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**RANDOM WALK OR MEAN
REVERSION: THE DANISH STOCK
MARKET SINCE WORLD WAR I**

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SINCE WORLD WAR I***

By

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Abstract:

This paper contributes to the growing literature on mean reversion in stock markets by examining a newly constructed Danish data set for the period 1922-95. Variance ratio tests clearly reject the random walk hypothesis at the 2-year horizon, that is, the riskiness of a 2-year investment is significantly less than twice the risk of a 1-year investment. Variance ratio tests for 3- and 4-year horizons are not significant under conventional significance levels, whereas autocorrelation tests of the joint hypothesis that there is departure from random walk at all horizons tend to reject the random walk hypothesis and support the mean reversion hypothesis.

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1. Introduction.

The purpose of this paper is to test whether the Danish stock market behaves like a random walk vis-à-vis the hypothesis that the market has a tendency to mean revert. In order to motivate this, let us consider the recent behaviour of the Danish market index. Like in so many other countries, the market has been on an upward trend in recent years. Thus, the (compounded) stock index went up by 6.3 per cent in 1995, by 30.5 per cent in 1996 and by 44.5 per cent in 1997. Altogether, that is more than a 80 per cent increase over three years and that is in times with low inflation. However, according to the random walk hypothesis this historical piece of evidence should not influence the forecast of the next years index change, that is, the expected change is independent of past movements. By contrast, the mean reversion hypothesis asserts that the market after a number of good years is likely to exhibit less exciting and perhaps even depressing returns depending of course on the exact form of the mean reversion process.¹

According to the random walk hypothesis, the change in the compounded stock index is random, which is tantamount to saying that the 1-period returns are uncorrelated over time. Due to that property, the variance of the k-period return is k times the variance of the 1-period return, and it is this property that is applied in the variance ratio testing methodology due to Cochrane (1988). By using this test or a closely related methodology, the random walk hypothesis has been tested for a number of countries using long horizon returns, in most cases 1- to 10-year returns, see Fama and French (1988), Lo and MacKinlay (1988) and Poterba and Summers (1988). The bottom line of these papers is that stock markets do not seem to behave like a random walk. There is mild support for the mean reversion hypothesis over longer time horizons; “mild” because it is not always possible to get significant test results as noted by Poterba and Summers, but that is hardly surprising given that a very substantial deviation from the random walk hypothesis would be implausible because future returns would then be highly predictable, using only the information in past returns.

¹ The research underlying this paper was completed before markets started to fall. The recent stock market decline may be seen as yet another observation that supports the main thesis of this paper.

More recently, Kim, Nelson and Startz (1991) have argued that the mean reverting behaviour in US stock prices reported in previous papers is mainly a pre World War II phenomenon as indicated also in Fama and French (1988), and is not particularly relevant anymore. Campbell, Lo and MacKinlay (1997) conclude their survey chapter by arguing that upon a closer examination of the literature there is little evidence for mean reversion in long horizon returns which, however, may be a symptom of small sample sizes rather than conclusive evidence against mean reversion. Their conclusion is influenced by the work of Richardson and Stock (1989) who show that the traditional variance ratio tests may be misleading when the return horizon divided by the sample period is not a small number. Due to that criticism this paper only reports the results for 2, 3 and 4 year returns and not for longer horizon returns as it was common earlier.

This paper contributes to this growing literature by examining the Danish stock returns since World War I using a newly constructed dataset, see Nielsen and Risager (1998). To set the scene, section 2 briefly outlines the random walk hypothesis and the variance ratio methodology. Section 3 presents the test results. Section 4 adds complimentary material by examining autocorrelation properties of the returns. Section 5 concludes the paper.

2. Random Walk and Variance Ratio Test.

The compounded return denoted X_{t+k} of a k-year investment in the market portfolio of stocks at time t is given by (1), where the R_s are the simple 1-year returns consisting of a dividend yield and a capital gain component. Under the assumption that the log of one plus the annual returns equal a constant μ plus a White Noise random term ϵ as shown by (2), it is straightforward to show that the log of the k-period return (henceforth just the k-period return) denoted x_{t+k} equals k times the expected annual return plus the k innovation terms, see (3). Moreover, the k-period return follows a random walk with drift μ , see (4).

$$X_{t+k} = (1+R_t)(1+R_{t+1})\dots(1+R_{t+k}) \quad (1)$$

$$\ln(1+R_t) = \mu + \epsilon_t, \quad E(\epsilon_t) = 0, \quad \text{cov}(\epsilon_t, \epsilon_s) = 0 \quad t \neq s \\ = \sigma^2 \quad t = s \quad (2)$$

$$x_{t+k} = k\mu + \epsilon_t + \dots + \epsilon_{t+k} \quad (3)$$

$$x_{t+k} = \mu + x_{t+k-1} + \epsilon_{t+k} \quad (4)$$

By using (3) it can be shown that the expected k-period return is k times the annual mean return, whereas the variance is k times the annual variance, see (5).

$$E(x_{t+k}) = k\mu, \quad \text{Var}(x_{t+k}) = k\sigma^2 \quad (5)$$

The variance ratio methodology tests the hypothesis that the variance of multiperiod returns increases linearly with time. The variance of the k-year return to the variance of the 1-year return divided by k is unity under the random walk. The variance ratio is significantly below one under mean reversion, and above one under mean aversion, see (6).

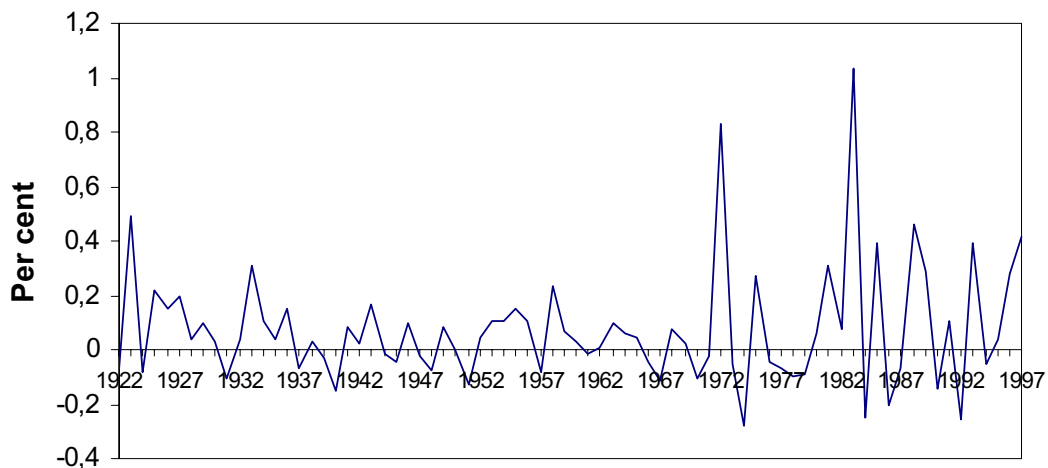
$$VR(k) = \frac{\text{Var}(x_{t+k})/k}{\text{Var}(x_t)} \begin{array}{l} =1 \text{ under random walk} \\ <1 \text{ under mean reversion} \\ >1 \text{ under mean aversion} \end{array} \quad (6)$$

3. Variance Ratio Tests.

Having sketched the theory we now proceed to test the random walk hypothesis. The sample $\{x\}_{t=0}^T$ runs from 1922($t=0$) to 1995($t=T$), that is, we have 74 ($T+1$) yearly returns. The variance estimator is developed by Cochrane (1988) and is based on overlapping multyear returns, see footnote 1 in Table 1. This estimator corrects for small sample bias, that is, the problem that the variance of the k-year return goes to zero as k approaches T. The variance ratios can, however, be biased when the return horizon is large relative to the sample period, see Richardson and Stock (1989) who develop an alternative statistical approach. Following the alternative approach in Richardson and Stock (1989), the expected variance ratio equals $(1-\delta)^2$ even under the random walk null hypothesis, where δ is the return horizon relative to the sample period. Thus for $\delta=2/74, 3/74, 4/74$, the expected variance ratio is 0.95, 0.92, 0.90, respectively. For a 10-year horizon the expected variance ratio equals 0.75 and hence is far from unity. This concern explains why we only report tests for 2, 3 and 4-year horizons.

Prior to reporting the results it is useful to look at the annual stock returns, which after all provide the basis for the tests, see Figure 1. It is interesting to observe that the process seems to be characterized by negative serial correlation, which means that good years tend to be followed by bad years, and vice versa. This phenomenon seems to be particularly strong towards the end of the sample. Thus, the very high return in 1972 is followed by a poor return in 1973 and particular in 1974. Following the poor 1974 we have a high return in 1975, which immediately is followed by several bad years. There is a strong recovery in the beginning of the 1980s, and 1983 is the year with the highest return recorded so far in the Danish stock market history, but this year is immediately followed by a very bad 1984, which again is followed by a bullish 1985. This tendency for the market to jump up and down continues throughout the sample period and is of course an important motivator for formally testing the mean reversion hypothesis.

Figure 1: 1-Year Real Stock Return in Denmark, 1922-97



The first row in Table 1 gives the variances at different investment horizons. The variance of the 1-year real return equals 3.21 per cent, the variance of the 2-year return equals 4.65 and so forth. Obviously, the variances do not increase linearly with time. The variance ratios are reported in line 2, and they are all below unity. The third row presents the test statistic, which is asymptotically standard normal distributed and the corresponding prob-values are given in

line 4. At the 2-year horizon there is a clear rejection of the random walk, that is, we can reject the hypothesis at the 5 per cent level. We also reject at the 3-year horizon, whereas results are less clear at the 4-year horizon.

Table 1: Variance Ratio Tests for Danish Real Stock Returns, 1922-95

	Investment Horizon (years)			
	1	2	3	4
1) Variance ($\hat{\sigma}_k^2$)	3.21	4.65	6.41	7.99
Variance Ratio $\left(\frac{\hat{\sigma}_k^2/k}{\hat{\sigma}_1^2} \right)$	-	0.72	0.66	0.62
2) Variance Ratio Test	-	-2.36	-1.92	-1.73
Prob Value	-	0.02	0.05	0.08
3) Variance Ratio Test under Heteroscedasticity	-	-1.99	-1.56	-1.39
Prob Value	-	0.05	0.12	0.16

Notes: 1)
$$\hat{\sigma}_k^2 = \frac{T}{(T-k)(T-k+1)} \sum_{t=k}^T \left(x_t - x_{t-k} - \frac{k}{T} (x_T - x_0) \right)^2$$

2)
$$Z(k) = \left[\frac{\hat{\sigma}_k^2/k}{\hat{\sigma}_1^2} - 1 \right] \left[\frac{2(2k-1)(k-1)}{3Tk^2/k} \right]^{-1/2}$$

3)
$$Z_H(k) = \left[\frac{\hat{\sigma}_k^2/k}{\hat{\sigma}_k^2} - 1 \right] \left[\sum_{j=1}^{k-1} \left[\frac{2(k-j)}{k} \right]^2 \frac{\sum_{t=j+1}^T a_{0t} a_{jt}}{\left(\sum_{t=1}^T a_{0t} \right)^2} \right]^{-1/2}, a_{jt} = \left(x_{t-j} - x_{t-j-1} - \frac{1}{T} (x_T - x_0) \right)^2$$

So far it has been assumed that the distribution of the returns is constant over time, but Figure 1 indicates that the variability of the 1-year returns seems to have changed, that is, there is evidence of increasing volatility towards the end of the sample. This motivates tests that allow for heteroscedasticity. We follow the approach taken in Lo and MacKinlay (1988). Tests that allow for general forms of heteroscedasticity are given in line 5. It now becomes more difficult to reject the random walk hypothesis. However, there is still rejection at the 2-year horizon using the conventional 5 per cent significance level. The result that the 2-year variance is significantly less than twice the 1-year variance is tantamount to saying that there is significant negative first order serial correlation, which is something we shall return to.

The variance ratio tests for the nominal returns are reported in Table 2. Again, there is a tendency to mean reversion, but the results are not statistically significant. The stronger mean reversion in real returns than in nominal returns is likely to reflect slow adjustment to inflation shocks. Thus, in the short term stock returns do not provide a perfect inflation hedge, whereas stocks are a much better hedge in the longer term, see also the recent study by Olesen (1998) using the same data for Denmark. A sudden increase (fall) in inflation is therefore followed by a drop (rise) in the real return in the short term and subsequently by an increase (decline) to the level that normally applies. That is in accordance with the mean reversion hypothesis.

Table 2: Variance Ratio Tests for Danish Nominal Stock Returns, 1922-95

	Investment Horizon (Years)			
	1	2	3	4
¹⁾ Variance ($\hat{\sigma}_k^2$)	3.19	4.79	6.85	8.56
Variance Ratio $\left(\frac{\hat{\sigma}_k^2/k}{\hat{\sigma}_k^2} \right)$	-	0.75	0.72	0.67
²⁾ Variance Ratio Test	-	-2.12	-1.63	-1.50
Prob Value	-	0.04	0.11	0.14
³⁾ Variance Ratio Test under Heteroscedasticity	-	-1.83	-1.34	-1.21
Prob Value	-	0.07	0.18	0.23

Notes: ¹⁾ see note 1) to Table 1

²⁾ see note 2) to Table 1

³⁾ see note 3) to Table 1

Kim, Nelson and Startz (1991) found that the tendency to mean reversion in the US is entirely due to the pre World War II period, which implies that the hypothesis is of little relevance today. We examine this issue by only using the sample 1946-95. Line 6 in Table 3 presents the test statistics also corrected for heteroscedasticity. The results for the 2-year horizon show that we can only reject the random walk at the 10 per cent level. At first glance this finding seems to weaken the mean reversion hypothesis, but the results from simple correlation tests stand in sharp contrast to the outcome of variance ratio tests, cf. below. Moreover, it should also be noted that it is harder to get statistical significant results in small samples as can be seen by inspecting the test statistic, and we suspect that the small sample plays a role for the result.

Table 3: Variance Ratio Tests for Danish Yearly Data, 1946-95

	Investment Horizon (Years)			
	1	2	3	4
¹⁾ Variance ($\hat{\sigma}_k^2$)	4.03	5.87	7.85	9.57
Variance Ratio $\left(\frac{\hat{\sigma}_k^2 / k}{\hat{\sigma}_k^2} \right)$	-	0.73	0.65	0.59
²⁾ Variance Ratio Test	-	-1.90	-1.65	-1.52
Prob Value	-	0.066	0.102	0.126
³⁾ Variance Ratio Test under Heteroscedasticity	-	-1.69	-1.40	-1.28
Prob Value	-	0.096	0.150	0.179

Notes: ¹⁾ see note 1) to Table 1

²⁾ see note 2) to Table 1

³⁾ see note 3) to Table 1

4. Autocorrelation Tests.

The above results rejected the random walk at the 2-year horizon. The results were less clear at longer horizons. Due to this mixture of findings it is desirable to test the joint hypothesis that there is departure from random walk at all horizons. This is accomplished by the Ljung-Box autocorrelation test designed for small samples. The null hypothesis is that all autocorrelation coefficients are zero, whereas the alternative is departure from zero autocorrelation at all lags. Note that to apply the Ljung-Box test we do not need to assume normality of returns. We have looked at the serial correlation pattern when there are 12, 10, 8, 6, 5, 4, 3, 2, 1 lags. The results show that we reject the random walk in about two third of the cases, see Table 4.

Table 4: Autocorrelation Tests of 1-Year Real Returns¹⁾

LB-Test With 74 Observations	
12	25.05128**
10	23.77147**
8	14.15947*
6	7.193352
5	6.880049
4	6.758254
3	6.459494*
2	6.39740**
1	6.33033**

¹⁾Ljung-Box statistic = $N(N + 2) \sum_{k=1}^m \frac{\rho^2(k)}{N - k}$, where N is sample, $\rho(k)$ is autocorrelation

of lag k. ** means significant at the 5% level, i.e. rejection of the Random Walk.

* means significant at the 10% level.

Further insight can be obtained by estimating AR-processes. We begin by estimating an AR(12) for the whole sample period. The process is subsequently simplified by throwing the variable out that is least significant according to the t-statistic; the model is then reestimated and the variable that is least significant is then omitted and so forth. To apply the t-test, the residual has to be normal and that is why we include dummies for 1972 and for 1983, which are highly unusual in the Danish market history, see Figure 1 and Nielsen and Risager (1998). Next, we estimate an AR(10) and gradually simplify until there are only significant variables in the equation. By repeating this procedure also for AR(8), AR(6), AR(4), AR(2) we arrive at the result that in all cases, except the AR(10) case, there is significant negative first order serial correlation, see (7) below.² That result is of course in line with the variance ratio rejection at the 2-year horizon. The interpretation is that news to the market often gives rise to overshooting, which afterwards is corrected by the market.

²In the AR(10) case we end up with an equation with negative serial correlation at lag 1 that is offset by positive serial correlation at lag 10.

$$\ln(1 + R)_t = 0.047 - 0.275\ln(1 + R)_{t-1} + 0.55D72 + 0.68D83 \quad (7)$$

(0.017) (0.093) (0.14) (0.14)

Sample 1923-95, $R^2 = 0.41$, Normality $\text{CHI}^2(2) = 2.955$

As mentioned earlier, there is apparently much less mean reversion after World War II in the US. When the AR(12) is estimated only for the period 1946-95, we end up with much stronger negative serial correlation in the Danish case (not shown - to save space). However, 12 lags are quite demanding and we have therefore estimated AR(6) processes for 1928-55, 28-65, 28-75, 28-85, 28-95. It turns out that the strong negative serial correlation is a phenomenon that is statistically significant in particular from the 1970s and onwards. Moreover, the parsimonious specification since the mid 1970s is an AR(1) with significant negative serial correlation. Table 5 reports the parsimonious equations.

Table 5: Parsimonious Real Return Equations From AR(6) Processes

Sample	Most significant explanatory variables
1928-55	- 0.227 $\ln(1+R)_{t-4}$ (1.32)
1928-65	- 0.159 $\ln(1+R)_{t-4}$ (1.07)
1928-75	- 0.362 $\ln(1+R)_{t-2}$ + 0.264 $\ln(1+R)_{t-3}$ (3.23) (2.42)
1928-85	- 0.227 $\ln(1+R)_{t-1}$ (2.39)
1928-95	- 0.265 $\ln(1+R)_{t-1}$ (2.85)

Note: The AR(6) process is sequentially simplified by eliminating the insignificant variables. We only report those variables that are nearly significant (in case none, we report the model which we end up with). Constant and dummy variables are not reported. T-statistics are in brackets.

5. Conclusions

This paper contributes to the growing literature on mean reversion in stock markets by examining a newly constructed Danish data set for the period 1922-95. The variance ratio tests show statistically significant mean reversion at the second year of the investment horizon, that is, the riskiness of a 2-year investment is significantly less than twice the risk of a 1-year investment. Hence, at the 2-year horizon the random walk hypothesis is clearly rejected. Results for 3- and 4-year horizons are in line with mean reverting behaviour, but they are not statistically significant under conventional significance levels. Whether this is evidence against mean reversion and support for the random walk hypothesis or just a reflection of a small sample size is something we cannot tell. However, it should be noted that simple autocorrelation tests of the joint hypothesis that there are departures from random walk at all horizons reject the random walk hypothesis in the majority of cases.

Moreover, in contrast to previous results for the US, the strong negative first order serial correlation in real returns does not hinge on pre-World War II data as it apparently does in the US, see Kim, Nelson and Startz (1991). On the contrary, the serial correlation tests show that mean reversion in the Danish stock market is a stronger phenomenon in modern times³. One possible interpretation of the negative serial correlation in the data is that market overreacts to news, that is, following the first dramatic reaction to a shock, the market falls back to normal. This view is consistent with the permanent/transitory components model, which basically says that stock markets are driven by a fundamental component that reflects the efficient market price and behaves like a random walk, whereas there is also a temporary component that captures deviations from efficiency and this component reverts to something that is close to zero in the long term.

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³The variance ratio statistics are, however, not so significant for the post War period as for the whole sample period. We suspect that this is due to the short sample period.

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