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**INFORMAL UNEMPLOYMENT AND EDUCATION**

**Ann-Sofie Kolm      Birthe Larsen**

# Informal Unemployment and Education\*

Ann-Sofie Kolm<sup>†</sup> and Birthe Larsen<sup>‡</sup>

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## Abstract

This paper develops a four sector equilibrium search and matching model with informal sector employment opportunities and educational choice. We show that underground activities reduce educational attainments if informal employment opportunities mainly are available to low educated workers. More zealous enforcement policy will in this case improve educational incentives as it reduces the attractiveness of remaining a low educated worker. Characterizing the optimal enforcement policies, we find that relatively more audits should be targeted towards the sector employing low educated workers, otherwise a too low stock of educated workers is materialized.

JEL-codes: H26, I21, J64

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<sup>†</sup>Stockholm University. E-mail address: ann-sofie.kolm@ne.su.se

<sup>‡</sup>Insead, CEBR and Copenhagen Business School, E-mail address: bl.eco@cbs.dk

# 1 Introduction

Researchers have been puzzled by the fact that observed tax evasion, despite low audit rates and fairly modest fines, is substantially lower than what is predicted by theory. Andreoni et al (1998), argue that this discrepancy is most likely explained by non-economic factors, such as moral, guilt, and shame. However, Kleven et al (2011) whom conduct a large field experiment in Denmark, suggests that this discrepancy is explained by the degree of third party reporting. As incomes for individuals are not self-reported, rather reported by a third party such as the employer, it is difficult, and thus costly, to evade taxes. These costs, both due to third-party reporting, or even moral, guilt or shame, tends to reduce the profitability of evading taxes and limits the size of the informal sector, although the expected punishment fees are low relative to taxes.

In this paper we argue that these types of costs may explain why highly educated workers to a lesser extent evade taxes and work informally than low educated workers. If highly educated workers to a smaller extent work in industries which handles cash-payments and to a larger extent are subject to third party reporting, it will be more difficult, and thus more costly, for these workers to evade taxes.

This is supported by data. Evidence indicates that manual workers, or workers with a lower level of formal education, to a substantially larger degree face informal employment opportunities compared to highly educated workers. Pedersen (2003), using the same questionnaire design for Germany, Great Britain, Denmark, Norway, and Sweden confirms that skilled blue colour workers carry out more informal market activities than others. Figure 1 shows the extent of informal activities in the five countries by industry. Most informal work are carried out in the construction sector, followed by the agricultural sector, and in hotels and restaurants.

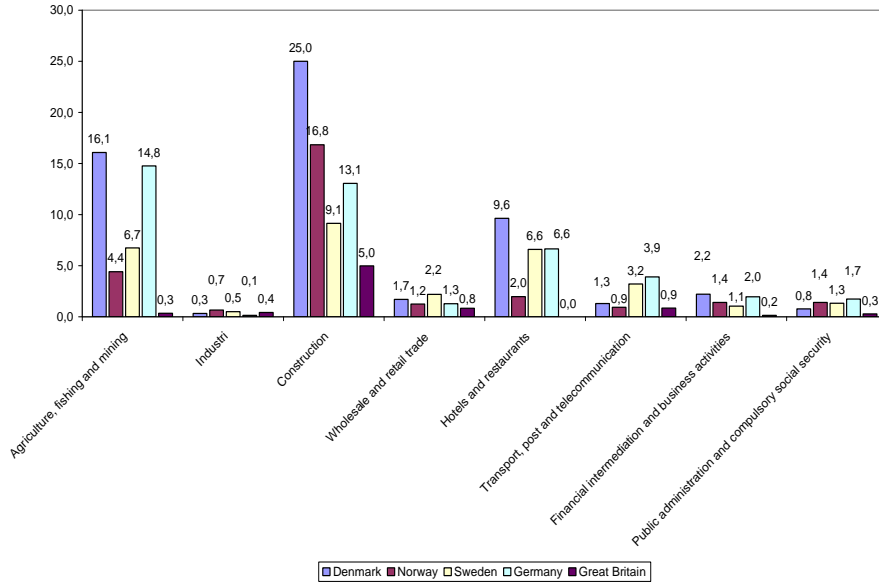


Figure 1: Fraction of informal sector work by industry. Pedersen 2003.

Furthermore, performing logistic regressions for the five countries, Pedersen (2003) confirms that the likelihood of informal market activities falls with the length of education. In addition, Boeri and Garibaldi (2002) show for Sicily, that mainly workers at the lower end of the skill distribution engage in informal activities.

The fact that mainly low educated workers seem to work in the informal sector suggests that the choice of educational attainment is potentially distorted by informal sector employment opportunities. The aim of this paper is to investigate the equilibrium impact of underground activities on labour market performance and educational attainment, as well as to characterize the optimal enforcement policy. To that end, we develop a four sector general equilibrium model featuring matching frictions on the labour market. Unemployed workers search for jobs in both a formal and an informal sector, and workers decide whether or not to acquire higher education based on

their ability levels.

In order to isolate the mechanisms and increase the transparency of the model, we keep the differences between the sectors at a minimum. The only dissimilarity between the formal and the informal sector is that taxes are not paid in the latter. Instead of paying taxes, informal sector firms have to pay a fine in case they are hit by an audit and detected as tax cheaters. In addition, firms in the informal sector are assumed to face concealment costs. In our model, we let concealment costs capture costs associated with concealing taxable income due to third party reporting or even moral, guilt or shame. The costs reduce the profitability of evading taxes and limits the size of the informal sector although the expected punishment fees are low relative to taxes. In line with Kleven et al (2010), we also let these costs be higher the more income that is hidden from the tax authorities.

Early theoretical analyses of tax evasion are provided by Allingham and Sandmo (1972) and Srinivasan (1973), where under-reporting of income is modelled as a decision made under uncertainty.<sup>1</sup> There has, however, been a switch in research focus from individual decision making on the extent of evasion, towards a modelling strategy where firms under-report their true profits, sales and wages paid, as advanced economies nowadays make extensive use of third-party information reporting.<sup>2</sup> There has also been a movement towards considering the case where firms or worker do not report at all. This characterizes the informal sector in focus in this paper.

The present paper extends the recent strand of the tax evasion literature, which departs from the assumption of imperfectly competitive labour markets, by incooperating involuntary unemployment through the inclusion

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<sup>1</sup>Subsequent papers have since then enhanced the basic model of individual behavior by, for example, incorporating endogenous labour supply decisions. See for example Sandmo (1981) for an early contribution of endogenous labour supply and underreporting of income.

<sup>2</sup>Also equilibrium models with tax evasion have been developed. For example see the early study by Cremer and Gahvari (1993) and the recent study by Tonin (2010).

of search friction. See for example Fugazza and Jacques (2004), Kolm and Larsen (2006), and Boeri and Garibaldi (2002).<sup>3</sup> As apposed to these previous studies, we do not need to rely on asymmetries between the formal and the informal sector, such as heterogeneity in moral, in order to generate existence of both a formal and an informal sector.

We find that underground activities reduce the incentives to acquire higher education if informal employment opportunities mainly are available to low educated workers. More zealous enforcement policies will in this case improve educational incentives as it reduces the attractiveness of remaining a low educated worker. However, if also highly educated workers to a large extent are exposed to informal employment opportunities, the incentives to acquire higher education may fall with stricter enforcement policies as underground work pays off better to workers with high productivity. Moreover, we find that the actual unemployment rate most likely increases with stricter enforcement policies, although the official unemployment rate falls. Finally, characterizing the optimal enforcement policies, we find that relatively more audits should be targeted towards the sector employing low educated workers, elsewise the outcome is a too low stock of educated workers.

The paper is organized as follows. In section 2 the model is set-up. Section 3 offers a comparative statics analysis of an increase in the expected punishment fee. Simulations are presented in Section 4, whereas Section 5 considers optimal policy, and finally Section 6 concludes.

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<sup>3</sup>Some papers have also investigated informal employment from a non western economic point of view. Albrecht et al (2009) considers the impact of payroll taxes and severance pay on unemployment in the presence of an informal sector from a Latin American perspective. The informal sector can be seen as an unregulated sector which is not affected by payroll taxes and other formal policies.

## 2 The model

This section develops a four sector general equilibrium model with formal and informal sector employment opportunities and educational choice. Workers differ in ability to acquire education. Abilities,  $e$ , are uniformly distributed between 0 and 1,  $e \in [0, 1]$ , and the cost of higher education,  $c(e)$ , is decreasing in ability. Thus, workers with a high level of ability will find it more than worthwhile to attain higher education, whereas workers with low ability will not. Workers not attaining higher education will from now on be referred to as manual workers.

Both manual and highly educated workers allocate search effort optimally between the formal and the informal sector. Once matched with a firm, they bargain over the wage. The economy thus consists of four sectors; the formal and informal sector for manual workers, (denoted  $F, m$  and  $I, m$ ), and the formal and informal sector for highly educated workers (denoted  $F, h$  and  $I, h$ ).

### 2.1 Matching

Manual and highly educated workers search for jobs in both a formal and an informal sector. For simplicity, we assume that only unemployed workers search for jobs. This is a simplification, i.e. we do not acknowledge that the connection to the labour market given by working in the formal sector may bring about job opportunities not available while unemployed. The matching functions for the four categories of jobs are given by  $X_l^j = (v_l^j)^{\frac{1}{2}} \left( (\sigma_l^j)^\gamma u_l \right)^{\frac{1}{2}}$ , where  $X_l^j$  is the sectorial matching rate,  $v_l^j$ , is the sectorial vacancy rate, and  $u_l$ , is the unemployment rate,  $j = F, I$  and  $l = m, h$ . The rates are defined as the numbers relatively to the labour force of manual and highly educated workers, respectively. The exponents in the matching function is set to be equal to a half in order to simplify the welfare analysis where we derive the

optimal tax and punishment system when we have imposed the traditional Hosios condition. In that case we can disregard congestion externalities as the elasticity of the expected duration of a vacancy is equal to the bargaining power of workers in a symmetric Nash bargaining situation.

Workers allocate search effort optimally across the formal and the informal sector. Each worker's total search intensity is exogenously given and normalized to unity, where  $\sigma_l^I = \sigma_l$ , denotes search effort directed towards the informal sector, and  $\sigma_l^F = 1 - \sigma_l, l = m, h$ , denotes search effort directed towards the formal sector. The parameter  $\gamma < 1$  captures that the effectiveness of search falls with search effort, i.e., the first unit of search in one sector is more effective than the subsequent units of search. This could capture that different search methods are used when searching for a job in a market. The more time that is used in order to search in a market, the less efficient search methods have to be used.<sup>4</sup>

The transition rates into informal and formal sector employment for a particular worker  $i$ , are  $\lambda_{iI}^I = \sigma_{iI}^\gamma (\theta_{iI}^I)^{\frac{1}{2}}$  and  $\lambda_{iI}^F = (1 - \sigma_{iI})^\gamma (\theta_{iI}^F)^{\frac{1}{2}}$ , where  $\theta_{iI}^I = v_{iI}^I / (\sigma_{iI}^\gamma u_{iI})$  and  $\theta_{iI}^F = v_{iI}^F / ((1 - \sigma_{iI})^\gamma u_{iI})$  are labour market tightness,  $l = m, h$ , measured in effective search units. The rates at which vacant jobs become filled are  $q_{iI}^j = (\theta_{iI}^j)^{-\frac{1}{2}}, j = F, I, l = m, h$ .

## 2.2 Value functions

Let  $U_l, E_l^F$ , and  $E_l^I$  denote the expected present values of unemployment and employment for manual and highly educated workers. The value functions

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<sup>4</sup>This particular modelling strategy of search effort has a close resemblance to how search is modelled in van den Berg and van der Klaauw (2006), where search for a job can be conducted using different search channels.



for worker  $i$  then reads:

$$rU_{li} = R + \lambda_{li}^F(E_{li}^F - U_{li}) + \lambda_{li}^I(E_{li}^I - U_{li}) - I(l)c(e_i), l = m, h, \quad (1)$$

$$rE_{li}^F = R + w_{li}^F + s(U_l - E_{li}^F) - I(l)c(e_i), l = m, h, \quad (2)$$

$$rE_{li}^I = R + w_{li}^I + s(U_l - E_{li}^I) - I(l)c(e_i), l = m, h, \quad (3)$$

where  $r$  is the exogenous discount rate,  $w_l^j$  is the sector wage, and  $s$  is the exogenous separation rate.  $R$  is a lump-sum transfer that all individuals receive from the government which reflects that the government has some positive revenue requirements.<sup>5</sup> Highly educated workers pay the individual educational costs  $c(e_i)$ , where  $e_i$  is the worker's ability,  $e_i \in [0, 1]$ ,  $c'(e_i) < 0$  and  $c''(e_i) > 0$ . The indicator function  $I(l)$ ,  $l = m, h$  takes the value zero for manual workers and the value one for highly educated workers, hence  $I(m) = 0$  and  $I(h) = 1$ .<sup>6</sup> For simplicity, we disregard unemployment benefits.

Let  $J_l^F$  and  $V_l^F$  represent the expected present values of an occupied job and a vacant job in the formal sector, respectively. The arbitrage equations for the formal sector of a job paying the wage  $w_{li}^F$  and a vacant job are then

$$rJ_{li}^F = y_l - w_{li}^F(1 + z) + s(V_l^F - J_{li}^F), l = m, h, \quad (4)$$

$$rV_l^F = q_l^F(J_l^F - V_l^F) - ky_l, l = m, h, \quad (5)$$

where  $z$  is the payroll tax rate. Vacancy costs are denoted  $ky_l$ . Analogous

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<sup>5</sup>Everyone receives this transfer. The government cannot exclude the informal sector workers as the government does not know who the informal sector workers are (if it did, it could punish all of them).

<sup>6</sup>It is assumed to be costless to become a manual worker, but that workers who get a higher education find it costly to do so. This is a normalisation and has no consequences for the results. Moreover, we assume that the educational cost is a cost to acquire and maintain skill. This is a simplifying assumption and is not important for the results. The assumption enables us to use a model without having workers continuously being born and dying. Such a model would, however, generate the same qualitative expressions.

notation for the informal sector yields:

$$rJ_l^I = y_l - w_{li}^I(1 + p\alpha + \kappa_l) + s(V_l^I - J_l^I), l = m, h, \quad (6)$$

$$rV_l^I = q_l^I(J_l^I - V_l^I) - ky_l, l = m, h, \quad (7)$$

where  $p$  is the auditing rate which captures the probability of being detected employing a worker in the informal sector, and  $\alpha$  is the associated firm punishment fee rate. The  $\kappa_l, l = h, m$  captures concealment costs. These concealment costs capture that it is costly to hide income from the tax authorities. The costs could, for example, capture what Kleven et al (2011) refer to as third party reporting. When there is third party reporting of income, such as the firm reporting the wage payments directly to the tax authorities, this has to be agreed upon also by the worker, which is costly. These concealment costs could also be other direct costs associated with concealing evasion, as well as moral costs associated with evading taxes.

If firms hiring highly educated workers have a harder time concealing their activities than firms hiring manual workers, then  $\kappa_h > \kappa_m$ . This is the case if, for example, third party reporting is more common for highly educated or, as assumed in Kleven et al (2011), the marginal costs of evasion increases with amount of income evaded. Although this is likely to be the case, we do not a priori impose any restriction on the values of  $\kappa_l, l = h, m$ .

In order to improve the transparency of the model, we disregard taxation, expected punishment and concealment costs on the worker side. This is of no importance for the results.

The unemployed worker  $i$  allocates search between the two sectors,  $\sigma_{li}$ , in order to maximize the value of unemployment,  $rU_{li}$ . A necessary condition for an interior solution is that  $\gamma < 1$ , which holds by assumption. The first order condition can be written as:

$$\frac{(1 - \sigma_{li})^{1-\gamma}}{(\sigma_{li})^{1-\gamma}} = \left( \frac{\theta_l^F}{\theta_l^I} \right)^{\frac{1}{2}} \frac{E_l^F - U_{li}}{E_l^I - U_{li}}, l = m, h. \quad (8)$$

Workers allocate their search between sectors to equalize the net marginal returns to search effort across the two sectors.

### 2.3 Wage determination

When a worker and firm meet they bargain over the wage,  $w_{li}^j$ , taking economy wide variables as given. The first order conditions from the Nash bargaining with equal bargaining power for workers and firms, can be written as:

$$J_l^F = (E_l^F - U_l)(1 + z), l = m, h, \quad (9)$$

$$J_l^I = (E_l^I - U_l)(1 + p + \kappa_l), l = m, h, \quad (10)$$

where we have imposed symmetry and the free entry condition,  $V_l^j = 0$ ,  $j = F, I$ ,  $l = m, h$ .

We can now derive an equation determining how search is allocated between the formal and the informal sectors in a symmetric equilibrium by substituting (9) and (10) into (8) and using that  $J_l^F = \frac{ky}{q_l^F}$  and  $J_l^I = \frac{ky}{q_l^I}$  from (5) and (7) together with free entry. This yields:

$$\frac{(1 - \sigma_l)^{1-\gamma}}{(\sigma_l)^{1-\gamma}} = \frac{\theta_l^F}{\theta_l^I} \psi_l, \quad (11)$$

where

$$\psi_l = \frac{1 + p\alpha + \kappa_l}{1 + z}$$

is the cost wedge between the informal sector and the formal sector. When workers allocate their search between the formal and the informal sectors in equilibrium, they account for the wedge,  $\psi_l$ , and for the formal relative to the informal sectorial tightness,  $\theta_l^F/\theta_l^I$ . It follows that relatively more search will be directed towards the formal sector if expected punishment plus concealment costs are higher than the tax payments, i.e. if  $\psi_l > 1$ , and

if formal sector tightness exceeds informal sector tightness, i.e.,  $\theta_l^F/\theta_l^I > 1$ . And vice versa when  $\psi_l < 1$  and  $\theta_l^F/\theta_l^I < 1$ .

By use of equation (1)-(7) and (11) in equations (9) and (10), equilibrium producer wages,  $\omega_l^j$ , are given by:

$$\omega_l^F = w_l^F (1 + z) = \frac{1}{2} y_l \left( 1 + k \frac{\theta_l^F}{(1 - \sigma_l)^{1-\gamma}} \right), l = m, h, \quad (12)$$

$$\omega_l^I = w_l^I (1 + p\alpha + \kappa_l) = \frac{1}{2} y_l \left( 1 + k \frac{\theta_l^I}{\sigma_l^{1-\gamma}} \right), l = m, h. \quad (13)$$

An increase in tightness,  $\theta_l^j$ , makes it easier for an unemployed worker to find a job, and at the same time harder for a firm to fill a vacancy. This improves the worker's relative bargaining position, resulting in higher wage demands. An increase in search, will instead increase the firm's relative bargaining position. This follows as firms then will find it easier to match with a new worker in case of no agreement. The improved bargaining position for firms moderates wage pressure.

## 2.4 Labour market tightness

Labour market tightness for the formal sector and the informal sector are determined by equation (4),(5), (6) and (7) using the free entry condition and the wage equations (12) and (13):

$$k(r + s) (\theta_l^F)^{\frac{1}{2}} = \frac{1}{2} \left( 1 - \frac{k\theta_l^F}{(1 - \sigma_l)^{1-\gamma}} \right), l = m, h, \quad (14)$$

$$k(r + s) (\theta_l^I)^{\frac{1}{2}} = \frac{1}{2} \left( 1 - \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right), l = m, h, \quad (15)$$

By use of the equilibrium search allocation equation in (11), where  $\frac{\theta_l^I}{(\sigma_l)^{1-\gamma}} = \frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \psi_l$ , in (15) it becomes clear that the wedge,  $\psi_l$ , is the crucial factor

determining the size of the formal sector in relation to the informal sector.<sup>7</sup> When  $\psi_l > 1$ , and thus expected punishment plus concealment costs are higher than payroll taxes, informal sector producer wages are higher than formal sector producer wages. In this case it is relatively more attractive for firms to enter the formal sector, which makes formal sector tightness exceed informal sector tightness. Hence, we obtain that  $\theta_l^F > \theta_l^I$  and  $\sigma_l < \frac{1}{2}$ ,  $l = m, h$  if  $\psi_l > 1$ . And vice versa when  $\psi_l < 1$ .

As the formal sector exceeds the informal sector in size in most western countries, it is most realistic to consider the case when  $\psi_l > 1$ . This implies considering the situation when the expected punishment rate plus concealment costs exceed the tax rate, i.e.,  $p\alpha + \kappa_l > z$ , which does not seem unrealistic given a broad interpretation of concealment costs. In fact, as discussed in the introduction, positive concealment costs  $\kappa_l > 0$  such that  $p\alpha + \kappa_l > z$  could potentially explain the puzzle of why we observe a relatively small informal sector although we, at the same time, observe rather low audit rates and fairly modest fines, i.e.,  $p\alpha < z$ . However, we do not a priori impose any restrictions on the size of  $\psi_l, p\alpha$  or  $\kappa_l$  when deriving the results in this paper. When discussing results that depends on the size of  $\psi_l$ , however, we focus the discussion on, what we believe is the most realistic case.

## 2.5 Education

When workers decide whether to acquire higher education or remain a manual worker, they compare the value of unemployment as an educated worker to the value of unemployment as a manual worker. Workers with low ability find it too costly in terms of effort to acquire higher education, whereas

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<sup>7</sup>Relative tightness is determined by:  $\theta_l^F/\theta_l^I = \left( \frac{1-k\theta_l^F(1-\sigma_l)^{\gamma-1}}{1-k\theta_l^I(1-\sigma_l)^{\gamma-1}\psi_l} \right)^2$   $\begin{matrix} > 1 \text{ if } \psi_l > 1 \\ = 1 \text{ if } \psi_l = 1 \\ < 1 \text{ if } \psi_l < 1 \end{matrix}$

high ability workers find it more than worthwhile to do so since they face lower costs of education. The marginal worker has an ability level,  $\hat{e}$ , which makes him just indifferent between acquiring higher education and remaining a manual worker. We write the condition determining the ability level of the marginal worker as:

$$rU_h(\hat{e}) = rU_m. \quad (16)$$

By using equations (1)-(3), it is clear that workers proceed to higher education if the expected income gain of education exceed their cost of education. However, as wages are endogenous, we can use equations (1) and (16) together with the first order conditions for wages, and equations (5), (7), (11), together with the free entry condition. This gives the following rewriting of condition (16):

$$c(\hat{e}) = \frac{k}{1+z} (y_h o_h - y_m o_m), \quad (17)$$

where  $o_l = \theta_l^F / (1 - \sigma_h)^{1-\gamma}$ ,  $l = h, m$ . Equation (17) gives  $\hat{e}$  as a function of the endogenous variables  $\theta_l^F$  and  $\sigma_l$ ,  $l = m, h$ . Workers with  $e \leq \hat{e}$ , choose not to acquire education, whereas workers with  $e > \hat{e}$  acquire education. Hence,  $\hat{e}$  and  $1 - \hat{e}$  constitute the manual and educated labour forces, respectively.

The right hand side in equation (17) is the expected income gain of attaining education. This gain needs to be positive in order for, at least some, worker to proceed to higher education. The fact that productivity is higher for highly educated workers, which gives rise to an educational wage premium, provides incentives for higher education. However, higher education may potentially also be associated with losses in expected income. For example, if concealment costs are higher for highly educated workers, i.e.,  $\kappa_h > \kappa_m$ , relatively more attractive informal employment opportunities for manual workers will be foregone in case of higher education. This reduces the incentives for education.

Clearly, in order to study the non-trivial case, where at least some workers proceed to higher education, it is necessary to assume that there is a net gain in expected income of higher education. Thus, we need to assume that productivity differences between manual and highly educated workers are sufficiently high, i.e.,  $y_h/y_m > o_m^F/o_m^I$ . Moreover, to guarantee a non-trivial interior solution where at least some, but not all, individuals choose to acquire education, the individual with highest ability face a very low costs of education, more specifically  $c(1) = 0$ , and the individual with the lowest ability face very high cost of education, i.e.,  $\lim_{e \rightarrow 0} c(e) = \infty$ . See the appendix for the proof of existence of  $\hat{e} \in (0, 1)$ .

## 2.6 Employment and Unemployment

The equations determining the employment rates in the formal sector and the informal sector,  $n_l^F, n_l^I$ , and the unemployment rates,  $u_l, l = m, h$ , are given by the flow equilibrium equations and the labour force identity.<sup>8</sup> The official unemployment rate  $u_l^o$ , is given by  $u_l^o = u_l + n_l^I$ . Solving for the employment and unemployment rates yield:

$$n_l^I = \frac{\lambda_l^I}{s + \lambda_l^I + \lambda_l^F}, n_l^F = \frac{\lambda_l^F}{s + \lambda_l^I + \lambda_l^F}, l = m, h, \quad (18)$$

$$u_l = \frac{s}{s + \lambda_l^I + \lambda_l^F}, u_l^o = \frac{s + \lambda_l^I}{s + \lambda_l^I + \lambda_l^F}, l = m, h. \quad (19)$$

A comparison of the unemployment rates for manual and highly educated workers requires assumptions about the size of the concealment costs. If concealments cost are higher for educated workers, i.e.,  $\kappa_h > \kappa_m$ , the official unemployment rate is always lower for highly educated workers than

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<sup>8</sup>For highly educated workers  $\lambda_h^I u_h \hat{e} = s n_h^I \hat{e}$ ,  $\lambda_h^F u_h \hat{e} = s n_h^F \hat{e}$ , and  $n_h^F + n_h^I = 1 - u_h$ , and for manual workers  $\lambda_m^I u_m (1 - \hat{e}) = s n_m^I (1 - \hat{e})$ ,  $\lambda_m^F u_m (1 - \hat{e}) = s n_m^F (1 - \hat{e})$ , and  $n_m^F + n_m^I = 1 - u_m$

for manual workers, i.e.,  $u_h^o < u_m^o$ . This is also what is observed in data. However, if furthermore,  $\psi_l > 1, l = h, m$  and hence the informal sector is smaller than the formal sector, the actual unemployment rate is higher for the highly educated workers,  $u_h > u_m$ , i.e. in this case, manual workers have a lower actual unemployment rate than highly educated workers if we also include workers in the informal sector into the employment measure.

The following proposition summarizes the results:

**Proposition 1** *The official unemployment rate is lower for highly educated workers,  $u_h^o < u_m^o$ , if they face higher concealment costs,  $\kappa_h > \kappa_m$ . The actual unemployment rate is higher (lower) for highly educated workers,  $u_h > u_m$  ( $u_h < u_m$ ), if they face higher concealment costs,  $\kappa_h > \kappa_m$ , and these concealment costs are high (low) enough to induce  $\psi_l > 1$  ( $\psi_l < 1$ ),  $l = m, h$ .*

The actual and official total number of unemployed workers are given by:

$$\begin{aligned} U_{TOT} &= \hat{e}u_m + (1 - \hat{e})u_h, \\ U_{TOT}^o &= \hat{e}u_m^o + (1 - \hat{e})u_h. \end{aligned}$$

### 3 Comparative statics

This section is concerned with the impact of more severe punishment of informal activities on labour market performance and educational attainment. We only consider fully financed changes in enforcement policies. Hence, changes in the audit rate and the punishment fees are always followed by adjustments in the tax rate so as to balance the government budget constraint given by:

$$\hat{e} \sum_{j=F,I} n_m^j w_m^j z + (1 - \hat{e}) \sum_{j=F,I} n_h^j w_h^j p\alpha = R, \quad (20)$$

where  $R$  is the exogenous revenue requirement.



From (20) it follows that an increase in the audit rate or the punishment fee,  $p$  or  $\alpha$ , or an increase in the tax rate,  $z$ , will, for a given tax base, always increase government revenues. The tax base may, however, fall and thereby reduce revenues. Throughout the analysis we will assume that we are located on the positively sloped side of the "Laffer curves". This implies that the direct effect of taxation and punishment on government revenues will always dominate the impact on revenues due to that the tax base may be reduced. An increase in the audit or punishment rate then always calls for a reduction in the tax rate in order to regain a balanced government budget. A fully financed increase in the punishment of the informal sector then induces  $\psi_l$  to increase both because  $p\alpha$  increases and because  $z$  falls.

In the government budget restriction in (20), potential auditing costs are left out. To include auditing costs will not affect any of the results derived in the positive analysis below. However, it affects the welfare analysis as it tends to favour costless taxation and punishment fees at the expense of auditing. The welfare analysis thus includes a separate section deriving the implications for the case including auditing costs.

Proofs of all Propositions follow from straightforward comparative statics and are available upon request.

### 3.1 Sector Allocation

The effects on the allocation of search and employment across the formal and the informal sector are summarized in the following proposition.

**Proposition 2** *A fully financed increase in the audit rate,  $p$ , or the punishment fee,  $\alpha$ , will reallocate search intensity and employment towards the formal sector, i.e.,  $\sigma_l$  falls,  $n_l^F$  increases, and  $n_l^I$  falls.*

More zealous enforcement will make informal work less attractive, inducing unemployed workers to reallocate their search effort towards the formal

sector. However, when search is reallocated towards the formal sector, the wage bargaining position strengthens for firms in the formal sector whereas it falls for firms in the informal sector. The lower producer wages in the formal sector stimulates formal firms to open vacancies, while at the same time, informal firms are discouraged to open new vacancies as they now face higher producer wages. As a consequence of that both vacancies and search effort is reallocated towards the formal sector, the formal sector employment rate increases at the expense of informal employment.

### 3.2 Unemployment Rates

As became clear in proposition 2, employment in the formal sector increases at the expense of employment in the informal sector following more severe punishment of the informal sector. While this is somewhat expected, it is a priori not clear what would happen to the unemployment rates. We have the following results:

**Proposition 3** *A fully financed increase in the audit rate,  $p$ , or in the punishment fee,  $\alpha$ , will always cause the official unemployment rate ( $u_l^o$ ) to fall, whereas the actual unemployment rate ( $u_l$ ) increases if  $\psi_l > 1$  (falls if  $\psi_l < 1$ ).*

The actual unemployment rates increase with more severe punishment of informal work if  $\psi_l > 1$ . The reason for this is that the large concealment costs discourages workers from searching, and firms from opening vacancies, in the informal sector. In fact, too few firms and too little search are allocated into the informal sector from an efficiency point of view. Increased punishment of the informal sector will encourage further reallocation of search and workers away from the informal sector, where relatively efficient search methods are used, towards the formal sector. Total search efficiency then falls, inducing unemployment to increase.

The fact that search becomes less efficient when reallocated towards the formal sector also has an impact on unemployment working through wage formation and tightness. As search is reallocated towards the formal sector, the wage demand is moderated in the formal sector and exaggerated in the informal sector. As search efficiency in the formal sector increases by less than search efficiency in the informal sector is reduced, the informal sector wage push will dominate the formal sector wage moderation. Thus, the incentives to open up a vacancy in the formal sector subseeds the disincentives to open up a vacancy in the informal sector; formal sector tightness will increase by less than informal sector tightness falls when  $\psi_l > 1$ .

The opposite holds if  $\psi_l < 1$ . In this case too much search, and too many firms, are allocated into the informal sector as there is a relative cost advantage of producing underground. Total search efficiency would then improve when the government tries to combat the informal sector.

The official unemployment rate always falls with more harsh punishment of informal activities as workers to a larger extent becomes formally employed. In this unemployment measure, workers in the informal sector were counted as unemployed to start with.

### 3.3 Education

From (17) it is clear that more severe punishment of the informal sector affects the number of educated workers as such policy increases  $\psi_l$ . This effect is further reinforced if the tax rate is reduced in order to assure a balanced government budget as the increase in  $\psi_l$  is reinforced by a reduction in  $z$ . However, a reduced payroll tax rate will also have a direct effect on the stock of educated workers. More specifically, a reduction in the tax rate,  $z$ , for a given wedge, will increase the number of educated workers. This follows as taxation is more harmful to high income earners, and consequently a tax

reduction will improve the income relatively more for high income earners.

However, let us first consider the impact of a more zealous enforcement policy on education, for a given tax rate. We have the following results:

**Proposition 4** *An increase in the audit rate,  $p$ , or the punishment fee,  $\alpha$ , will increase (reduce) the number of educated workers if the relative productivity of education is in the following range  $\frac{y_h}{y_m} \in \left[ \frac{o_m}{o_h}, g(\kappa_h, \kappa_m) \frac{o_m}{o_h} \right] \left( \frac{y_h}{y_m} \in \left[ g(\kappa_h, \kappa_m) \frac{o_m}{o_h}, \infty \right) \right)$  where  $g(\kappa_h, \kappa_m) > 1$  if  $\kappa_h > \kappa_m$ .*

**Proof.** We know from above that the existence of an interior solution of  $\hat{e}$  requires that  $y_h/y_m > o_m/o_h$ . Differentiating the educational equation with respect to expected punishment reveals that the impact on education is determined by the sign of  $y_m |do_m/d(p\alpha)| - y_h |do_h/d(p\alpha)|$  which is equal to the sign of  $y_h/y_m - g(\kappa_h, \kappa_m)(o_m/o_h)$ . See the appendix for a full proof. ■

The impact of a more zealous enforcement policy on educational attainment depends on how attractive underground work is to manual and educated workers, respectively. When concealment costs are higher for highly educated workers, more zealous enforcement policies tend to induce more workers to educate themselves. This follows as  $\kappa_h > \kappa_m$  implies that manual workers to a larger extent face informal labour market opportunities. Therefore, more zealous enforcement policies, which makes it less attractive to work in the informal sector, will be more harmful to manual workers.

This effect may, however, be counteracted by the fact that highly educated workers have higher productivity and therefore also earn higher wages. As also informal activities are highly productive for these workers, this implies that more harsh punishment, in this perspective, are more harmful for highly educated worker. Thus, even if highly educated workers face less informal employment opportunities, these opportunities are more profitable. This reduces educational incentives.

Which of the two effects dominate will thus depend on how sizable the differences in informal employment opportunities and productivity are. If underground employment opportunities in an economy foremost are available to manual workers, more harsh punishment of underground activities will push more workers into education. Thus increasing the stock of educated workers in the economy. However, if these employment opportunities to a large extent also are available for highly educated workers, harder punishment will harm highly educated workers more as these opportunities are more profitable to productive workers. This leads to that less workers educate themselves.

Note that proposition 4 only provides the sufficient conditions for when the educational stock increases and when it falls with more harsh punishment of the informal sector without considering the financing of the reform. Provided that we are located on the positively sloped side of the Laffer curve, we can conclude the following:

**Proposition 5** *If an increase in the audit rate,  $p$ , or in the punishment rate,  $\alpha$ , increases the number of educated workers as given by proposition 4, the financing of the reform will further increase the stock of educated workers.*

This simply follows as taxation as a direct effect is more harmful for high income earners, and consequently a tax reduction, in order to maintain a balanced government budget, will be more beneficial for high income earners, thus encouraging educational attainments.

### 3.4 Unemployment

From proposition 3, 4 and 5, it follows that more severe punishment of the informal sector potentially increases the total number of unemployed workers. If the formal sector is larger than the informal sector, the unemployment

rates for both manual and highly educated workers are augmented. Moreover, if informal employment opportunities to a significantly larger extent are available for manual workers, more workers will attain higher education when informal activities is punished more severely. This tends to increase total unemployment as the actual unemployment rate, including informal work, is higher for highly educated workers. Thus, in this case, total unemployment increases both because the unemployment rates for all workers increase, and because workers are reallocated towards the sector where the unemployment rate is highest. More generally, the proposition summarizes the result:

**Proposition 6** *A fully financed increase in the audit rate,  $p$ , or in the punishment fee,  $\alpha$ , increases(decreases) the number of unemployed workers if the relative productivity of education is in the following range  $\frac{y_h}{y_m} \in \left[ \frac{o_m}{o_h}, g(\kappa_h, \kappa_m) \frac{o_m}{o_h} \right]$  where  $g(\kappa_h, \kappa_m) > 1$  if  $\kappa_h > \kappa_m$ , and  $\psi_l > 1 (< \psi_l)$ .*

## 4 Numerical Simulations

In this section we perform simulations of the model in order to quantify the effects of enforcement policies on labour market performance and educational attainments. The educational costs are captured by the following cost function:  $c(e) = g(1 - e)^t$ , where the parameters  $g$  and  $t$  are used to match the OECD average fraction of 25 to 64 years old with tertiary education of 27.5 percent (OECD 2009). The year is used as the basic time unit, and the productivity of manual workers are normalized to unity,  $y_m = 1$ . The discount rate is set to  $r = 0.08$  as is the separation rate,  $s = 0.08$  (see Millard and Mortensen 1997).

The productivity of highly educated workers  $y_h$ , the hiring cost parameter,  $k$ , the search efficiency parameter,  $\gamma$ , and the concealment costs,  $\kappa_h$  and  $\kappa_m$ , are set so as to match an average observable unemployment rate of

10%, with an observable unemployment rate for manual workers of slightly less than 12%, and for highly educated workers of about 6%. These official unemployment rates are slightly higher than the pre-financial crises average in the OECD countries. The reason for this is partly that the model overestimates the official unemployment rates as all informally employed workers are assumed to be unemployed job searchers.<sup>9</sup> The employment rates for the informal sector are chosen to be between 5% and 15%. The parameter values we use are the following :

$r$	$s$	$\kappa_h$	$\kappa_m$	$k$	$\gamma$	$y_h$	$y_m$	$p\alpha$	$M$	$g$	$t$
0.08	0.08	0.9	0.75	0.4	0.35	1.25	1	0.05	3	0.5	1.1

In this case we obtain the values given in Table 1. The endogenous payroll tax rate,  $z$ , which always balances the government budget constraint is initially equal to 20%.

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<sup>9</sup>Based on labour market surveys, the official unemployment rate is measured by  $u^o = \frac{U^o}{LF^o}$ , where the  $U^o$  is the total number of unemployed job searchers and  $LF^o$  is the official labour force. In the model, everyone who is employed in the informal sector belongs to the labour force which implies that  $u^o = \frac{U^o + E^I}{U + E^F + E^I}$ . However, if some workers employed in the informal sector is counted as out of the labour force, which is likely the case in the labour statistic, the observed unemployment rate in the labour statistics measure  $u^o = \frac{U^o + \alpha E^I}{U + E^F + \alpha E^I}$ , where  $\alpha$  is the share of the informal workers being out of the labour force. Thus an  $\alpha < 1$  implies that the model value of the official unemployment rate should match to a higher value of unemployment than suggested by the labour statistics. This is of no importance for the analytical results, although it biases the values slightly in the numerical analysis.

Table 1:	Impact of higher expected punishment		
	$p\alpha = 0.05$	$p\alpha = 0.07$	$p\alpha = 0.09$
$u_h, u_m$	0.0215, 0.0204	0.0217, 0.0207	0.0219, 0.0210
$u_h^o, u_m^o$	0.0574, 0.1169	0.0469, 0.1005	0.0380, 0.0850
$1 - \hat{e}$	0.2750	0.2780	0.2811
$U_{TOT}, U_{TOT}^o$	0.0207, 0.1005	0.0210, 0.0856	0.0212, 0.0718
$n_h^I/n_h^F, n_m^I/n_m^F$	0.0381, 0.1092	0.0265, 0.0888	0.0168, 0.0700
$z$	0.2000	0.1899	0.1793

When expected punishment of informal activities become more zealous, unemployment increases but as informal sector employment falls, observable unemployment decreases too. In this case education increases. The impact of stricter punishment policies are illustrated in Figure 2.

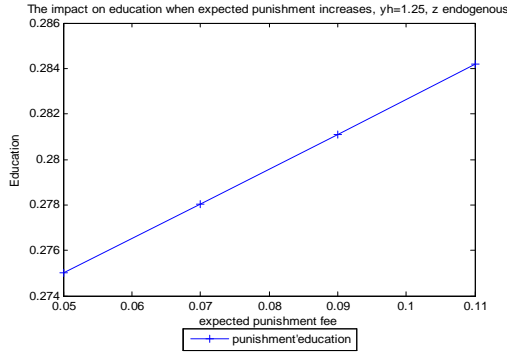


Figure 2

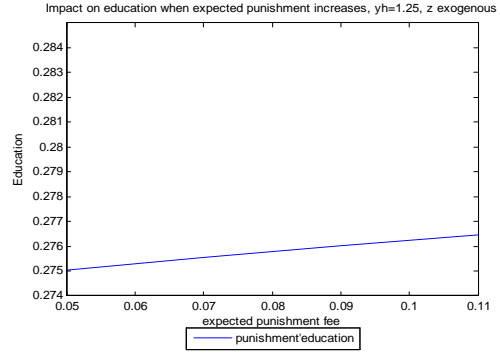


Figure 3

The analytical part confirmed that tighter enforcement policy had an ambiguous effect on educational attainments. Harsher punishment tends to increase educational attainments as manual workers face more informal employment opportunities when  $\kappa_h > \kappa_m$ , i.e., the "employment opportunity effect". On the other hand, more severe punishment tends to reduce educational attainments because highly educated workers are more productive,



i.e., the "productivity effect". In equilibrium (when  $z$  adjusts so to balance the government revenue), a reduction in  $z$  reinforces the employment opportunity effect. We observe in our benchmark case, see Figure 3, that when  $z$  is exogenously fixed, educational attainments increase with stricter punishment of informal work, although the effect is smaller.

We showed analytically that the relative importance for the two counter-acting effects on educational attainments hinged on differences in  $\kappa_l$  and  $y_l$ . But even for very high productivity levels facing the highly educated workers, education increases with  $p\alpha$  for a given  $z$ . We increase the parameter  $g$  simultaneously so to keep the initial educational level at  $1 - \hat{e} = 0.275$ . As shown analytically in proposition 4, letting  $\kappa_m$  increase to the same size as  $\kappa_h$ , causes the productivity effect to dominate, leading to a reduction in the educational attainments for a given  $z$ . This is, however, a less plausible outcome in terms of the relative size of informal-formal employment.

Sensitivity analysis with respect to the other parameters of the model, reveals that the results are sensitive to changes in  $\gamma$ . When  $\gamma$  is relative small, i.e.  $\gamma = 0.25$ , and  $\kappa_m$  increases to  $\kappa_m = 0.85$  education falls for fixed  $z$ . See Figure 4. However allowing for  $z$  to adjust to maintain government budget balance reestablishes the positive impact on education.

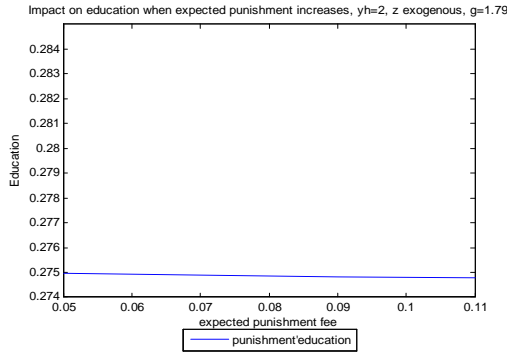


Figure 4

## 5 Welfare

This section is concerned with welfare analysis and the optimal design of punishment policies. As shown above, increasing the punishment fees or the audit rates affect the number of educated workers as well as the allocation of search and jobs across the formal and informal sectors. This is essential when considering the impact on welfare.

We make use of a utilitarian welfare function, which is obtained by adding all individuals' steady state flow values of welfare. This accounts for that both the formal and the informal economy generate welfare in the economy. The social welfare function is written as:

$$W = \hat{e}\tilde{W}_m + \int_{\hat{e}}^1 \tilde{W}_h de,$$

where

$$\begin{aligned} \tilde{W}_m &= u_m r U_m + \sum_{j=F,I} n_m^j r E_m^j + \sum_{j=F,I} n_m^j r J_m^j + \sum_{j=F,I} v_m^j r V_m^j + n_m^I r J_m^{law}, \\ \tilde{W}_h &= u_h r U_h + \sum_{j=F,I} n_h^j r E_h^j + \sum_{j=F,I} n_h^j r J_h^j + \sum_{j=F,I} v_h^j r V_h^j + n_h^I r J_h^{law}. \end{aligned}$$

We assume that firms are owned by "renters" who do not work. This explains the presence of  $\sum_{j=F,I} n_m^j r J_m^j + \sum_{j=F,I} v_m^j r V_m^j$  and  $\sum_{j=F,I} n_h^j r J_h^j + \sum_{j=F,I} v_h^j r V_h^j$  in the welfare function. Moreover, we assume that the concealment costs for tax evasion facing firms are payments to "lawyers" who only engage in concealing taxable income for other firms. The welfare function therefore includes  $n_m^I r J_m^{law} = n_m^I w_m^I \kappa_m$  and  $n_h^I r J_h^{law} = n_h^I w_h^I \kappa_h$ . This assumption enables us to disregard from the waste associated with tax evasion if firms only pay these expenses to nobody.

By making use of the asset equations, imposing the flow equilibrium conditions as well as the government budget restriction in (20), and considering the case of no discounting, i.e.,  $r \rightarrow 0$ , we can write the welfare function as:

$$W = \hat{e}W_m + \int_{\hat{e}}^1 W_h de, \quad (21)$$

$$W_m = (1 - u_m)y_m - u_mk y_m \Theta_m, \quad (22)$$

$$W_h = (1 - u_h)y_h - u_hk y_h \Theta_h - c(e), \quad (23)$$

where  $\Theta_l = (1 - \sigma_l)^\gamma \theta_l^F + \sigma_l^\gamma \theta_l^I$ ,  $l = m, h$ . This welfare measure is analogous to the welfare measure described in, for example, Pissarides (2000) as it includes aggregate production minus total vacancy costs, i.e. note that  $u_l \Theta_l k = (v_l^F + v_l^I) k$ ,  $l = m, h$ . With the assumption of risk neutral individuals, we ignore distributional issues and hence wages will not feature in the welfare function.

Let us first derive the socially optimal choice of tightness, search and stock of educated workers by maximizing the welfare function in (21)-(23) with respect to  $\theta_m^F, \theta_m^I, \theta_h^F, \theta_h^I, \sigma_m, \sigma_h$  and  $\hat{e}$ . The socially optimal solution are solved from the following seven conditions:

$$\sigma_l^{*\gamma-1} - (1 - \sigma_l^*)^{\gamma-1} = 0, \quad \rightarrow \quad \sigma_l^* = \frac{1}{2} \quad l = m, h \quad (24)$$

$$-sk (\theta_l^{*I})^{\frac{1}{2}} + \frac{1}{2} \left[ 1 - \frac{k\theta_l^{*I}}{\left(\frac{1}{2}\right)^{1-\gamma}} \right] = 0, \quad l = m, h, j = F, I \quad (25)$$

$$(y_h - y_m) \frac{k\theta_l^{*I}}{\left(\frac{1}{2}\right)^{1-\gamma}} - c(\hat{e}^*) = 0. \quad (26)$$

We can now compare the socially optimal solution with the market outcome. From (11), (14), and (15) it follows that the market solution for search and tightness coincides with the socially optimal allocation if the imposed tax and punishment policy are such that  $\psi_m = \psi_h = 1$ .<sup>10</sup>

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<sup>10</sup>When  $\psi_m = \psi_h = 1$  is imposed on the private solution, it follows from (14), (15) that tightness in the formal and the informal sector is equal, and that search must be split

This conclusion is intuitive as any policy that induces a deviation of the  $\psi^j, j = m, h$ , from unity, implies a favourable treatment of the formal or the informal sector which, in turn, induces a distortion in the sectorial allocation of search and tightness between the formal and informal sectors. For example, if search to a larger extent is allocated to the formal or informal sector instead of the other, search is inefficiently used as less efficient search methods in that sector needs to be used. Moreover, as discussed in relation to proposition 3, a favourable treatment of either the formal or the informal sector induce too many firms to open vacancies in that sector without accounting for the externality they impose on others. In fact, unemployment is minimized when the allocation of search and tightness across the formal and informal sector is equal, and so is vacancy costs. Thus, welfare is maximized when search and tightness are allocated equally across the formal and the informal sector.

Now let us compare the socially optimal stock of educated workers with the educational outcome induced by the market. As the market outcome in terms of sectorial allocation of search and tightness coincided with the socially optimal one when the government lets the market face  $\psi_m = \psi_h = 1$ , we evaluate also the private outcome of education under these conditions. This yields the following market outcome of the stock of educated workers:

$$(y_h - y_m) \frac{k\theta_l^I}{(1+z)\left(\frac{1}{2}\right)^{1-\gamma}} - c(\hat{e}) = 0. \quad (27)$$

It immediately follows that a tax and punishment policy which implies that  $\psi_m = \psi_h = 1$ , will not provide incentives to the market to generate a socially optimal stock of educated workers. Comparing (26) and (27), in fact, reveals that the market outcome induces too few workers to educate themselves if formal and informal sector jobs face uniform treatment in terms 

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equally between the formal and the informal sector, i.e.,  $\sigma = \frac{1}{2}$  from (11). Imposing  $\sigma = \frac{1}{2}$  and  $\theta_l^F = \theta_l^I, l = m, h$ , and assuming  $r = 0$  in (14) and (15), yields the same expression as (25).

of  $\psi_m = \psi_h = 1$ . This follows as taxes, captured by  $(1 + z)$  in (27), hits highly educated workers more severely than manual workers, which reduces the incentives of education. From this we can conclude that welfare would increase if more workers chose to educate themselves when  $\psi_m = \psi_h = 1$ .

This discussion brings us to the government's explicit choice of tax and punishment policy. How should the government punish informal work in order to maximize welfare?

## 5.1 Optimal punishment policy

The welfare analysis above indicates that it may be optimal to punish tax evading activities carried out by manual workers more severely than those carried out by highly educated workers. For example, if concealment costs are higher for highly educated workers, a punishment policy with  $\psi_m = \psi_h = 1$  is only possible if the manual workers to a larger extent than highly educated workers face punishment of informal activities. That is,  $p\alpha$  have to be set relatively higher for manual workers if  $\kappa_m < \kappa_h$  in order to induce  $\psi_m = \psi_h = 1$ .

This raises the question of whether it is possible or not to target the punishment fees and audit rates towards the sector employing manual vs highly educated workers. While governments potentially could, and in fact do,<sup>11</sup> target their audits to specific sectors, i.e. allowing for  $p_m$  to differ from  $p_h$ , this is less likely the case for the fee rates.

To find the socially optimal choice of audit rates for the sector employing manual workers and the sector employing highly educated workers, the welfare function in (21)-(23) is maximized by choice of  $p_m$  and  $p_h$  subject to the market reactions given by (11), (14), (15), (17) and (19) and the government budget restriction in (20). This yields the following first order conditions:

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<sup>11</sup>See, for example, Kleven et al 2011.

$$\frac{dW}{dp_m} = \hat{e} \frac{dW_m}{d\psi_m} \frac{d\psi_m}{dp_m} + \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_m} = 0, \quad (28)$$

$$\frac{dW}{dp_h} = (1-\hat{e}) \frac{dW_h}{d\psi_h} \frac{d\psi_h}{dp_h} + \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_h} = 0, \quad (29)$$

where  $\frac{dW_l}{d\psi_l} = \left[ \sum_{j=F,I} \frac{dW_l}{d\theta_l^j} \frac{d\theta_l^j}{d\psi_l} + \frac{dW_l}{d\sigma_l} \frac{d\sigma_l}{d\psi_l} \right]$ ,  $j = m, h$ . Evaluating the first order conditions at the levels of  $p_m$  and  $p_h$  ensuring that  $\psi_m = \psi_h = 1$  turns out to be very convenient and gives:

$$\frac{dW}{dp_m} \Big|_{\psi_m=1} = \frac{dW}{d(1-\hat{e})} \frac{d(1-\hat{e})}{dp_m} > 0, \quad (30)$$

$$\frac{dW}{dp_h} \Big|_{\psi_h=1} = \frac{dW}{d(1-\hat{e})} \frac{d(1-\hat{e})}{dp_h} < 0. \quad (31)$$

Provided that we are located on the positively sloped side of the Laffer curves, we can conclude that:

**Proposition 7** *Welfare is maximized when the sector employing manual workers are audited to a larger extent than the sector employing highly educated workers, i.e.,  $p_m > p_h$  so to get  $\psi_h^* < 1 < \psi_m^*$  if  $\kappa_h \geq \kappa_m$ .*

**Proof.** Evaluate the first order conditions (28) and (29) at  $\psi_m = \psi_h = 1$ . From the socially optimal allocation of search and tightness,  $\psi_l = 1$  implies that  $\frac{dW_l}{d\theta_l^F} = \frac{dW_l}{d\theta_l^I} = \frac{dW_l}{d\sigma_l} = 0$ ,  $l = m, h$ . Then  $\frac{dW}{dp_m} \Big|_{\psi_m=1} = \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_m} > 0$ , and  $\frac{dW}{dp_h} \Big|_{\psi_h=1} = \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_h} < 0$  as  $\frac{dW}{d(1-e)} > 0$  from (26) and (27) and  $\frac{d(1-\hat{e})}{dp_m} > 0$ ,  $\frac{d(1-\hat{e})}{dp_h} < 0$  from (17). Thus welfare improves by reallocation of audits towards the manual sector. If  $\kappa_h = \kappa_m$ ,  $p_m = p_h$  at  $\psi_m = \psi_h = 1$ , welfare improves by setting  $p_m > p_h$ . If  $\kappa_h > \kappa_m$ , the results are reinforced as  $p_m > p_h$  already when  $\psi_m = \psi_h = 1$ , and welfare improves by further increasing  $p_m$  and reducing  $p_h$ . ■

The result in proposition 7 follows straightforwardly from the first order conditions when evaluated at the  $p_m$  and  $p_h$  which induces  $\psi_m = \psi_h = 1$ . The first term on the right hand side of equations (28) and (29) then disappears as the distortions in search and allocation of tightness across the formal and the informal sectors are fully eliminated. In this case there are no other distortions present except for that too few workers have chosen to educate themselves. Recall that this is a consequence of that taxation harms high income earners relatively more. This distortion can, however, be corrected for by increasing the audits in the manual sector and reducing them in the sector for highly educated workers, which is captured by the right hand side in (30) and (31). As informal sector work for manual workers becomes less attractive when the government increases the number of audits, manual workers are encouraged to acquire higher education. Similarly, less audits in the highly educated sector further encourages workers to acquire higher education.

If concealment costs are higher in the sector employing highly educated workers, i.e.,  $\kappa_h > \kappa_m$ , there are even further incentives for the government to focus their audits on the manual sector. This follows as high concealment costs work as a self-regulating punishment of informal sector activities. Thus, if concealment costs are higher in the sector employing highly educated workers, this sector will be in less need of audits as concealment costs will do part of the job limiting the size of the informal sector.

Moreover it follows that:

**Corollary 8** *The stock of educated workers is below its socially optimal value when the audit rates are chosen so to maximize welfare.*

**Proof.** See appendix. ■

When deciding on the optimal audit rates the government face a trade-off between two distortions, and it is never optimal to fully eliminate one of them. When the stock of educated workers is at its socially optimal level, there is an

inefficient allocation of search and jobs across the formal and informal sectors. Welfare then improves as the stock of educated workers is reduced below its socially optimal level as this will only be a second order effect in comparison to the improved welfare following a more efficient sectorial allocation.

## 5.2 Optimal punishment policy when concealment costs are high

In deriving the optimal audit rates in the previous section, it was implicitly assumed that audit rates could be chosen freely without restrictions. For example, according to proposition 7, the audit rates should be chosen such that  $p_m^* > p_h^*$  so to get  $\psi_h^* < 1 < \psi_m^*$ . However, this is only possible if concealment costs are not too high. If, for example,  $\kappa_h > z$  then  $\psi_h > 1$  even when  $p_h$  is very small. Replacing the first order condition in (29) with the appropriate Kuhn-Tucker conditions,  $\frac{dW}{dp_h} + \mu = 0$ ,  $p_h \geq 0$ , and  $\mu p_h = 0$ , where  $\mu$  is the Lagrange multiplier for the constraint  $p_h \geq 0$ , then suggest that the audit rate in the sector should be set as low as possible when  $\kappa_h > z$ . Concealment costs are simply high enough to self-regulate the size of the informal sector facing highly educated workers, and there is no need for additional audits of this sector.<sup>12</sup>

Taking off in real world observations from western economies, this may not be an unrealistic scenario. Evidence indicates that manual workers, or workers with a lower level of formal education, to a substantially larger degree face informal employment opportunities compared to highly educated workers. Pedersen and Smith (1998) using comprehensive survey data, find that almost half of the informal sector activities in Denmark are carried out within the construction sector. They also find that around 70 percent of the

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<sup>12</sup>This clearly holds also for the manual sector if concealment cost are higher than the tax rate.



total hours performed in the informal sector is carried out within the service sector or construction sector.

Potential explanations for why manual, in contrast to highly educated, workers engage in informal activities are that manual workers to a larger extent work in industries which handles cash-payments or are to a lesser extent subject to third-party reporting. Firms and workers in industries dealing with cash-payments, or which to a lesser extent are subject to third-party reporting, will find it easier, and thus less costly, to conceal their tax evasion. Taking this at face value implies that concealment costs for highly educated workers,  $\kappa_h$ , could be very large. If  $\kappa_h$  is assumed to approach infinity, informal employment opportunities facing highly educated workers will become infinitely small, leading to that basically no firms will post informal sector vacancies to highly educated workers and none of the highly educated workers will allocate search effort into the informal sector. All the results derived in propositions 1 to 6 accounts for this special case, including the now clear cut result that higher punishment fees, or a general increase in the audit rate, encourage more workers to educate themselves. This follows as less workers will remain as manual workers as the forgone informal employment opportunities when attaining education has become less attractive. Moreover, the socially optimal audit rate is again being determined by an audit rate which implies that  $p_m^*$  is set large enough so to get  $\psi_m^* > 1$ , although not high enough to induce an efficient stock of educated workers.

### 5.3 Optimal punishment policy including auditing costs

Adding costs of auditing to the government budget constraint in (20) yields:

$$\hat{e} \sum_{j=F,I} n_m^j w_m^j z + (1 - \hat{e}) \sum_{j=F,I} n_h^j w_h^j p \alpha - \varphi(p) = R, \quad (32)$$

where  $\varphi(p)$  is the cost of audits, and  $p$  is the total intensity of audits,  $p = p_m + p_h$ . Adding costs of auditing will have no impact on the results derived in the positive analyses. However, the welfare function in (21) is now equal to

$$W = \hat{e}W_m + \int_{\hat{e}}^1 W_h de - \varphi(p),$$

where the first order conditions for optimal audit rates in the sectors employing manual and highly educated workers are given by:

$$\frac{dW}{dp_m} = \hat{e} \frac{dW_m}{d\psi_m} \frac{d\psi_m}{dp_m} + \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_m} - \varphi'(p) = 0, \quad (33)$$

$$\frac{dW}{dp_h} = (1-\hat{e}) \frac{dW_h}{d\psi_h} \frac{d\psi_h}{dp_h} + \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_h} - \varphi'(p) = 0, \quad (34)$$

where, again,  $\frac{dW_l}{d\psi_l} = \left[ \sum_{j=F,I} \frac{dW_l}{d\theta_l^j} \frac{d\theta_l^j}{d\psi_l} + \frac{dW_l}{d\sigma_l} \frac{d\sigma_l}{d\psi_l} \right]$ ,  $j = m, h$ . The inclusion of auditing costs reduces the optimal level of audits in both sectors. The result from proposition 7, that welfare is maximized when the government to a larger extent targets its audits to the manual sector, i.e.,  $p_m > p_h$  if  $\kappa_h \geq \kappa_m$ , will however still hold.

## 6 Conclusion

There has recently been an intensified focus on issues related to tax evasion and informal activities from both a policy and research perspective.<sup>13</sup> The study by Kleven et al (2011), which conducted a large field experiment in Denmark, made it possible to address, and convincingly answer, a number of

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<sup>13</sup>The OECD recently initiated the "Global forum of transparency and exchange of information for tax purposes" (OECD, 2010), whereas the European commission conducted the first EU wide comparable questionnaire in order to increase the knowledge about tax evasion in Europe (see EC, 2007).

questions related to tax compliance behaviour, that had not been answered before.

This paper uses this knowledge to investigate the general equilibrium implications of informal sector activities on economic performance. A number of questions can be asked. How will informal employment opportunities affect labour market performance and educational attainments? Can informal jobs really be turned into formal jobs by more zealous punishment policies? And if so, to what extent will formal sector jobs replace jobs in the informal sector?

In order to address these questions, we develop a four sector equilibrium search and matching model with informal sector employment opportunities and educational choice. We find that informal activities reduces the incentives to acquire higher education if informal employment opportunities mainly are available to low educated workers. More zealous enforcement policies will in this case improve educational incentives as it reduces the attractiveness of remaining a low educated worker. Moreover, we find that stricter enforcement policies will create new jobs in the formal sector, although most likely to a lesser extent than the number of jobs destructed in the informal sector. This will lead to an increase in the actual unemployment rates although the official unemployment rates fall. Finally, characterizing the optimal enforcement policies, we find that relatively more audits should be targeted towards the sector employing low educated workers, otherwise a too low stock of educated workers could materialize.

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## 7 Appendix

### 7.1 Tightness relatively to search intensity

We show that  $\frac{\theta_t^F}{(1-\sigma_t)^{1-\gamma}} < \frac{\theta_x^F}{(1-\sigma_x)^{1-\gamma}}$  when  $\kappa_t > \kappa_x$  in the following way. Differentiating equations (14),(15) and (11) with respect to  $\kappa_l$  gives around the equilibrium:

$$\frac{d\theta_l^F}{d\kappa_l} = \frac{(1-\gamma) \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \frac{1}{2} \left(1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}}\right) \frac{1}{\theta_l^I}}{D_l} (1+z) > 0, \quad (35)$$

$$\frac{d\theta_l^I}{d\kappa_l} = -\frac{\frac{1}{2} \left(1 + \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}}\right) \frac{1}{\theta_l^F} (1-\gamma) k\theta_l^I \sigma_l^{\gamma-2}}{D_l} (1+z) < 0, \quad (36)$$

$$\frac{d\sigma_l}{d\kappa_l} = -\frac{\frac{1}{2} \left(1 + \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}}\right) \frac{1}{\theta_l^F} \frac{1}{2} \left(1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}}\right) \frac{1}{\theta_l^I}}{D_l} (1+z) < 0, \quad (37)$$

where  $D_l = \frac{(1-\gamma)}{\theta_l^F \theta_l^F 4(1-\sigma_l)\sigma_l} \left( \left( \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} + 1 \right) \left( 1 - \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) (1 - \sigma_l) + \left( \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} + 1 \right) \sigma_l \left( 1 - \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \right) > 0$ . Now, differentiating  $\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}$  with respect to  $\kappa_l$  gives:

$$\begin{aligned} \frac{d\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}}{d\kappa_l} &= \frac{d(1-\sigma_l)^{\gamma-1} \theta_l^F}{d\kappa_l} = (1-\sigma_l)^{\gamma-1} \theta_l^F \left( (1-\gamma)(1-\sigma_l)^{-1} \frac{d\sigma_l}{d\kappa_l} + \frac{1}{\theta_l^F} \frac{d\theta_l^F}{d\kappa_l} \right) \\ &= \frac{(1-\sigma_l)^{\gamma-1} \theta_l^F}{2} \left( - (1-\gamma)(1-\sigma_l)^{-1} \frac{\left( 1 + \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \right) \frac{1}{\theta_l^F} \frac{1}{2} \left( 1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \frac{1}{\theta_l^I}}{D_l} \right. \\ &\quad \left. + \frac{\frac{(1-\gamma)}{1-\sigma_l} \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \left( 1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \frac{1}{\theta_l^I}}{\theta_l^F D_l} \right) (1+z) \\ &= -\frac{(1-\sigma_l)^{\gamma-1} \theta_l^F (1-\gamma)}{4D_l} \frac{1}{1-\sigma_l} \left( 1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \frac{1}{\theta_l^I} \frac{1}{\theta_l^F} \left( 1 - \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \right) (1+z) < 0. \end{aligned}$$

Hence, if  $\kappa_t > \kappa_x$  then  $\frac{\theta_t^F}{(1-\sigma_t)^{1-\gamma}} < \frac{\theta_x^F}{(1-\sigma_x)^{1-\gamma}}$ .

## 7.2 Existence of $\hat{e} \in (0, 1)$ .

Consider the educational equation

$$c(\hat{e}) = \frac{k}{1+z} (y_h o_h - y_m o_m),$$

where  $o_l = \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}}$ ,  $l = h, m$ . For a non-trivial solution, there need to be a net gain in expected income of higher education. Thus,  $y_h/y_m > o_m/o_h$ . Moreover, to guarantee a non-trivial interior solution where at least some, but not all, individuals choose to acquire education, the individual with highest ability face a very low costs of education, more specifically  $c(1) = 0$ , and the individual with the lowest ability face very high cost of education, i.e.,  $\lim_{e \rightarrow 0} c(e) = \infty$ .

In case  $\kappa_h \leq \kappa_m$  then  $o_m/o_h < 1$  and hence  $y_h/y_m > o_m/o_h$  holds as  $y_h > y_m$ . If educated workers face higher concealment costs than manual workers  $\kappa_h > \kappa_m$ , then we need to assume that  $y_h/y_m > o_m/o_h$  holds, which is possible to ensure as the right hand side is independent of  $y_l$ .

### 7.3 Formal versus informal sector employment

Relative employment for formal versus informal workers depends on concealment costs  $\kappa_l, l = h, m$ . We consider the impact on relative employment of a higher  $\kappa_l$ . Noting that  $n_l^F/n_l^I = \lambda_l^F/\lambda_l^I = \left( (1 - \sigma_h)^\gamma (\theta_l^F)^{\frac{1}{2}} \right) / \left( \sigma_l^\gamma (\theta_l^I)^{\frac{1}{2}} \right)$  we need to examine the impact on relative transition rates from higher  $\kappa_l$ . Using equations (35)-(37) we can conclude that

$$\frac{d(n_l^F/n_l^I)}{d\psi_l} = \frac{\left( (1 - \sigma_l)^\gamma (\theta_l^F)^{\frac{1}{2}} \right) / \left( \sigma_l^\gamma (\theta_l^I)^{\frac{1}{2}} \right)}{d\psi_l} > 0.$$

Hence, when  $\kappa_t > \kappa_x$ , formal relatively to informal sector, is higher for  $t$  workers than for  $x$  workers,  $n_t^F/n_t^I > n_x^F/n_x^I$ .

### 7.4 Relative unemployment rates

Unemployment is increasing in concealment costs if  $\psi_l > 1$ . Hence if  $\kappa_t > \kappa_x$  then  $u_t > u_x$  if  $\psi_l > 1$ . We show that in the following way,  $u_t > u_x$  if and only if

$$\frac{s}{s + \lambda_x^F + \lambda_x^I} < \frac{s}{s + \lambda_t^F + \lambda_t^I}$$

if and only if

$$\lambda_t^F + \lambda_t^I < \lambda_x^F + \lambda_x^I$$

Hence, the condition holds if

$$\begin{aligned} \frac{d(\lambda_l^F + \lambda_l^I)}{d\kappa_l} &= \frac{(1 - \sigma_h)^\gamma (\theta_h^F)^{\frac{1}{2}} + \sigma_h^\gamma (\theta_h^I)^{\frac{1}{2}}}{d\kappa_l} = \\ &= \gamma \left( - (1 - \sigma_l)^{\gamma-1} (\theta_l^F)^{\frac{1}{2}} + \sigma_l^{\gamma-1} (\theta_l^I)^{\frac{1}{2}} \right) \frac{d\sigma_l}{d\kappa_l} + \frac{1}{2} \left( \frac{(1 - \sigma_h)^\gamma d\theta_l^F}{(\theta_h^F)^{\frac{1}{2}} d\kappa_l} + \frac{\sigma_h^\gamma d\theta_l^I}{(\theta_h^I)^{\frac{1}{2}} d\kappa_l} \right) < 0 \end{aligned}$$

We substitute for the derivatives and the first order condition for search intensity to obtain the condition

$$\begin{aligned} & \gamma \left( \frac{1}{\psi_l} \frac{1}{(\theta_l^F)^{\frac{1}{2}}} - \frac{1}{(\theta_l^I)^{\frac{1}{2}}} \right) \left( 1 + \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \left( 1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \\ & + (1-\gamma) \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \left( \frac{1}{(\theta_l^F)^{\frac{1}{2}}} \frac{1}{\psi_l} \left( \frac{1}{\psi_l} + \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) - \frac{1}{(\theta_l^I)^{\frac{1}{2}}} \left( 1 + \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \right) \end{aligned}$$

which is negative when  $\psi_l > 1$ , as then  $(\theta_l^F) > (\theta_l^I)$  giving  $\frac{1}{\psi_l} \frac{1}{(\theta_l^F)^{\frac{1}{2}}} < \frac{1}{(\theta_l^I)^{\frac{1}{2}}}$  and  $\left( \frac{1}{\psi_l} + \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) < \left( 1 + \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right)$ . Hence unemployment increases with  $\psi_l$  and hence  $u_t > u_x$  when  $\kappa_t > \kappa_x$ .

The official unemployment rate facing  $t$  workers is higher than the official unemployment rate facing  $x$  workers,  $u_t^o > u_x^o$  if and only if

$$\frac{s + \lambda_t^I}{s + \lambda_t^F + \lambda_t^I} > \frac{s + \lambda_x^I}{s + \lambda_x^F + \lambda_x^I}.$$

This holds if an only if

$$\lambda_x^F (s + \lambda_t^I) > \lambda_t^F (s + \lambda_x^I),$$

which is true when  $\lambda_x^F > \lambda_t^F$  and  $\lambda_x^I > \lambda_t^I$ , that is when  $\kappa_t > \kappa_x$ .

## 7.5 Impact of higher punishment on sector allocation

Raising the audit rate  $p_l$  or the punishment fee  $\alpha$ , increases the wedge,  $\psi_l = (1 + p\alpha + \kappa_l) / (1 + z)$ . Differentiating equations (14),(15) and (11) with respect to  $\psi_l$  gives around the equilibrium:

$$\begin{aligned} \frac{d\theta_l^F}{d\psi_l} &= \frac{\frac{(1-\gamma)}{1-\sigma_l} \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \frac{1}{2} \left( 1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \frac{1}{\theta_l^I} \frac{1}{\psi_l}}{D_l} > 0, l = m, h \\ \frac{d\theta_l^I}{d\psi_l} &= -\frac{\frac{1}{2} \left( 1 + \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \right) \frac{1}{\theta_l^F} (1-\gamma) k\theta_l^I \sigma_l^{\gamma-2}}{D_l} \frac{1}{\psi_l} < 0, l = m, h \\ \frac{d\sigma_l}{d\psi_l} &= -\frac{\frac{1}{2} \left( 1 + \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \right) \frac{1}{\theta_l^F} \frac{1}{2} \left( 1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \frac{1}{\theta_l^I} \frac{1}{\psi_l}}{D_l} < 0, l = m, h \end{aligned}$$



where  $D_l = \frac{(1-\gamma)}{\theta_l^F \theta_l^I 4(1-\sigma_l)\sigma_l} \left( \left( \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} + 1 \right) \left( 1 - \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) (1-\sigma_l) + \left( \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} + 1 \right) \sigma_l \left( 1 - \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \right) > 0$ . Hence, as  $\lambda_{li}^I = \sigma_{li}^\gamma (\theta_l^I)^{\frac{1}{2}}$  and  $\lambda_{li}^F = (1-\sigma_{li})^\gamma (\theta_l^F)^{\frac{1}{2}}$ , by inspection of equation (18) it follows that

$$dn_l^F/d\psi_l > 0, dn_l^I/d\psi_l < 0, l = m, h.$$

The impact on wages is then

$$\begin{aligned} \frac{d\omega_l^F}{d\psi_l} &= \frac{1}{2} y_l k \frac{d \frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}}{d\psi_l} < 0, l = m, h, \\ \frac{d\omega_l^I}{d\psi_l} &= \frac{1}{2} y_l k \frac{d \frac{\theta_l^I}{\sigma_l^{1-\gamma}}}{d\psi_l} > 0, l = m, h. \end{aligned}$$

as

$$\begin{aligned} \frac{d \frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}}{d\psi_l} &= \frac{d \left( (1-\sigma_l)^{\gamma-1} \theta_l^F \right)}{d\psi_l} = (1-\sigma_l)^{\gamma-1} \theta_l^F \left( (1-\gamma) (1-\sigma_l)^{-1} \frac{d\sigma_l}{d\psi_l} + \frac{1}{\theta_l^F} \frac{d\theta_l^F}{d\psi_l} \right) \\ &= -\frac{(1-\sigma_l)^{\gamma-1} (1-\gamma)}{4D_l} \frac{1-\gamma}{1-\sigma_l} \left( 1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \frac{1}{\theta_l^I} \left( 1 - \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \right) \frac{1}{\psi_l} < 0. \quad (38) \end{aligned}$$

and

$$\begin{aligned} \frac{d \frac{\theta_l^I}{\sigma_l^{1-\gamma}}}{d\psi_l} &= \frac{d \left( \sigma_l^{\gamma-1} \theta_l^I \right)}{d\psi_l} = \sigma_l^{\gamma-1} \theta_l^I \left( - (1-\gamma) \sigma_l^{-1} \frac{d\sigma_l}{d\psi_l} + \frac{1}{\theta_l^I} \frac{d\theta_l^I}{d\psi_l} \right) \\ &= \frac{\sigma_l^{\gamma-1} (1-\gamma)}{\psi_l 4D_l} \frac{1-\gamma}{\sigma_l} \left( 1 + \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} \right) \frac{1}{\theta_l^F} \left( 1 - \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) > 0 \end{aligned}$$

## 7.6 Impact of higher punishment on unemployment rates

Raising the audit rate  $p$  or the punishment fee  $\alpha$ , increases the wedge,  $\psi_l = (1 + p\alpha + \kappa_l) / (1 + z)$ . Differentiating equation (19) with respect to  $\psi_l$  gives:

$$\frac{du_l}{d\psi_l} = -\frac{s}{(s + \lambda_l^I + \lambda_l^F)^2} \left( \frac{d\lambda_l^F}{d\psi_l} + \frac{d\lambda_l^I}{d\psi_l} \right)$$

where

$$\begin{aligned} \frac{d(\lambda_l^F + \lambda_l^I)}{d\psi_l} &= \frac{(1 - \sigma_h)^\gamma (\theta_h^F)^{\frac{1}{2}} + \sigma_h^\gamma (\theta_h^I)^{\frac{1}{2}}}{d\psi_l} = \\ &= \gamma \left( -(1 - \sigma_l)^{\gamma-1} (\theta_l^F)^{\frac{1}{2}} + \sigma_l^{\gamma-1} (\theta_l^I)^{\frac{1}{2}} \right) \frac{d\sigma_l}{d\psi_l} + \frac{1}{2} \left( \frac{(1 - \sigma_h)^\gamma d\theta_l^F}{(\theta_h^F)^{\frac{1}{2}} d\psi_l} + \frac{\sigma_h^\gamma d\theta_l^I}{(\theta_h^I)^{\frac{1}{2}} d\psi_l} \right) \end{aligned}$$

Substitute for the derivatives and the first order condition for search intensity we obtain that  $du_l/d\psi_l$  has the same sign as

$$\begin{aligned} &-\gamma \left( \frac{1}{\psi_l} \frac{1}{(\theta_l^F)^{\frac{1}{2}}} - \frac{1}{(\theta_l^I)^{\frac{1}{2}}} \right) \left( 1 + \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \left( 1 + \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \\ &- (1 - \gamma) \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \left( \frac{1}{(\theta_l^F)^{\frac{1}{2}}} \frac{1}{\psi_l} \left( \frac{1}{\psi_l} + \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) - \frac{1}{(\theta_l^I)^{\frac{1}{2}}} \left( 1 + \frac{1}{\psi_l} \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} \right) \right). \end{aligned}$$

Hence

$$\frac{du_l}{d\psi_l} \begin{matrix} \leq \\ \geq \end{matrix} 0 \text{ if and only if } \psi_l \begin{matrix} \leq \\ \geq \end{matrix} 1.$$

The impact on the official unemployment rate resulting from an increase in the audit rate or the punishment fee, corresponds to

$$\frac{du_l^o}{d\psi_l} = \frac{(s + \lambda_l^I + \lambda_l^F) \frac{d\lambda_l^I}{d\psi_l} - (s + \lambda_l^I) \left( \frac{d\lambda_l^F}{d\psi_l} + \frac{d\lambda_l^I}{d\psi_l} \right)}{(s + \lambda_l^I + \lambda_l^F)^2} = \frac{\lambda_l^F \frac{d\lambda_l^I}{d\psi_l} - (s + \lambda_l^I) \left( \frac{d\lambda_l^F}{d\psi_l} \right)}{(s + \lambda_l^I + \lambda_l^F)^2} < 0 \forall \psi_l, l = m, h.$$

## 7.7 Impact of higher punishment on education

A closer examination of (17) reveals that changes in the audit rates or punishment rates affect the share of educated workers,  $1 - \hat{e}$ , through  $\psi_l$  only, whereas changes in the tax rate,  $z$ , have a direct effect on  $1 - \hat{e}$  in addition to the effects working through  $\psi_l$ . Therefore, in order to consider the effects of a fully financed change in the punishment rates on the number of educated

workers, we have to account for repercussions on  $1 - \hat{e}$  following adjustments in the tax rate. However, let us first consider the impact on  $1 - \hat{e}$  of a change in the tax and expected punishment separately:

$$\begin{aligned}\frac{\partial(1-\hat{e})}{\partial(p\alpha)}\Big|_z &= -\frac{k}{c'(e)(1+z)}\left(y_h\frac{d\frac{\theta_h^F}{(1-\sigma_h)^{1-\gamma}}}{d(p\alpha)}-y_m\frac{d\frac{\theta_m^F}{(1-\sigma_m)^{1-\gamma}}}{d(p\alpha)}\right) \\ \frac{\partial(1-\hat{e})}{\partial z}\Big|_{p\alpha} &= \psi_l\frac{\partial(1-\hat{e})}{\partial(p\alpha)}\Big|_z-\frac{c(\hat{e})}{c'(\hat{e})(1+z)}\end{aligned}$$

Using equation (38) we obtain

$$\frac{d\frac{\theta_l^F}{(1-\sigma_l)^{1-\gamma}}}{d(p\alpha)}=-\frac{\frac{1}{\psi_l}\frac{k\theta_l^I}{\sigma_l^{1-\gamma}}}{\left(\frac{\frac{1}{\psi_l}\frac{k\theta_l^I}{\sigma_l^{1-\gamma}}+1\right)\left(1-\frac{k\theta_l^I}{\sigma_l^{1-\gamma}}\right)}\frac{1}{\psi_l}\frac{1}{1+z},l=h,m,$$

$$\frac{\left(\frac{1}{\psi_l}\frac{k\theta_l^I}{\sigma_l^{1-\gamma}}\right)\left(1-\frac{k\theta_l^I}{\sigma_l^{1-\gamma}}\right)}{\left(1+\frac{k\theta_l^I}{\sigma_l^{1-\gamma}}\right)\left(1-\frac{1}{\psi_l}\frac{k\theta_l^I}{\sigma_l^{1-\gamma}}\right)}\frac{1-\sigma_l}{\sigma_l}+1$$

whereby the educational impacts become

$$\begin{aligned}\frac{\partial(1-\hat{e})}{\partial(p\alpha)}\Big|_z &= -\frac{1}{c'(e)(1+z)^2}\left(y_h\frac{do_h}{d\psi_h}-y_m\frac{do_m}{d\psi_m}\right), \\ \frac{\partial(1-\hat{e})}{\partial z}\Big|_{p\alpha} &= -\psi_l\frac{\partial(1-\hat{e})}{\partial(p\alpha)}\Big|_z+\frac{c(\hat{e})}{c'(\hat{e})(1+z)},\end{aligned}$$

where  $o_l = \frac{1}{\psi_l}\frac{k\theta_l^I}{\sigma_l^{1-\gamma}} = \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}}, l = h, m$  and

$$\frac{do_l}{d\psi_l} = -\frac{\frac{1}{\psi_l}o_l}{\frac{(o_l+1)(1-\psi_lo_l)(1-\sigma_l)}{(\psi_lo_l+1)(1-o_l)}\frac{1-\sigma_l}{\sigma_l}+1} < 0, l = h, m \quad (39)$$

For existence of an interior solution for education we need  $y_h o_h - y_m o_m > 0$ . Hence, education increases if  $y_h \frac{do_h}{d\psi_h} - y_m \frac{do_m}{d\psi_m} > 0$ . As  $\frac{do_l}{d\psi_l}, l = h, m$  is negative, and  $y_h/y_m > o_m/o_h$ , then for existence of an interior solution for  $\hat{e}$ , if

$$\left|\frac{do_m}{d\psi_m}\right|/\left|\frac{do_h}{d\psi_h}\right| > y_h/y_m > o_m/o_h \quad (40)$$

then education increases with  $p\alpha$ . Consider the case where  $\kappa_h > \kappa_m$ . As  $\psi_l$  increases with  $\kappa_l$ , then for such a solution to exist, we need that  $\left| \frac{do_l}{d\psi_l} \right|$ ,  $l = h, m$  is decreasing in concealment costs whereby  $\left| \frac{do_m}{d\psi_m} \right| > \left| \frac{do_h}{d\psi_h} \right|$ . We first show that that is the case. Multiply the numerator and denominator by  $\psi_l$  to obtain

$$\left| \frac{do_l}{d\psi_l} \right| = \frac{o_l}{\frac{(1+o_l)(1-\psi_l o_l)}{\left(\frac{1}{\psi_l} + o_l\right)(1-o_m)} \frac{1-\sigma_l}{\sigma_l} + \psi_l}.$$

Substituting for the tightness equations,  $1 - \frac{k\theta_l^F}{(1-\sigma_l)^{1-\gamma}} = 1 - o_l = k(r+s)(\theta_l^F)^{\frac{1}{2}} 2$  and  $1 - \frac{k\theta_l^I}{\sigma_l^{1-\gamma}} = 1 - \psi_l o_l = k(r+s)(\theta_l^I)^{\frac{1}{2}} 2$  and use the fact that  $\frac{1-\sigma_l}{\sigma_l} = \left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{1-\gamma}} \psi_l^{\frac{1}{1-\gamma}}$  according to the search equation to obtain

$$\left| \frac{do_l}{d\psi_l} \right| = \frac{o_l}{A_l \left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{1-\gamma} - \frac{1}{2}} \psi_l^{\frac{1}{1-\gamma}} + \psi_l},$$

where  $A_l = \frac{(1+o_l)}{\left(\frac{1}{\psi_l} + o_l\right)}$ . Differentiating this expression with respect to  $\psi_l$  we obtain

$$\frac{d \left| \frac{do_l}{d\psi_l} \right|}{d\psi_l} = \frac{\frac{do_l}{d\psi} \left( A_l \left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{1-\gamma} - \frac{1}{2}} \psi_l^{\frac{1}{1-\gamma}} + \psi_l \right) - o_l \psi_l^{\frac{1}{1-\gamma}} \left( \frac{\theta_l^F}{\theta_l^I}^{\frac{1}{1-\gamma} - \frac{1}{2}} \left( \frac{dA_l}{d\psi_l} + \frac{A_l}{\psi_l} \left( \frac{1}{1-\gamma} \right) \right) + A_l \frac{d \frac{\theta_l^F}{\theta_l^I}^{\frac{1}{1-\gamma} - \frac{1}{2}}}{d\psi} + 1 \right)}{\left( A_l \left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{1-\gamma} - \frac{1}{2}} \psi_l^{\frac{1}{1-\gamma}} + \psi_l \right)^2}$$

as substituting for  $\frac{do_l}{d\psi_l}$  using the expression from equation (39) gives

$$\frac{dA_l}{d\psi_l} = \frac{\frac{do_l}{d\psi_l} \left( \frac{1}{\psi} - 1 \right) + \frac{1+o_l}{\psi^2}}{\left( \frac{1}{\psi_l} + o_l \right)^2} = \frac{\frac{1}{\psi_l} o_l + \frac{1+o_l}{\psi^2} \frac{(o_l+1)(1-\psi_l o_l)}{(\psi_l o_l+1)(1-o_l)} \frac{(1-\sigma_l)}{\sigma_l} + \frac{1}{\psi^2}}{\left( \frac{1}{\psi_l} + o_l \right)^2 \left( \frac{(o_l+1)(1-\psi_l o_l)}{(\psi_l o_l+1)(1-o_l)} \frac{(1-\sigma_l)}{\sigma_l} + 1 \right)} > 0,$$

and from the equilibrium equations we have  $d(\theta_h^F/\theta_h^I)/d\psi_l > 0$  and  $do_l/d\psi_l < 0$ .

Hence as  $\frac{d\left|\frac{do_l}{d\psi_l}\right|}{d\psi_l} < 0$  then  $\frac{d\left|\frac{do_l}{d\kappa_l}\right|}{d\kappa_l} < 0$  so when  $\kappa_h > \kappa_m$  then  $\left|\frac{do_m}{d\psi_m}\right| > \left|\frac{do_h}{d\psi_h}\right|$ .

We observe  $\frac{d\left|\frac{do_l}{d\psi_l}\right|}{d\psi_l} < 0$  both because the numerator decreases with  $\psi_l$  and the denominator increases with  $\psi_l$ . Rewriting the expression determining the sign of  $\frac{\partial(1-\hat{e})}{\partial(p\alpha)}|_z$ , equation (40) as

$$\frac{\frac{o_m}{A_m \left(\frac{\theta_m^F}{\theta_m^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}} \psi_m^{\frac{1}{1-\gamma}} + \psi_m}}{\frac{o_h}{A_h \left(\frac{\theta_h^F}{\theta_h^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}} \psi_h^{\frac{1}{1-\gamma}} + \psi_h}} = g(\kappa_h, \kappa_m) o_m/o_h > y_h/y_m > o_m/o_h,$$

where

$$g(\kappa_h, \kappa_m) \equiv \frac{D \frac{do_h}{d\psi_h}}{D \frac{do_m}{d\psi_m}} = \frac{A_h \left(\frac{\theta_h^F}{\theta_h^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}} \psi_h^{\frac{1}{1-\gamma}} + \psi_h}{A_m \left(\frac{\theta_m^F}{\theta_m^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}} \psi_m^{\frac{1}{1-\gamma}} + \psi_m} > 1,$$

when  $\kappa_h > \kappa_m$  as the denominator of  $\left|\frac{do_l}{d\psi_l}\right|$  increases with  $\psi_l$ . We conclude that if  $\frac{y_h}{y_m} \in \left[\frac{o_m}{o_h}, g(\kappa_h, \kappa_m) \frac{o_m}{o_h}\right]$  education increases with  $p\alpha$  and when  $\frac{y_h}{y_m} \in \left[g(\kappa_h, \kappa_m) \frac{o_m}{o_h}, \infty\right]$  education falls with  $p\alpha$ .

## 7.8 Impact of higher punishment on unemployment

Raising the audit rate  $p$  or the punishment fee  $\alpha$ , increases the wedge,  $\psi_l = (1 + p_l\alpha + \kappa_l) / (1 + z)$ . Differentiating total unemployment with respect to  $\psi_l$  gives

$$\frac{dU_{TOT}}{d\psi_l} = \frac{d\hat{e}}{d\psi_l} (u_m - u_h) + \hat{e} \frac{du_m}{d\psi_l} + (1 - \hat{e}) \frac{du_h}{d\psi_l}.$$

The last two terms are positive ( $\leq 0$ ) when  $\psi_l$  is larger than one ( $\leq 1$ ). The first term is positive if  $\frac{y_h}{y_m} \in \left[\frac{o_m}{o_h}, g(\kappa_h, \kappa_m) \frac{o_m}{o_h}\right]$  where  $g(\kappa_h, \kappa_m) > 1$  if  $\kappa_h > \kappa_m$ , and  $\psi_l > (=) 1$ . as then  $(u_m - u_h) < (=) 0$  and  $\frac{d\hat{e}}{d\psi_l} < 0$ . However, when  $\psi_l < 1$  and  $\kappa_h > \kappa_m$  then  $(u_m - u_h) > 0$  and in case  $\frac{d\hat{e}}{d\psi_l} < 0$  then

unemployment falls,  $\frac{dU_{TOT}}{d\psi_l} < 0$ . If,  $\frac{y_h}{y_m} \in \left[ \frac{\sigma_m}{\sigma_h}, g(\kappa_h, \kappa_m) \frac{\sigma_m}{\sigma_h} \right]$  then  $\frac{d\hat{e}}{d\psi_l} > 0$  and  $\frac{dU_{TOT}}{d\psi_l}$  has an ambiguous sign.

Total official unemployment changes according too

$$\frac{dU_{TOT}^o}{d\psi_l} = \frac{d\hat{e}}{d\psi_l} (u_m^o - u_h^o) + \hat{e} \frac{du_m^o}{d\psi_l} + (1 - \hat{e}) \frac{du_h^o}{d\psi_l} < 0,$$

where the last two terms are negative and therefore when  $\kappa_h > \kappa_m$  we obtain  $\frac{dU_{TOT}^o}{d\psi_l} < 0$  when  $\frac{d\hat{e}}{d\psi_l} \leq 0$  as  $(u_m^o - u_h^o) > 0$ . When  $\frac{d\hat{e}}{d\psi_l} > 0$  the sign of  $\frac{dU_{TOT}^o}{d\psi_l}$  is ambiguous.

## 7.9 Socially optimal solution for $\theta_m^F, \theta_m^I, \theta_h^F, \theta_h^I, \sigma_m, \sigma_h$ and $\hat{e}$ .

From the first order conditions for tightness in the formal and informal sector for manual and highly educated workers, i.e.,  $\frac{\partial W}{\partial \theta_l^I} = 0, \frac{\partial W}{\partial \theta_l^F} = 0, l = m, h$ , we get the following conditions:  $2sk(\theta_l^I)^{\frac{1}{2}} = u_l[1 + k\Theta_l]$  and  $2sk(\theta_l^F)^{\frac{1}{2}} = u_l[1 + k\Theta_l], l = m, h$ , which gives  $\theta_l^F = \theta_l^I$ . Substitute  $\theta_l^F = \theta_l^I$  into the first order condition for search effort,  $\frac{\partial W}{\partial \sigma_l} = 0$ , and the following condition determines the social optimal level of search:  $\sigma_m^{\gamma-1} - (1 - \sigma_m)^{\gamma-1} = 0$ . This yields  $\sigma_l = \frac{1}{2}, l = m, h$ . Substitute  $\sigma_l = \frac{1}{2}, l = m, h$  into  $2sk(\theta_l^I)^{\frac{1}{2}} = u_l[1 + k\Theta_l]$  and  $2sk(\theta_l^F)^{\frac{1}{2}} = u_l[1 + k\Theta_l], l = m, h$ , which yields the four equations in (25) determining  $\theta_m^F, \theta_m^I, \theta_h^F$ , and  $\theta_h^I$  in equilibrium. The socially optimal educational stock is determined by:  $\frac{\partial W}{\partial (1-\hat{e})} = W_h(\hat{e}) - W_m = y_h[1 - u_h[1 + k\Theta_h]] - y_m[1 - u_m y_m[1 + k\Theta_m]] - c(\hat{e}) = 0$ . Now use the equations determining the optimal levels of tightness,  $2sk(\theta_l^I)^{\frac{1}{2}} = u_l[1 + k\Theta_l]$  and  $2sk(\theta_l^F)^{\frac{1}{2}} = u_l[1 + k\Theta_l], l = m, h$ , and the equation for the optimal educational level given by (26). To show that we have a global maximum we

differentiate  $W$  with respect to  $\sigma_l, \theta_l^I, \theta_l^F, l = m, h$  and  $1 - \hat{e}$  to obtain

$$\begin{aligned} \sigma_l^{*\gamma-1} - (1 - \sigma_l^*)^{\gamma-1} &= 0, l = m, h, \\ -sk (\theta_l^{*I})^{\frac{1}{2}} + \frac{1}{2} \left[ 1 - \frac{k\theta_l^{*I}}{(\sigma_l)^{1-\gamma}} \right] &= 0, l = m, h, \\ -sk (\theta_l^{*F})^{\frac{1}{2}} + \frac{1}{2} \left[ 1 - \frac{k\theta_l^{*F}}{(1 - \sigma_l)^{1-\gamma}} \right] &= 0, l = m, h, \\ \left( y_h \frac{k\theta_h^{*I}}{(\sigma_h)^{1-\gamma}} - y_m \frac{k\theta_m^{*I}}{(\sigma_m)^{1-\gamma}} \right) - c(\hat{e}^*) &= 0. \end{aligned}$$

The associated Hessian matrix is then

$$\begin{vmatrix} (\gamma - 1) S_m & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{k\theta_m^I}{2\sigma_m^{2-\gamma}} (\gamma - 1) & \Delta_m^I & 0 & 0 & 0 & 0 & 0 \\ \frac{k\theta_m^I}{2(1-\sigma_m)^{2-\gamma}} (\gamma - 1) & 0 & \Delta_m^F & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\gamma - 1) S_h & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{k\theta_h^I}{2\sigma_h^{2-\gamma}} (\gamma - 1) & \Delta_h^I & 0 & 0 \\ 0 & 0 & 0 & \frac{k\theta_h^I}{2(1-\sigma_h)^{2-\gamma}} (\gamma - 1) & 0 & \Delta_h^F & 0 \\ -y_m (\gamma - 1) \frac{k\theta_m^{*I}}{(\sigma_m)^{2-\gamma}} & -y_m k \sigma_m^{\gamma-1} & 0 & (\gamma - 1) y_h \frac{k\theta_h^{*I}}{(\sigma_h)^{2-\gamma}} & y_h k (\sigma_h)^{\gamma-1} & 0 & c'(\hat{e}^*) \end{vmatrix}$$

where  $S_l = (\sigma_l^{\gamma-2} + (1 - \sigma_l)^{\gamma-2})$ ,  $l = m, h$ ,  $\Delta_l^I = -\frac{1}{2} \left( sk (\theta_l^I)^{-\frac{1}{2}} + k\sigma_l^{\gamma-1} \right)$ ,  $l = m, h$  and  $\Delta_l^F = -\frac{1}{2} \left( sk (\theta_l^F)^{-\frac{1}{2}} + k(1 - \sigma_l)^{\gamma-1} \right)$ ,  $l = m, h$ .

Therefore,  $H_1 = (\gamma - 1) (\sigma_m^{\gamma-2} + (1 - \sigma_m)^{\gamma-2}) < 0$  and the principal minors alternate in sign, for all variable values, i.e.,  $H_2 = -(\gamma - 1) (\sigma_m^{\gamma-2} + (1 - \sigma_m)^{\gamma-2}) \Delta_m^I > 0, \dots, H_7 = (\gamma - 1) (\sigma_m^{\gamma-2} + (1 - \sigma_m)^{\gamma-2}) \Delta_m^I \Delta_m^F (\gamma - 1) (\sigma_h^{\gamma-2} + (1 - \sigma_h)^{\gamma-2}) \Delta_h^I \Delta_h^F c'(\hat{e}^*) < 0$ , by which we have a global maximum.

## 7.10 Optimal not to induce the socially efficient stock of education.

Evaluating (28) and (29) at  $p_m^e$  and  $p_h^e$  such that the socially optimal level of education is reached, i.e.,  $\frac{dW}{d(1-e)} = 0$ . From 7 this requires that  $\psi_m^e > 1 > \psi_h^e$ .

This yields:  $\frac{dW}{dp_m} |_{\psi_m^e > 1} = \hat{e} \left[ \frac{dW}{d\theta_m^F} \frac{d\theta_m^F}{d\psi_m} + \frac{dW}{d\theta_m^I} \frac{d\theta_m^I}{d\psi_m} + \frac{dW}{d\sigma_m} \frac{d\sigma_m}{d\psi_m} \right] \frac{d\psi_m}{dp_m}$  and  $\frac{dW}{dp_h} |_{\psi_h^e < 1} = (1 - \hat{e}) \left[ \frac{dW}{d\theta_h^F} \frac{d\theta_h^F}{d\psi_h} + \frac{dW}{d\theta_h^I} \frac{d\theta_h^I}{d\psi_h} + \frac{dW}{d\sigma_h} \frac{d\sigma_h}{d\psi_h} \right] \frac{d\psi_h}{dp_h}$ . From the derivations of the socially optimal solution for  $\theta_m^F, \theta_m^I, \theta_h^F, \theta_h^I, \sigma_m, \sigma_h$  it follows that  $\frac{dW}{d\theta_l^F} |_{\psi_l > 1} < 0$ ,  $\frac{dW}{d\theta_l^I} |_{\psi_l > 1} > 0$ ,  $\frac{dW}{d\sigma_l} |_{\psi_l > 1} > 0$  and  $\frac{dW}{d\theta_l^F} |_{\psi_l < 1} > 0$ ,  $\frac{dW}{d\theta_l^I} |_{\psi_l < 1} < 0$ ,  $\frac{dW_l}{d\sigma_l} |_{\psi_l < 1} < 0$  as the welfare function is maximized at  $\psi_l = 1$ , i.e.,  $\frac{dW}{d\theta_l^F} |_{\psi_l=1} = \frac{dW_l}{d\theta_l^F} |_{\psi_l=1} = \frac{dW_l}{d\sigma_l} |_{\psi_l=1} = 0$ . It then follows that  $\frac{dW}{dp_m} |_{\psi_m^e > 1} < 0$  and  $\frac{dW}{dp_h} |_{\psi_h^e < 1} > 0$ .