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**ON THE RELATIONSHIP BETWEEN THE
DANISH STOCK AND BOND MARKET IN
THE MEDIUM AND LONG TERM**

Jan Overgaard Olesen Ole Risager

On the Relationship between the Danish Stock and Bond Market in the Medium and Long Term

by

Jan Overgaard Olesen¹ & Ole Risager
(jo.eco@cbs.dk) (or.eco@cbs.dk)

Department of Economics
Copenhagen Business School
Solbjerg Plads 3 (5th)
DK-2000 Frederiksberg
Denmark

Abstract

This short paper studies the empirical relationship between realized stock returns and bond yields at the 5- and 10-year investment horizons, respectively. Using annual Danish data since 1927, we find that stock returns and bond yields are closely linked in the medium and long term, as we estimate strong cointegrating relations at both horizons. Hence, at the 5- and 10-year investment horizons a high bond yield tends to go hand in hand with a high stock return, and vice versa. Results show that stock returns tend to respond less than one-to-one to changes in the bond yield.

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1. Introduction

This short paper studies the relationship between Danish stock returns and government bond yields in the period 1927 to 1997. The paper is concerned with the relationship between the two markets in a medium term perspective, defined as an investment horizon of 5 years, and in a long term perspective, defined as a 10-year horizon. The specific question we want to address is whether the stock returns and bond yields at the 5- and 10-year horizon, respectively, form cointegrating relationships² ?

If the stock and the bond market are interdependent, an expected abnormal high return in one of the markets is likely to attract funds from the other market, which in turn will result in an equilibrating price increase in the first market and in a declining price in the latter market. In that case, expected returns in the first market will decline whereas returns in the other market will go up. As a result of this arbitrage process, the return gap will decline³. The question we address is whether the returns in the two markets are closely linked in the long run ? Because of the interdependence between the two markets, it is appropriate to apply a cointegration technique that allows for interdependence or simultaneity in the jargon of econometrics. We use the VAR method of Johansen (1996). Another advantage of the Johansen method is that it uses both the short and long-run information in the data to extract the long-run relationship between the two asset markets.

The paper is in 7 sections. The next section briefly describes the historical movements of the stock returns and bond yields. The third section outlines the simple theoretical arbitrage type framework that helps us organize our thoughts about the relationship between stock and bond returns. The fourth section sketches the cointegration method. In section 5 and 6, we present the empirical results. Section 7 concludes.

² For an introduction to the concept of cointegration, see Engle and Granger (1991, Chapter 1).

³ The gap between the expected returns on stocks and bonds is related but not exactly identical to the notion of a risk premium. It is not exactly a risk premium in the conventional sense (a premium relative to a riskfree asset) because bonds are also risky due to inflation and default risk and (because we use the yield-to-maturity on bonds) the reinvestment risk attached to the coupons payed before maturity.

2. A Look at the Data

Figure 1 shows the 5-year nominal stock return and the 5-year yield-to-maturity on government bonds in the period 1927-92, using overlapping annual observations⁴. All returns in this paper are annualized logarithmic returns, that is, they are defined as the logarithm to one plus the annualized return. Moreover, they are forward-looking and relate to investments by the end of the year, which means that a return recorded in say year t measures the realized return in the periods $t+1$ through $t+5$. Hence, the 5-year observation for 1992 covers the years 1993 to 1997. The data source is Nielsen, Olesen and Risager (1998).

< Insert Figure 1 around here >

The 5-year interest rate is relatively constant from the 1920s to the beginning of the 1960s, where it starts climbing up. It reaches a peak in 1982. In 1983 the interest rate displays the largest fall observed in the entire sample period. The considerable fall is usually attributed to the shift in economic policy regime that took place in late 1982, see e.g. Andersen and Risager (1987). Following the dramatic fall in 1983, the interest rate continues to decline until it stabilizes towards the end of the sample.

The 5-year stock return is obviously much more volatile than the interest rate, but follows also a pattern that resembles the interest rate. Thus, stock returns oscillate around a fairly constant mean until the beginning of the 1960s. Thereafter, stock returns tend to increase. There is a drastic fall in stock returns in the beginning of the 1980s. The decline sets in at a time with very high oil prices and large wage increases and where it is widely recognized that the overall macroeconomic policy stance is unsustainable. In the remainder of the 1980s, stock returns tend to decline. Moreover, both the 1970s and the 1980s are decades where bond yields often exceed stock returns.

⁴ The yield-to-maturity concept assumes essentially a flat yield curve or that coupons can be reinvested at the (constant) yield-to-maturity rate. This is, of course, a weakness of this measure and the alternative return on a zero-coupon bond would, in principle, have been a superior measure of the return on bond investments. However, data for zero-coupon rates are not available over the historical sample. Moreover, what we focus on in this study are the long-run, non-stationary movements in the level of bond rates, and for this purpose, the yield-to-maturity measure should suffice.

Figure 2 illustrates the stock return and yield-to-maturity at the 10-year investment horizon in the period 1927-1987 (the last observation covers the 10-year period from 1988 to 1997). The correlation between the two series is now much stronger. This shows that stock returns and bond yields are very closely connected in the long term. It is also interesting to observe that in this time perspective, stocks do not appear to be more volatile. In a 10-year perspective, the ups and downs in the stock market tend to cancel out and this explains why there is not much difference between the two series in terms of volatility, see Nielsen and Risager (1999).

< Figure 2 >

A formal test of the stationarity properties of the above series, see the Appendix, shows that they are integrated of order 1 (I(1)) within the sample period, i.e., the series are non-stationary in levels but stationary in first differences. We treat the series as such, notwithstanding the fact that the I(1) assumption may appear counter-intuitive on more economic grounds. The point is that in our small (or rather, finite) sample the returns behave *as if* they are I(1) series and treating them as such should improve the small sample properties of the statistical methods. Because of the non-stationarity of returns, we use cointegration methods.

3. Arbitrage between Stocks and Bonds

Economic theory suggests that there is a simple no-arbitrage relation between stocks and bonds,

$$(1) \quad E(S_t | I_t) = E(B_t | I_t) + PR_t$$

which says that the expected forward-looking return on stocks $E(S_t | I_t)$ equals the expected forward-looking bond return $E(B_t | I_t)$ plus an additive risk premium PR_t on stocks relative to bonds. The returns relate to the same (5- or 10-year) investment horizon and the expectations are conditioned on the available information set I_t . Over the business cycle, the risk premium is likely to vary, whereas it seems plausible that the premium is constant in the long term. Because this paper focuses entirely on the long run relationship between the two markets, we assume that the risk premium is constant ⁵.

⁵ The risk premium could, in principle, be related to past or (for the predetermined yield-to-maturity on bonds) current levels of returns. One possibility is that the risk premium declines if the

For an empirical analysis of the variation in the risk premium in the short run, see Olesen and Risager (1999).

Equation (1) is an ex-ante equilibrium relationship that is consistent with the idea that the marginal investor will move money in or out of the markets until the expected stock return equals the expected bond yield plus a risk premium. For a rational marginal investor, the above equation should incorporate investor taxation insofar as income from the two assets is taxed at different rates ⁶. In theory, the marginal investor is a well defined agent, whereas the marginal investor is harder to identify in practice and in particular over a long historical time period. Candidates to the title are numerous. In modern times, it can be large Danish institutional investors like pension funds. Over the recent decades, it may also be foreign investors, reflecting the capital account liberalizations concerning stock investments which took place in the 1970s. It could also be wealthy private citizens, foundations and so forth even though it seems plausible that these investors played a bigger role in the past.

stock market has done well in the past because investors' memory may be short. In this case, investors may become less risk averse because they tend to forget how risky the market can be. Similarly, the risk premium could also be negatively related to the level of the bond rate. Thus, in a situation where bonds may be perceived as being more risky, e.g. due to increased default or inflation risk, we should at the same time expect a higher bond rate and (for given risk attitude towards stocks) a lower risk premium on stocks relative to bonds. However, as high bond rates may be associated with a high rate of business failure due to firms facing a higher interest burden, stocks could also be expected to be more risky and the net effect on the relative risk premium on stocks is unclear. These concerns are, however, most likely to be of greatest relevance over the business cycle. Due to our focus on the long run, we take a simple approach to the risk premium.

⁶ With taxation of investment income, equation (1) is replaced by an after-tax no-arbitrage relation, saying that the expected after-tax return on stocks is equal to the expected after-tax return on bonds plus a risk premium. By rearranging this relation, we get the following equation for the before-tax expected stock return (ignoring the risk premium),

$$(1') \quad E(S_t | I_t) = \frac{1 - t_B}{1 - t_S} E(B_t | I_t)$$

where J_S (J_B) is the representative marginal investor's rate of tax on stock (bond) returns. If the tax rate on bond returns is higher than on stock returns, i.e., $J_B > J_S$, the coefficient to the expected bond return $E(B_t | I_t)$ in (1') is less than one, and *vice versa*.

As Danish banks traditionally have held both stocks and bonds in large quantities, the banking sector may also at times have been the marginal investor.

The bottom line is that the marginal investor is not well defined in practice. Due to these complexities, we have not attempted to include taxation in the analysis.

However, for now, it is important to be aware that the general picture is that stock returns often have been taxed at a lower rate than bond yields. Thus, pension funds were from 1984 to the end of the sample not taxed on stocks but taxed on their interest income at a varying rate, related to the rate of inflation in the economy. Private households have in the same period been taxed lighter on their income from stocks. To the extent that banks acquire stocks and bonds for their own deposits, they are taxed uniformly at the rate applicable to taxable earnings in the banking sector. As regards the arbitrage process, one could at an informal level argue that it is the agent with the highest tax rate on bond investments, i.e., the agent who is taxed in the most asymmetric way, that will dominate the scene and hence become the marginal investor. This agent will have the highest reservation price for stocks. Assuming that short selling in bonds is allowed (or that there are no restrictions on borrowing in banks), this agent will tend to buy out the other agents in the stock market and bid up the market price to his reservation price⁷. In practice, there are limitations to the marginal investor's willingness to take extreme positions, including liquidity constraints, constraints stemming from risk aversion and possibly legal quantitative constraints on the allocation of portfolios. However, this very simple line of reasoning suggests that there is an inherent tendency for the agent suffering the highest tax rates on bond investments to become the marginal investor. As a result, we might expect that the coefficient to the bond yield is below one in the estimated relation between stock returns and bond yields.

⁷ If we allow for short selling in stocks, we could, actually, turn the argument around and conclude that it might be the agent with the lowest tax rate on bond investments that will become the marginal investor in the stock market, not as a holder of stocks but as a supplier of stocks. Thus, this agent will have a high reservation price for *bonds* and will have an incentive to buy bonds financed by a sale of stocks. If both short selling in stocks and bonds is allowed, no definite equilibrium exists. However, in practice, short selling in stocks has not been possible (or customary) in the Danish stock market over the historical period.

Equation (1) assumes rational expectations. In general, the realized return on an asset equals the expected return plus a component reflecting forecast errors, and under rational expectations the mean of this component will be zero over a long time period. Hence, realized returns will also be related to each other in a linear fashion. The error term will reflect forecast errors in both markets but have a mean that is zero in the long run. However, because we use overlapping observations, the error term may be serially correlated. This is purely a statistical artifact.

Given this, we can now turn to the cointegration analysis with the purpose of detecting whether there exists a linear long run relationship between realized stock and bond returns.

4. Cointegration Analysis

As mentioned earlier, the VAR method of Johansen is a simultaneous equation method that allows for interdependence between the stock and the bond market. It is also a full information maximum likelihood (FIML) estimation method which uses both the short and long run information in the data. In error-correction form, the dynamic Vector Autoregressive (VAR) system to be estimated is

(2)

$$\Delta Z_t = \mathbf{t}_1 \Delta Z_{t-1} + \dots + \mathbf{t}_{k-1} \Delta Z_{t-k+1} + \mathbf{p} Z_{t-1} + \mathbf{m} + d83_t + \mathbf{e}_t$$

where $Z_t = (S_t, B_t)'$ is the (2×1) vector of the endogenous stock and bond returns, μ is a (2×1) vector of constants, $d83_t$ is a (2×1) vector of impulse dummies for the year 1983⁸, and \mathbf{e}_t is a (2×1) vector of white noise errors. The lag length k is chosen such that the residuals satisfy the white noise assumptions of being serially uncorrelated and homoskedastic. In the 5-year horizon model, we choose $k=5$ and in the 10-year model, we set $k=3$, cf. below. These lag lengths can, furthermore, be validated by formal testing of the significance of individual lag lengths in a general-to-specific-procedure. The impulse dummy for 1983 has, likewise, been included to ensure that the 5- and 10-year models are well specified. Thus, without this dummy the residuals show serial correlation in both models.

⁸ Each impulse dummy has a value of one for 1983 and is zero, otherwise.

Because of the shift in the economic policy regime and the resulting dramatic fall in interest rates in 1983, the dummy is also warranted on economic grounds. In the estimation, we restrict the constant terms in μ to lie within the cointegrating space. This precludes a deterministic time trend in the endogenous variables, cf. Johansen (1996), which appears to be consistent with the data, cf. Figures 1 and 2. The parameter matrix that this paper is concerned with is B . This matrix can be decomposed in a Γ and a β matrix according to

$$(3) \quad p = \Gamma \beta'$$

where Γ and β are $2 \times r$ matrices, where r is the rank of B . Notice that if B is of full rank ($r=2$), the long run solution for Z_t is unique and equal to a vector of constants. However, since Z_t is $I(1)$ (and not stationary), this is false, and B cannot be of full rank. As explained by e.g. Johansen (1996), the rank of B determines the number of cointegrating vectors. Below, we find that there for both investment horizons exists one cointegrating vector β ($r=1$). The elements of this β -vector are the long run coefficients which we focus on in this paper. The elements of the estimated Γ vector measure the average speed of adjustment towards long run equilibrium. These parameters also have an interpretation related to the concept of weak exogeneity, which we return to.

5. Results for the 5-Year Horizon

We first estimate the dynamic system (2) on the basis of overlapping 5-year returns for Danish stocks and government bonds. As it is important for the inference that the error term in (2) fulfills the white noise assumptions, we perform a number of specification tests, see Table 1. Table 1 reports the outcome of single-equation specification tests for serial correlation, heteroskedasticity (ARCH) and normality. Both equations pass the tests at the conventional 5% significance level. We have also performed specification tests using a system approach (multivariate tests for serial correlation, normality and heteroskedasticity, not reported for expositional reasons); the outcome is, again, that there is no sign of misspecification at the 5% level. We therefore conclude that the dynamic model is well specified.

< Table 1 >

Tests for the rank of B can then be performed, see Table 2. We report both the standard asymptotic trace test, cf. Johansen (1996), and the small-sample-adjusted trace test, as suggested by Reimers (1992). The critical values for the trace tests are simulated using the simulation program DisCo, cf. Johansen and Nielsen (1993), to take account of the inclusion of the impulse dummy for 1983⁹. The conclusion is very clear; there is one and only one cointegrating vector, i.e., $r=1$. Furthermore, the evidence of cointegration is strong; based on the simulated test values, the critical significance level for the null that there is not cointegration is virtually zero.

< Tables 2 and 3 >

Table 3 reports the estimates of the α and β vector under the restriction that there is one cointegrating vector. The B matrix is also reported. The estimated α vector leads to the following long-run equilibrium relation between stock and bond returns at the 5-year horizon (indicative standard errors of coefficient estimates in parentheses):

$$(4) \quad S_t = 0.86 B_t + 0.026 \quad (5 \text{ year})$$

$\begin{matrix} (0.09) & (0.007) \end{matrix}$

The coefficient to the bond yield, estimated to be 0.86, is clearly significant. We furthermore note that this point estimate is below one. Equation (1) suggests a one-to-one relationship between stock and bond returns, and it is therefore of relevance to test the hypothesis that the ('true') coefficient is one¹⁰. The Likelihood Ratio (LR) test statistic for this hypothesis is 2.77; the asymptotic test distribution is $P^2(1)$ and the critical test value equals 3.84 at the 5% significance level. Hence, the hypothesis cannot be rejected at the conventional 5% level¹¹.

⁹ For comparison, the standard 95% critical test values are 20.0 (for $r=0$) and 9.1 ($r=1$), cf. Johansen (1996, Table 15.2).

¹⁰ Formally, we test the validity of the restricted cointegrating vector ($\alpha_{11}=1, \alpha_{21}=-1, \alpha_{31}$), where we have normalized on stock returns (this augmented 3×1 vector includes the constant term (α_{31}), restricted to be part of the cointegrating relation, cf. section 4).

¹¹ The critical significance level of the test is 9.6%, so at a strict 10% significance level we would reject the null.

However, there appears to be a tendency for the bond yield coefficient to be less than one. Based on the normal distribution and the indicative standard error of the coefficient estimate, the (indicative) 95% confidence interval can be shown to be (0.69,1.03). The fact that it is likely that the ('true') bond yield coefficient is below one may reflect the tendency for stocks to be taxed at a lower rate than bonds, cf. section 3.

Over the sample 1927-1992, the average difference between the 5-year return on stocks and the 5-year return on bonds is 1.4% per year. This long run return difference between the two assets is reflected in the constant term of (4), estimated to be 2.6% per year (slightly higher because the estimated bond yield coefficient is below one). The premium on stocks may be considered fairly low, in particular, by international standards. In judging the difference between stock and bond yields, it should be noted, though, that we are dealing with 5-year horizons, and that the 5-year bond rate on average is higher than a short Treasury Bill rate, which is a common estimator of the riskfree rate in the equity premium literature, see Kocherlakota (1996) for a survey of this literature.

Having estimated the long-run equilibrium relation between the stock and bond markets, we can test whether deviations from this relationship trigger adjustments in the bond yield and stock returns, respectively. These tests are concerned with the estimated adjustment coefficients in the (α) vector and amount to testing for weak exogeneity of bond and stock returns, cf. Johansen (1996). To begin with, we test the null hypothesis that the adjustment coefficient in the direction of bond yields (estimated to be -0.13, cf. Table 3) is zero¹². The LR test of this hypothesis has a critical significance level of 3.4%, so we reject the null at conventional significance levels. In other words, the bond yield is *not* exogenous as deviations from the long run stock and bond yield relationship trigger adjustments in the bond yield. Likewise, we can test whether stock returns are exogenous by testing the null that the adjustment coefficient in the direction of stock returns (estimated to be -1.45) is zero. This hypothesis is clearly rejected as the critical significance level of the corresponding LR test is effectively zero (the LR test statistic is 43 which should be compared to the $P^2(1)$ distribution).

¹² Formally, we test the validity of the restricted adjustment vector $(\alpha_{11}, \alpha_{21}=0)$.

Thus, stock returns take on a significant burden in the adjustment to long-run equilibrium between the stock and bond markets¹³.

6. Results for the 10-Year Horizon

The results for the 10-year horizon are reported in Tables 4, 5 and 6. We have performed the same specification tests as before and find that there are no signs of serial correlation or heteroskedasticity (ARCH) in the residuals of the dynamic model, using the conventional 5% significance level. We reject the hypothesis of normally distributed residuals in the equation for stock returns, a standard assumption underlying the statistical results of the Johansen method. However, as shown in Johansen (1996, Part II), this assumption is not crucial as the asymptotic inference of the Johansen method is also valid in the less restrictive case where residuals are identically (and not necessarily normally) distributed over time. We therefore conclude that the dynamic model (2) is acceptable from a statistical point of view.

< Tables 4, 5 and 6 >

We find one and only one cointegrating relation in the data and, again, the evidence of cointegration is strong. The estimated long-run equilibrium relationship between stock returns and the bond yield at the 10-year horizon is:

$$(5) \quad S_t = \underset{(0.05)}{0.71} B_t + \underset{(0.004)}{0.035} \quad (10 \text{ year})$$

¹³ In interpreting the results on exogeneity, it is important to recall that the yield-to-maturity on bonds, in contrast to stock returns, is a predetermined variable as it is determined and known at the beginning of any 5-year investment period. The apparent endogeneity of the bond yield, therefore, in principle, implies that future stock return *realizations* are significant in explaining the *current* bond yield. At the 10-year horizon we also find that the bond yield is endogenous. This result could tentatively be explained by expectational effects and a slow arbitrage process, cf. section 6. However, at the 5-year horizon, this interpretation is not so obvious. The estimated adjustment coefficients suggest that if stock returns are excessively high relative to bond yields, stock returns tend to decline *by more* than is necessary to restore equilibrium (as the adjustment coefficient in the direction of stock returns is negative and above one in absolute value) while at the same time the bond yield also tends to *decline* (as the adjustment coefficient is negative). This suggests a rather complex (and puzzling) dynamic adjustment to long-run equilibrium.

The point estimate of the coefficient to the 10-year bond yield is highly significant and equals 0.71. Thus, the coefficient is also below one at the 10-year horizon. By testing whether the coefficient is one, we get a clear rejection at any significance level (the LR test statistic is 22 with a critical significance level that is virtually zero). The indicative 95% confidence interval for the coefficient is given as (0.61,0.81). Hence, at this horizon there is a clear deviation from a unitary coefficient to the bond yield. The estimate of the constant equals 3.5% per year, slighter higher than at the 5-year horizon¹⁴.

Our results on the exogeneity status of the variables parallel the findings at the 5-year horizon, as both the stock return and the bond yield are endogenous¹⁵. Hence, deviations from the long run cointegrating relation trigger adjustments in both the stock return and the bond yield. The estimated adjustment coefficients show that if stock returns exceed the long run level implied by the cointegrating relation, stock returns tend to fall (negative adjustment coefficient) whereas the bond yield tends to increase (positive adjustment coefficient). Thus, the returns in both markets contribute to restore the long-run equilibrium relation.

Tentatively, the endogeneity of the predetermined bond yield may reflect expectational effects and a slow arbitrage process between the two markets. Thus, consider a situation where the representative investor expects that the stock return over the next 10-year investment period will be higher than the equilibrium level implied by the cointegrating relation. In an effective market, this expected abnormal return will lead to an immediate transfer of funds from the bond to the stock market which eliminates the arbitrage opportunity. However, if for some reason the arbitrage process is slow¹⁶, the expected stock return will remain high. Moreover, we will see a gradual adjustment in the expected stock return (which will decline) and the bond yield (which will increase) as funds are transferred across markets over time.

¹⁴ For comparison, the average difference between the 10-year stock return and the 10-year bond yield is 1.5% over the sample 1927-1987.

¹⁵ In both cases, the critical significance level of the relevant LR test is, for all practical purposes, zero.

¹⁶ Possible reasons for a slow arbitrage process could be the existence of transaction costs, a slow transmission of information in the market or legal constraints on the representative investor's portfolio allocation.

During this adjustment phase the expected excess return on stocks leads the adjustment in the bond yield. Because the realized stock return signals the expected stock return under rational expectations, we may as the outcome observe that an abnormally high *realized* stock return over the next 10-year period leads changes in the *current* bond yield. That is, the bond yield may appear to be endogenous even though it is predetermined.

7. Concluding Remarks

This short paper has shown that the realized stock returns and bond yields at the 5-year horizon and at the 10-year horizon form strong cointegrating relationships. Thus, a high bond yield tends to go hand in hand with a high stock return in the medium and long term, and *vice versa*. The stable relationships between the two markets are likely to reflect an arbitrage process that works in the medium and long term.

Our results also show that stock returns tend to respond less than one-to-one to changes in the interest rate. This result is strongest at the 10-year horizon where the deviation is statistically significant. In a world where investors are taxed symmetrically, arbitrage considerations suggest that there should be a one-to-one relationship between stock returns and bond yields. Hence, we think that the result reflects asymmetric taxation because bond yields often have been taxed at a higher rate than stock returns; that is the picture that applies to the majority of the investors.

To get a deeper understanding of the effects of taxation is, however, a large and complex project in itself. It requires first of all a careful study of the tax laws (that differ across investors) in the entire sample period. It also requires that one is able to identify the marginal investor at different points in time. This is an interesting project with many potential externalities. We therefore consider this issue to be an obvious topic for future research.

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Appendix: Unit Root Tests

We have examined the stationarity properties of each of the return series, using the unit root tests of Phillips and Perron (1988) (PP) (see Table A.1) and Kwiatkowski *et al.* (1992) (KPSS) (see Table A.2). The tests are introduced in the notes to the tables.

< Tables A.1 and A.2 >

The outcome for the 10-year stock return and the 5- and 10-year bond yields is clear; using conventional significance levels, all three series are concluded to be integrated of order 1 (I(1)), i.e., to be non-stationary in levels, but stationary in first differences. The outcome for the 5-year stock return is mixed, as the PP test indicates I(0) (i.e., that the series is stationary in levels) while the KPSS test suggests I(1). To provide further evidence, we have performed an augmented Dickey-Fuller test of the null that the process for the 5-year stock return contains a unit root, cf. Dickey and Fuller (1979). Based on a well-specified augmented Dickey-Fuller regression where we include the fifth lag of the first differences of the stock return (this lag structure is chosen by a general-to-specific procedure, eliminating the insignificant lags in a general regression), we get a test statistic of -2.5. Hence, at conventional significance levels, we can not reject the null of a unit root, i.e., the level of returns is non-stationary. Overall, we therefore conclude that also the 5-year stock return is I(1).

Finally, it should be noted that in order to use the (standard) Johansen method, all we need is that each return series is at most I(1). This is clearly accepted by the unit root tests.

Figure 1. 5-Year Stock and Bond Return, 1927-1992.

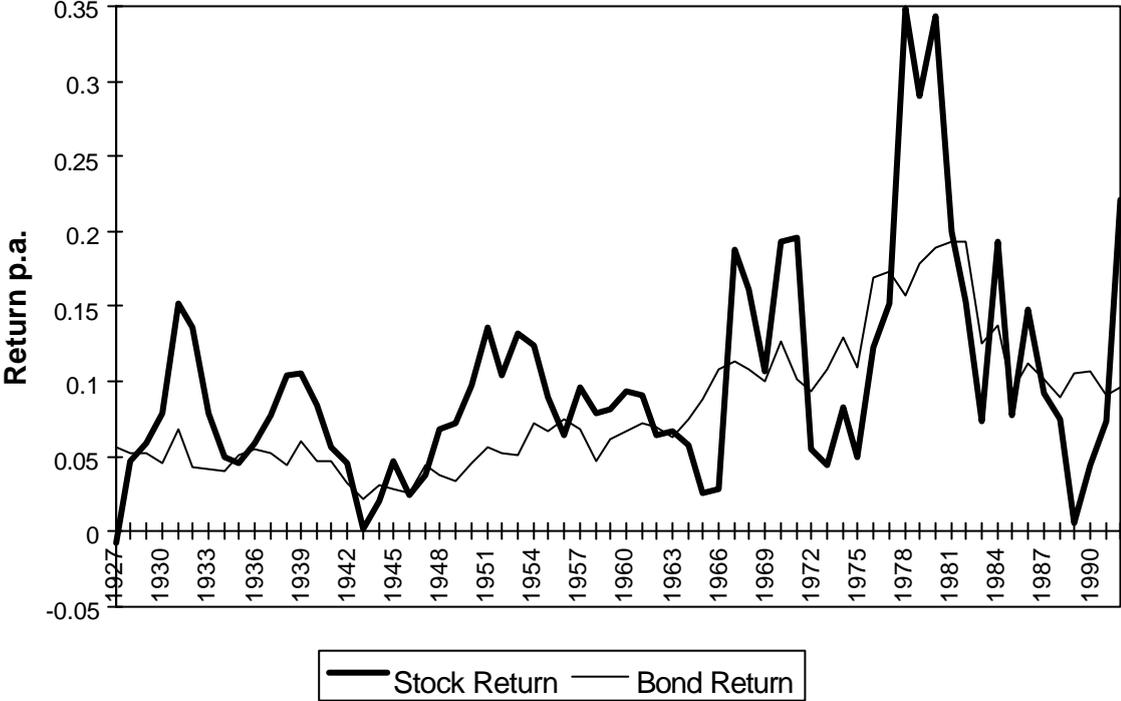


Figure 2. 10-Year Stock and Bond Return, 1927-1987.

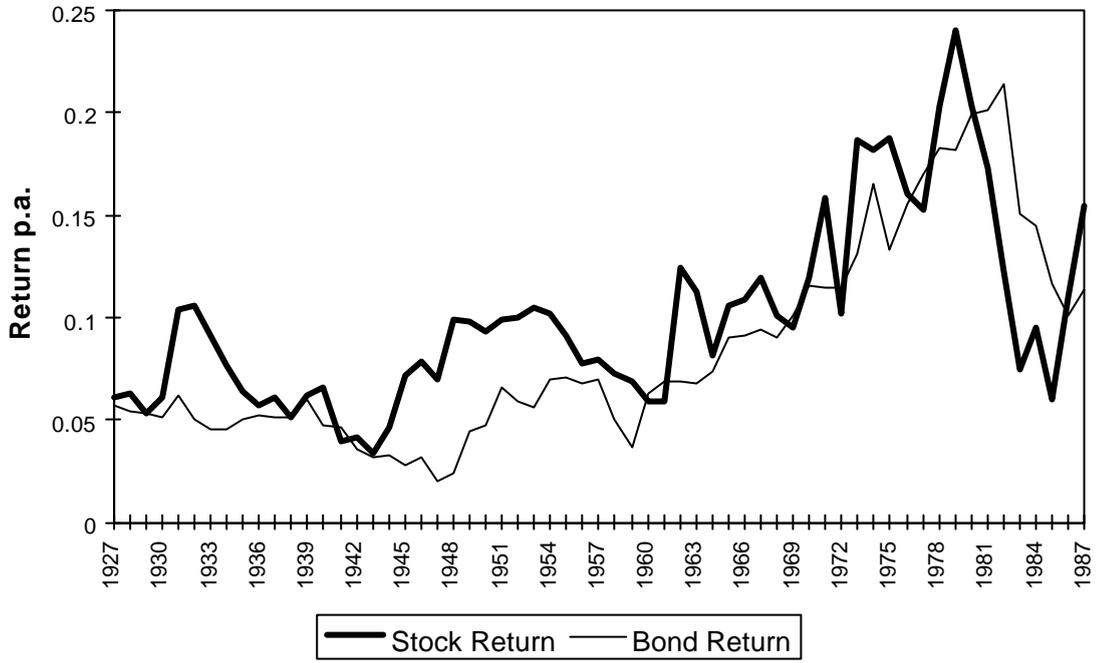


Table 1. Specification Tests of the VAR Model ¹⁾. 5-Year Horizon. 1927-92.

	Correl. Actual & Fitted ²⁾	S.E. ³⁾	Normality P ² (2)	ARCH, F-test				Autocorrelation, F-test			
				Lags				Lags			
				1 to 1	1 to 2	1 to 4	1 to 6	1 to 1	1 to 2	1 to 4	1 to 6
Eq.) S _t	0.845	0.037	0.52 [0.77]	0.40 [0.53]	2.50 [0.09]	1.30 [0.29]	1.12 [0.37]	2.90 [0.09]	1.67 [0.20]	1.01 [0.41]	0.71 [0.64]
Eq.) B _t	0.963	0.012	1.37 [0.50]	0.69 [0.41]	2.09 [0.13]	1.15 [0.35]	1.06 [0.40]	0.01 [0.93]	0.70 [0.50]	0.80 [0.53]	0.57 [0.75]

Note: The VAR model has a lag length of 5 and includes a constant term (restricted to the cointegrating space) and an impulse dummy for 1983 (enters unrestricted). F-tests are small sample approximations to Lagrange Multiplier tests (F-form). Normality test of Doornik and Hansen (1994), cf. also Doornik and Hendry (1997). Critical significance levels in brackets.

1) Single-equation specification tests for Normality, ARCH and serial correlation in residuals. We have also undertaken vector specification tests (based on the whole system) for Normality, heteroskedasticity (squares) and serial correlation (not reported). None of these signal misspecification problems at the 5% significance level when we apply the small sample F-test.

2) Coefficient of correlation between actual and fitted value for variable in levels.

3) Standard error of residuals.

Table 2. Tests for Cointegrating Rank. 5-Year Horizon. 1927-92.

Rank	Eigenvalue	Trace Statistic ¹⁾	Trace Statistic (small sample) ²⁾	Trace 95% Quantil ³⁾
r=0	0.49	45.08	38.25	16.5
r=1	0.01	0.93	0.79	5.9

Note: Maximum Likelihood estimation by method of Johansen (1996). 1) Asymptotic trace test statistic, cf. Johansen (1996).

2) Trace test statistic adjusted for small sample, cf. Reimers (1992) and Doornik and Hendry (1997). 3) 95% critical test value for trace test, simulated by the simulation program DisCo, cf. Johansen and Nielsen (1993). Allows for the impulse dummy for 1983.

Table 3. Restricted A-Matrix, (and \$-Vector (r=1). 5-Year Horizon. 1927-92.

(\$'	A				
-1.445 (0.20)	1	-0.864 (0.087)	-0.026 (0.007)	-1.445 (0.20)	1.249 (0.17)	0.037 (0.005)
-0.130 (0.07)				-0.130 (0.07)	0.113 (0.06)	0.003 (0.002)

Note: Maximum Likelihood estimation by method of Johansen (1996). Rank r restricted to 1. The \$-vector is normalized on stock returns and includes the constant term (restricted to the cointegrating space). Indicative standard errors of parameter estimates in parentheses.

Table 4. Specification Tests of the VAR Model ¹⁾. 10-Year Horizon. 1927-87.

	Correl. Actual & Fitted ²⁾	S.E. ³⁾	Normality P ² (2)	ARCH, F-test				Autocorrelation, F-test			
				Lags				Lags			
				1 to 1	1 to 3	1 to 5	1 to 7	1 to 1	1 to 3	1 to 5	1 to 7
Eq.) S _t	0.866	0.022	7.80 [0.02]	1.84 [0.18]	0.72 [0.54]	0.76 [0.59]	0.51 [0.82]	0.38 [0.54]	1.21 [0.32]	0.73 [0.61]	1.40 [0.23]
Eq.) B _t	0.981	0.009	0.06 [0.97]	2.91 [0.09]	1.72 [0.18]	1.36 [0.26]	1.20 [0.32]	0.96 [0.33]	1.64 [0.19]	0.99 [0.43]	0.69 [0.68]

Note: The VAR model has a lag length of 3 and includes a constant term (restricted to the cointegrating space) and an impulse dummy for 1983 (enters unrestricted). F-tests are small sample approximations to Lagrange Multiplier tests (F-form). Normality test of Doornik and Hansen (1994), cf. also Doornik and Hendry (1997). Critical significance levels in brackets.

1) Single-equation specification tests for Normality, ARCH and serial correlation in residuals. We have also undertaken vector specification tests (based on the whole system) for Normality, heteroskedasticity (squares) and serial correlation (not reported). None of these signal misspecification problems at the 5% significance level when we apply the small sample F-test.

2) Coefficient of correlation between actual and fitted value for variable in levels.

3) Standard error of residuals.

Table 5. Tests for Cointegrating Rank. 10-Year Horizon. 1927-87.

Rank	Eigenvalue	Trace Statistic ¹⁾	Trace Statistic (small sample) ²⁾	Trace 95% Quantil ³⁾
r=0	0.53	47.22	42.57	16.1
r=1	0.03	1.62	1.46	6.0

Note: Maximum Likelihood estimation by method of Johansen (1996). 1) Asymptotic trace test statistic, cf. Johansen (1996).

2) Trace test statistic adjusted for small sample, cf. Reimers (1992) and Doornik and Hendry (1997). 3) 95% critical test value for trace test, simulated by the simulation program DisCo, cf. Johansen and Nielsen (1993). Allows for the impulse dummy for 1983.

Table 6. Restricted A-Matrix, C- and \$-Vector (r=1). 10-Year Horizon. 1927-87.

C	\$'	A				
-0.571 (0.17)	1	-0.709 (0.050)	-0.035 (0.004)	-0.571 (0.17)	0.405 (0.12)	0.020 (0.006)
0.384 (0.07)				0.384 (0.07)	-0.272 (0.05)	-0.014 (0.003)

Note: Maximum Likelihood estimation by method of Johansen (1996). Rank r restricted to 1. The \$-vector is normalized on stock returns and includes the constant term (restricted to the cointegrating space). Indicative standard errors of parameter estimates in parentheses.

Table A.1. Phillips and Perron (1988) Z_t -Test for Unit Root

Series:	Lag length (l)						
	0	1	2	3	4	5	6
<i>5 Year Horizon (1927-1992):</i>							
S_t	-3.71	-3.71	-3.84	-3.82	-3.86	-3.70	-3.64
B_t	-1.62	-1.44	-1.44	-1.49	-1.51	-1.53	-1.56
\hat{S}_t	-9.14	-9.13	-9.12	-9.15	-9.17	-9.37	-9.61
\hat{B}_t	-10.51	-10.51	-10.48	-10.41	-10.38	-10.36	-10.36
<i>10 Year Horizon (1927-1987):</i>							
S_t	-2.27	-2.27	-2.25	-2.24	-2.20	-2.19	-2.17
B_t	-1.10	-1.08	-1.11	-1.14	-1.17	-1.19	-1.20
\hat{S}_t	-8.11	-8.11	-8.12	-8.14	-8.21	-8.28	-8.38
\hat{B}_t	-8.00	-8.00	-8.00	-8.00	-8.01	-8.01	-8.01
<i>Critical test values:</i>			<i>10%</i>	<i>5%</i>	<i>1%</i>		
Without trend			-2.59	-2.91	-3.54		

Note: The Phillips and Perron (1988) unit root test is based on the first order autoregression $x_t = \alpha + \rho x_{t-1} + u_t$ (without trend) where the disturbance term u_t has mean zero but can otherwise be heterogeneously distributed and serially correlated up to lag l , see also Hamilton (1994). The Z_t test statistic is a modified t-statistic for the null hypothesis of a unit root ($\rho=1$), correcting for the possible non-standard properties of u_t . The null is rejected in favor of the stationary alternative ($\rho<1$) if Z_t is negative and sufficiently large in absolute value. Critical test values are small-sample values calculated from MacKinnon (1991). Underlining indicates rejection of a unit root at the 5% significance level.

Table A.2. Kwiatkowski *et al.* (1992) Test for Unit Root

Series:	Lag length (l)						
	0	1	2	3	4	5	6
<i>5 Year Horizon (1927-1992):</i>							
S_t	1.20	0.75	0.58	0.50	0.46	0.45	0.44
B_t	4.54	2.37	1.61	1.24	1.01	0.86	0.75
\hat{S}_t	0.02	0.03	0.03	0.03	0.03	0.05	0.05
\hat{B}_t	0.07	0.09	0.10	0.10	0.10	0.10	0.10
<i>10 Year Horizon (1927-1987):</i>							
S_t	3.00	1.66	1.19	0.95	0.80	0.70	0.63
B_t	4.52	2.31	1.57	1.20	0.98	0.83	0.73
\hat{S}_t	0.04	0.04	0.04	0.05	0.05	0.06	0.07
\hat{B}_t	0.12	0.12	0.12	0.11	0.11	0.11	0.11
<i>Critical test values:</i>			<i>10%</i>	<i>5%</i>	<i>1%</i>		
Mean-stationarity			0.35	0.46	0.74		

Note: The Kwiatkowski *et al.* (1992) test for a unit root is a Lagrange Multiplier test of the null hypothesis that the series can be described by a stationary process around a constant mean, against the alternative that the process also includes a non-stationary random walk component. The null of stationarity is rejected in favor of the unit root alternative if the test statistic is sufficiently large. Critical values are from Kwiatkowski *et al.* (1992). Underlining indicates rejection of the null (i.e., a unit root is present) at the 5% significance level. The lag length l is the number of lags allowed for in the stationary component of the process.