.

Instead of (5), the value of the firm is now

$$V = \sum_{\tau=1}^{\infty} \left[\frac{(1-t)\bar{C}(1-\xi)^{\tau-1}}{(1+\rho)^{\tau}} + \frac{t(K+R)(1-\nu)^{\tau-1}}{(1+r)^{\tau}} \right] = \frac{(1-t)\bar{C}}{\rho+\xi} + \frac{t(K+R)}{r+\nu}. \quad (29)$$

Instead of (6), the criterion for accepting an (additional) investment is now

$$\frac{dV}{dI} = \frac{1-t}{\rho+\xi} \frac{d\bar{C}}{dI} + \frac{t(\nu+r(1-\eta))}{\nu+r} \geq 1. \quad (30)$$

Instead of (12), the minimum value of ρ_b is now ρ_b^* given by

$$\rho_b = \frac{d\bar{C}}{dI} - \xi \geq \rho \frac{1-t \left(1 - \eta \frac{r}{\nu+r}\right)}{1-t} + \frac{r\xi t\eta}{(\nu+r)(1-t)} \equiv \rho_b^*. \quad (31)$$

We can now find the effects on ρ_b^* of separate changes in ξ and ν . They are,

$$\frac{\partial \rho_b^*}{\partial \xi} = \frac{tr\eta}{(1-t)(\nu+r)} > 0, \quad (32)$$

and

$$\frac{\partial \rho_b^*}{\partial \nu} = \frac{-r\eta t(\rho+\xi)}{(1-t)(\nu+r)^2} < 0. \quad (33)$$

The signs of these could be seen fairly directly from (30). A higher ξ means that the operating cash flow decreases more rapidly. If ν is fixed, the operating cash flow must then start at a higher level (relative to dI , i.e., a higher $d\bar{C}/dI$) in order for the first term to exceed $1 - t(\nu+r(1-\eta))/(\nu+r)$. A higher $d\bar{C}/dI$ means a higher ρ_b^* . A higher ν , on the other hand, means that the value of the fraction $t(\nu+r(1-\eta))/(\nu+r)$ increases, and with ξ fixed this implies a lower ρ_b^* .

As a control, we can let $\xi = \nu$ and add the two effects. A small change in ξ alone, and then an equally small change in ν , amounts to the small simultaneous change considered in equation (13). True enough, the two partial effects in (32) and (33) add up to the effect in (13), which is negative. A partial increase in ξ will increase ρ_b^* , a partial increase in ν goes in the opposite direction, and the latter effect is the stronger if the two partial increases are of equal size.

In the concluding remarks of LA73 (p. 693), we find, “The higher the annual depreciation figure, the lower the firm’s value. At first glance, this result seems paradoxical, since the higher depreciation, the higher the certain tax benefit tK . However, one must recall that the greater the depreciation, the greater the replacement outlay that the firm is committed to make in order to assure a perpetual income stream.”

Two comments should be made. The first statement, about the firm’s value, is somewhat misleading. The whole analysis concerns a marginal investment, which has a net value of zero. If, for instance, the tax system is changed in an unfavorable direction, a partial effect of this may be to decrease the firm’s value. But the total effect in this kind of analysis is to increase the required rate of return before taxes so much that the net value of the marginal project is still zero.

The final statement in the quotation illustrates the dependence of LA73 on the assumption of commitment to perpetual reinvestment. This is the assumption which explains the seemingly paradoxical result. The assumption was not challenged by Bradford (1975).

IV Conclusion

When introducing depreciable assets into the model of Modigliani and Miller (1958, 1963), one must carefully consider whether to maintain their assumption of perpetual cash flows with constant expected values. For the purpose of investment decisions, the assumption implies that the firm commits to undertake reinvestments forever. This is unrealistic in most cases.

Without the assumption of reinvestment, the effect of depreciation on the cost of capital is reversed. This is true for the cost of capital both before and after corporate taxes. The strong positive effect found by Levy and Arditti (1973) is replaced by a weaker, but still significant, negative effect.

The findings have implications for the use of capital market data for deriving the cost of capital for investment decisions. Not only should observed rates of return be unlevered, but they should be adjusted for different depreciation rates and tax rates.

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Denominator:		A , the expression from MM63										
Numerator:		B of LA73					ρ_b^* of this paper					
	ξ	0.00	0.20	0.40	0.60	0.80	1.00	0.20	0.40	0.60	0.80	1.00
ρ												
0.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.06	1.00	1.44	1.89	2.33	2.78	3.22	3.66	0.96	0.95	0.95	0.95	0.95
0.07	1.00	1.76	2.52	3.29	4.05	4.81	5.57	0.92	0.92	0.91	0.91	0.91
0.08	1.00	2.00	3.00	4.00	5.00	6.00	7.00	0.90	0.89	0.88	0.88	0.88
0.09	1.00	2.19	3.37	4.56	5.74	6.93	8.11	0.88	0.87	0.86	0.86	0.86
0.10	1.00	2.33	3.67	5.00	6.33	7.67	9.00	0.87	0.85	0.85	0.84	0.84
0.11	1.00	2.45	3.91	5.36	6.82	8.27	9.72	0.85	0.84	0.83	0.83	0.83
0.12	1.00	2.56	4.11	5.67	7.22	8.78	10.33	0.84	0.83	0.82	0.82	0.81
0.13	1.00	2.64	4.28	5.92	7.56	9.21	10.97	0.84	0.82	0.81	0.81	0.80
0.14	1.00	2.71	4.43	6.14	7.86	9.57	11.61	0.83	0.81	0.80	0.80	0.80
0.15	1.00	2.78	4.56	6.33	8.11	9.89	12.25	0.82	0.80	0.79	0.79	0.79
0.20	1.00	3.00	5.00	7.00	9.00	11.00	15.00	0.80	0.78	0.77	0.76	0.76

Table I: Ratios of required expected rates of return before tax. Case with depreciable assets divided by case without. Reinvestment (LA73) or no reinvestment (this paper). 72 different combinations of ρ and ξ . Other parameters: $t = 0.5$, $r = 0.05$, $\eta = 0.5$.