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Constant Proportion Debt Obligations (CPDOs):

Modeling and Risk Analysis

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Abstract

Constant Proportion Debt Obligations (CPDOs) are structured credit derivatives which generate high coupon payments by dynamically leveraging a position in an underlying portfolio of investment grade index default swaps. CPDO coupons and principal notes received high initial credit ratings from the major rating agencies, based on complex models for the joint transition of ratings and spreads for all names in the underlying portfolio. We propose a parsimonious model for analyzing the performance of CPDO strategies using a top-down approach which captures the essential risk factors of the CPDO. Our approach allows to compute default probabilities, loss distributions and other tail risk measures for the CPDO strategy and analyze the dependence of these risk measures on various parameters describing the risk factors. We find that the probability of the CPDO defaulting on its coupon payments is found to be small–and thus the credit rating arbitrarily high– by increasing leverage, but the ratings obtained strongly depend on assumptions on the credit environment (high spread or low spread). More importantly, CPDO loss distributions are found to be bimodal with a wide range of tail risk measures inside a given rating category, suggesting that credit ratings are insufficient performance indicators for such complex leveraged strategies. A worst-case scenario analysis indicates that CPDO strategies have a high exposure to persistent spread-widening scenarios CPDO ratings are shown to be quite unstable during the lifetime of the strategy.

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Contents

1 Introduction

Constant Proportion Debt Obligations (CPDOs) are leveraged credit investment strategies which appeared in the low credit spread environment of 2006 with the aim of generating high coupons while investing in investment grade credit. The asset side of the CPDO contains two positions: a money market account and leveraged credit exposure via index default swaps on indices of corporate names, typically the ITRAXX and DJ CDX. The dynamically adjusted risky exposure is chosen such as to ensure that the CPDO generates enough income to meet its promised liabilities and also to cover for fees, expenses and credit losses due to defaults in the reference portfolio and mark-to-market losses linked to the fair value of the index default swap contract.

The CPDO strategy involves high initial leverage but adjust this leverage dynamically: leverage is reduced as the gap between portfolio value and present value of liabilities narrows and increased if losses are incurred, in order to regain some of the lost capital. With this leverage rule a CPDO has no upside potential but it has an added ability to recover from negative positions at the cost of not having principal protection, contrarily to the better-known portfolio insurance (CPPI) strategies [10]. The term "constant proportion" refers to the fact that it operates with a piecewise constant leverage rule (see Section 2.2).

The first CPDO launched by ABN Amro paid coupons at 100bp above Euribor and later versions of the CPDO paid spreads as high as 200bp above EURIBOR/LIBOR. Yet CPDO coupons and principal notes initially received top (AAA) ratings from the major rating agencies. This top rating gave rise to an intense discussion among market participants, because standard top-rated products such as treasury bonds pay significantly lower coupons and also because the pool of corporate names on which the CPDO sells protection has significantly lower average rating.

When first issued, there were several studies on the risk and performance of CPDOs conducted by rating agencies [25, 18] and by issuers [24]. The sensitivity analysis conducted in these studies suggested that the CPDO strategy is fairly robust and could overcome most historical credit stresses prior to the 2007–2008 financial crisis with low default rates [22]. However, one concern of agencies which chose not to rate this product was the potentially high level of model risk involved in the analysis of the CPDO strategy, given the large number of factors and parameters in these models. Another major concern was the limited extent of historical data for backtesting the strategy: spread data for the ITRAXX and CDX indices are only a few years in length (only a fraction of the risk horizon of CPDOs) and in this period credit markets had not been under serious stress. In hindsight this was a serious drawback since the 2007 credit crunch hit the markets quite suddenly and the following steep increase in ITRAXX and CDX spreads caused heavy CPDO losses. The continued market distress has forced many structures to unwind.

The methods used by rating agencies [5, 25, 23] to analyze CPDOs have been based on high-dimensional models for co-movements of ratings and spreads for all names in the reference portfolio. Defaults in the underlying index are generated through a detailed modeling of rating migrations of the underlying names and the index spread is modeled as a stochastic process depending on the average rating of the names in the index. This modeling approach leads to hundreds of state variables and is not accessible to entities other than rating agencies due to lack of historical data on ratings, not to mention the difficulty of calibrating such models with thousands of parameters.

We argue that such a complex framework may not be necessary and may in fact obscure the main risk factors influencing the CPDO strategy. We show that the main risk and performance drivers can be parsimoniously modeled using a *top-down* approach where the underlying credit portfolio is modeled in terms of its aggregate default loss. We model the rate of occurrence of defaults in the underlying index using a default intensity process, representing the rate of default in the underlying index. This setting allows to study the key risk factors associated with CPDOs, while keeping estimation and simulation of the model at a simple level and enabling a meaningful sensitivity analysis. Our analysis allows an independent assessment of the credit ratings assigned by agencies, allows to compute default probabilities, loss distributions and other tail risk measures for the CPDO strategy.

1.1 Summary of main findings

Besides illustrating the possibility of analyzing the risk of the CPDO strategy using a parsimonious top-down model, our study also leads to several interesting findings on the nature of this instrument:

- **CPDOs are path-dependent spread derivatives**. One of the insights of our study is to show that the main risk of a CPDO is not default risk but spread risk and interest rate risk: in fact, the worst case scenario for the CPDO investor is observed to be a spread-widening scenario, even in absence of defaults. Thus, a CPDO may be more appropriately viewed as path-dependent derivative on the spread of underlying CDS index.

- **Credit ratings are insufficient to characterize the risk of a CPDO**: the risk of a CPDO is not appropriately characterized by a "credit rating", based on either expected loss or default probability.

- **CPDO strategies lead to skewed loss distributions with considerable tail risk**. The simulated loss distribution generated by CPDO strategies is observed to have a highly asymmetric shape, which is not adequately characterized by a single statistic such as expected loss or the default probability. Examining risk measures such as Expected Shortfall (or Tail Conditional Expectation) leads to a different picture of the risk of CPDOs than the one portrayed by credit ratings.

- **CPDOs can achieve high ratings but at the price of higher exposure to "tail risk"**: a CPDO strategy may be adjusted to achieve a default probability lower than a given threshold but at the price of a higher Expected Shortfall. In particular, credit ratings based on default probability may be arbitrarily improved by "pushing the risk far enough into the tails".

- **CPDOs are highly exposed to model uncertainty**: ratings and risk measures associated to the CPDO strategy are observed to have values which are quite sensitive to model parameters, making it difficult to make precise statements. This contrasts with the precision implied by some agency ratings on such products.

1.2 Outline

The paper is organized as follows. Section 2 describes the CPDO strategy and the cash flows involved. Risk factors influencing these cash flows are analyzed in section 3 and based on this we setup a one factor top-down model for the default intensity. We perform a simulation-based analysis of the performance of CPDOs in section 4 by studying ratings and risk measures in different credit market environments, by conducting a sensitivity analysis and evaluating transition probabilities for ratings. Section 5 summarizes our results and discusses some implications of our analysis.

2 The CPDO strategy

A CPDO is a dynamically leveraged credit trading strategy which aims at generating high coupon payments (100–200 bps above LIBOR in the examples observed in the market) by selling default protection on a portfolio of investment-grade obligors with low default probabilities. The idea is to generate such high coupons by taking a leveraged position in a credit (CDS) index and dynamically adjusting this leveraged exposure as the value of the portfolio changes.

2.1 Description

An investor in a CPDO provides initial capital (normalized to 1 in the sequel) and receives periodic coupon payments of a contractual spread above the LIBOR rate until expiry T of the deal. The CPDO manager sells protection on some credit index via index default swaps on the notional which is leveraged up with respect to initial placement. The CPDO portfolio is composed of two positions: a short term investment, such as a money market account, denoted $(A_t)_{0\leq t\leq T}$ and a position in a T^I-year index default swap (typically the 5-year index default swap). The sum of the value of the swap contracts and the money market account is denoted by $(V_t)_{t\in[0,T]}$.

Initially, the notional paid by the investor, minus an eventual arrangement fee ($\simeq 1\%$) is invested in the money market account: $A_0 = 0.99$. The money market account earns interest at the LIBOR rate: we denote $L(t,s)$ the spot LIBOR rate quoted at t for maturity $s > t$.

The investor receives coupons at dates $\mathbf{CD} = \{t_l \leq T \,|\, l = 1, 2, ...\}$. CPDO coupons are paid out as a spread δ over LIBOR

$$c_{t_l} = \Delta(t_l)\big[L(t_{l-1}, t_l) + \delta\big],$$

where $\Delta(t) = t - \max\{t_l \in \mathbf{CD} |\, t_l < t\}$ is the time elapsed since last coupon payment date. The present value of these liabilities is called the *target value*:

$$TV_t = B(t,T) + \sum_{t_l \in \mathbf{CD} \cap [t,T]} E^Q \left[c_{t_l} e^{-\int_t^{t_l} r_s ds} \,|\mathcal{F}_t \right],$$

where $B(t,u) = E^Q \left[e^{-\int_t^u r_s ds} \,|\mathcal{F}_t \right]$ is the discount factor associated with some short rate process r and $\mathcal{F}_t$ is the market information at time t. If $V_t \geq TV_t$ then the CPDO manager can meet her obligations by simply investing (part of) the fund in the money market.

To be able to meet the coupon payments, the CPDO manager sells protection on a reference credit index (ITRAXX, CDX,...) by maintaining a position in index default swaps on the investor's notional that is leveraged by a factor m (the leverage ratio). This position generates income for the CPDO by earning a periodic spread, denoted $S(t, T^I)$ for the spread observed at time t of a swap expiring at time T^I. We denote by P_t the present value of these spread payments; i.e. P_t is equal to the present value of the premium leg of the index default swap at time t.

If a name in the underlying index defaults, the CPDO manager incurs a loss, which is magnified through leverage. We denote by $\mathbf{DT} = \{\tau_1 \leq \tau_2 \leq ... \leq \tau_{N_I}\}$ the set of default times in the index: τ_i represents the date of the i-th default event, N^I denotes the number of names in the underlying index ($N^I = 250$ for a CPDO referencing the ITRAXX and CDX), and $N_t = \sum_{i=1}^{N^I} \mathbf{1}_{\{\tau_i \leq t\}}$ is the number of defaults in the index up to time t.

The CPDO is said to *cash in* if the portfolio value reaches a value sufficient to meet future liabilities, i.e. $V_t \geq TV_t$. In this event all swap contracts are liquidated and the CPDO portfolio consists only of the money market account.

If, on the other hand, the value falls below a threshold k, $V_t \leq k$ (e.g. $k = 10\%$ of the investor's initial placement) the CPDO is said to *cash out*. In this case the CPDO unwinds all its risky exposures, ends coupon payments and returns the remaining funds to the investor.

A CPDO can default on its payments either by cashing out and thereby defaulting on both remaining coupon payments and principal note, or by simply failing to repay par to investor at maturity, in which case it defaults on its principal note. Default clustering in the reference portfolio or sudden spread-widening may result in a cash out event where the money market account is not sufficient to settle the swap contracts. This loss is covered by the CPDO issuer and the risk of such a scenario (known as "gap risk") is reduced by setting the cash out threshold strictly above zero.

Until expiry, a cash-in or a cash-out event occurs, the manager readjusts the leveraged position in index default swaps using a rule described in the next section.

2.2 Leverage rule

At initiation there is a shortfall between the net value V_t of assets and the target value TV_t: $TV_0 > V_0$. The target leverage m_t is chosen such that the income generated by the swap, P_t compensates the shortfall:

$$m_t = \beta \frac{TV_t - V_t}{P_t}. \tag{1}$$

β denotes a gearing factor that controls the aggressiveness of strategy.

The actual leverage is not adjusted continuously as this would involve significant trading costs in practice. The underlying index rolls into a new series every six months and it is therefore natural to update actual leverage $\left(\bar{m}^i\right)_{i=1,2,..}$ to equal target leverage on index roll dates **RD**:

$$\bar{m}^{i(t)} = m_t, \quad \text{for} \quad t \in \mathbf{RD} = \left\{ T_j \,\middle|\, T_j = \frac{j}{2},\, j = 1, ..., 2T \right\},$$

where $i(t) \in \mathbb{N}$ denotes the leverage factor index employed at time t. The leverage factor is also adjusted if it differs more than ε (usually $\varepsilon = 25\%$) from target leverage:

$$\bar{m}^{i(t)} = m_t, \quad \text{if} \quad \bar{m}^{i(t)-1} \notin \big[(1-\varepsilon)m_t, (1+\varepsilon)m_t\big].$$

The set of these rebalancing dates (excluding roll dates) will be denoted **RBD**. The actual leverage factor is automatically adjusted on default dates as the number of names in the underlying index is reduced by one until next roll date:

$$\bar{m}^{i(t)} = \frac{N^I - N_t}{N^I - N_{t-}} \bar{m}^{i(t)-1}, \quad \text{for} \quad t \in \mathbf{DT}.$$

The leverage factor is capped at a maximum level M in order to reduce the overall possible loss (usually $M = 15$).

By this strategy, the leverage factor employed by a CPDO is piecewise constant, hence the name "constant proportion" debt obligations. The leverage adjustment rule leads to an increase in leverage if losses occur in the index, and a decrease in leverage if the shortfall is reduced. It is therefore a "buy low, sell high" strategy as opposed, for instance, to more popular CPPI strategies [10], which lead to a "buy high, sell low" strategy.

2.3 Cash flow structure

Spread income generated by the CPDO is determined by the average spread on the swap contracts held. Contracted spread changes every time the CPDO enters new swap contracts and is thereby a piecewise constant process denoted $(\bar{S}^i)_{i=1,2,...}$. Initially, contracted spread is equal to observed spread: $\bar{S}^0 = S(0, T^I)$. On index roll dates existing swap contracts on the off-the-run index are liquidated and new on-the-run contracts are entered, i.e.

$$\bar{S}^{i(t)} = S(t, t + T^I) \quad \text{for} \quad t \in \mathbf{RD}.$$

At rebalancing dates on which the leverage factor is increased, the new contracts entered contribute to the contracted spread. For $t \in \mathbf{RBD}$

$$\bar{S}^{i(t)} = \begin{cases} \bar{S}^{i(t)-1}, & \bar{m}^{i(t)} < \bar{m}^{i(t)-1} \\ w\bar{S}^{i(t)-1} + (1-w)S\big(t, T_{j(t)} + T^I\big), & \bar{m}^{i(t)} > \bar{m}^{i(t)-1} \end{cases},$$

where $w = \frac{\bar{m}^{i(t)-1}}{\bar{m}^{i(t)}}$ is the relative weight of old contracts in the swap portfolio after releveraging, and $T_{j(t)}$ denotes the latest roll date prior to time t: $j(t) := \max\{j \,|\, T_j < t,\, T_j \in \mathbf{RD}\}$.

A change in the observed index default swap spread implies a change in the mark-to-market value, denoted MtM, of the swap contracts. Mark-to-market is the value of entering an offsetting swap with the same expiry and coupon dates:

$$MtM_t = \big(\bar{S}^{i(t)} - S(t, T_{j(t)} + T^I)\big) D_t^{\text{swap}},$$

where

$$D_t^{\text{swap}} := E^Q \left[\sum_{t_l \in \mathbf{CD} \cap [t, T^I]} e^{-\int_t^{t_l} r_s ds} \Delta(t_l) \left(1 - \frac{N_{t_l}}{N^I}\right) \Big| \mathcal{F}_t \right]$$

is the duration of the swap contract. The value of the CPDO portfolio is given as the sum of the money market account and the value of swap contracts: $V_t = A_t + MtM_t$.

Liquidating (part of) the position in swap contracts leads to a profit or loss which is balanced by the money market account. On roll dates the entire position of swap contracts is liquidated and the profit/loss is

$$\bar{m}^{i(t)} \Big(\bar{S}^{i(t)} - S(t, T_{j(t)} + T^I)\Big) D_t^{\text{swap}}, \quad t \in \mathbf{RD}.$$

Note that on roll date $t \in \mathbf{RD}$ the spread at which protection on the off-the-run is bought back is $S(t, t + T^I - \frac{1}{2})$, whereas the spread of new on-the-run contracts is $S(t, t + T^I)$; new contracts have six months longer to expiry.

At rebalancing dates on which the leverage factor is decreased ($t \in \mathbf{RBD} \cap \{m_t < \bar{m}^{i(t)-1}\}$) a part of the swap contracts are liquidated giving the following profit/loss to the money market account:

$$\big(\bar{m}^{i(t)-1} - m_t\big) \Big(\bar{S}^{i(t)} - S(t, T_{j(t)} + T^I)\Big) D_t^{\text{swap}}.$$

In summary the cash flows of a CPDO can be decomposed into:

1. **Interest payments** $t \in [0, T]$: $A_{t-\Delta} L(t - \Delta, t)\Delta$, where Δ is time between interest payment dates.

2. **Coupon payments** $t_l \in \mathbf{CD}$: $-c_{t_l}$.

3. **Spread income** $t_l \in \mathbf{CD}$: $\bar{m}^{i(t_l)} \bar{S}^{i(t_l)} \Delta(t_l)$ (assuming spread premiums are paid on the same dates as CPDO coupons).

4. **Default loss** $\tau \in \mathbf{DT}$: $-\bar{m}^{i(\tau)} \frac{(1-R)}{N^I}$, where R is the recovery rate on a single default event.

5. **Liquidation of swap contracts**:

$$\begin{aligned} &\bar{m}^{i(t)} \Big(\bar{S}^{i(t)} - S(t, T_{j(t)} + T^I)\Big) D_t^{\text{swap}} \mathbf{1}_{\mathbf{RD}}(t) \\ &+ \big(\bar{m}^{i(t)-1} - m_t\big) \Big(\bar{S}^{i(t)} - S(t, T_{j(t)} + T^I)\Big) D_t^{\text{swap}} \mathbf{1}_{(\mathbf{RBD} \cap \{m_t < \bar{m}^{i(t)-1}\})}(t). \end{aligned}$$

Given that the value of the money market account and the CPDO portfolio is known up to but not at time t, A_t and V_t can be calculated in the following way:

$$\begin{aligned} A_t &= A_{t-\Delta}\big(1 + L(t-\Delta, t)\Delta\big) + \Big(\bar{m}^{i(t)}\bar{S}^{i(t)}\Delta(t) - c_t\Big)\,\mathbf{1_{CD}}(t) \qquad (2) \\ &\quad - m^{i(t)}\frac{(1-R)}{N^I}\mathbf{1_{DT}}(t) + \bar{m}^{i(t)}\Big(\bar{S}^{i(t)} - S(t, T_{j(t)} + T^I)\Big)D_t^{\text{swap}}\mathbf{1_{RD}}(t) \\ &\quad + \big(\bar{m}^{i(t)-1} - m_t\big)\Big(\bar{S}^{i(t)} - S(t, T_{j(t)} + T^I)\Big)D_t^{\text{swap}}\mathbf{1}_{(\mathbf{RBD}\cap\{m_t < \bar{m}^{i(t)-1}\})}(t) \\ V_t &= A_t + MtM_t. \qquad (3) \end{aligned}$$

2.4 Risk factors

Based on the description above we can identify the following risk factors influencing the cash flows of the CPDO strategy:

- Spread risk
  The main determinant of the CPDO cash flows is the index default swap spread. The leverage rule is designed such that, if the index spread were constant, the CPDO would always cash in prior to expiry, given that there are no defaults in the underlying portfolio. Therefore a stochastic model for the swap spread is essential for capturing the spread risk of the strategy.

  An increase in the index spread increases the premium payments at each payment date, but results in an immediate loss in market value of the CPDO since the CPDO is selling protection on the index. This loss in market value materializes as a cash flow on roll and rebalancing dates. A sudden spread change will give rise to a single cash flow on roll dates, but it will have long term effects on the spread income.

  The index roll will typically result in a downward jump in the swap spread since the downgraded names that are removed contribute with higher spreads than the investment grade names they are replaced by. On average this negative jump implies a mark-to-market loss on roll dates.

- Default risk
  The default rate in the underlying portfolio determines the average number of defaults during the lifetime of the CPDO. A higher default rate is negative for the CPDO performance due to higher expected credit losses.

  The recovery level affects the size of credit losses incurred at default dates although this is to some extent offset by its effect on the spread income, since lower recovery level implies higher swap spread and thereby higher spread premium income to the CPDO. Since recovery data is sparse a constant recovery level $R = 0.4$ is chosen.

- Interest rates
  The term structure of interest rates has two main effects on the cash flows.

First, higher LIBOR rates imply higher coupon payments to the investor but this effect will more or less be offset by the higher interest accruing to the money market account. The interest rate also influences present value calculations via the discount factor, for example when determining the target value. The stochastic evolution of the interest rate can easily be incorporated in our framework but in the remainder of the paper we will focus on a constant term structure since the effect is of second order with respect to the credit spreads and their volatility.

- Liquidity risk
  The liquidity of the index default swaps also affects the cash flows via the bid/ask spread of the index. Note however that most CPDOs reference the most liquid indices, ITRAXX and DJ CDX. In the following we do not explicitly model liquidity risk though this can be done by introducing a bid/ask spread of the index at roll dates.

2.5 Rating of CPDOs

Credit ratings, issued by rating agencies, are routinely used as an indicative scale of credit risk for bonds. It has become market practice to also assign ratings to structured credit products. Such structured finance ratings are expressed using the same letter scale (AAA, AA, etc) as bonds and misleadingly tend to imply that such structured products have a risk profile similar to corporate bonds with identical ratings. As it will become clear from our discussion below, this is far from being true in the case of CPDOs.

A CPDO is a structured product with leverage effects, and it is not straightforward -and not necessarily meaningful- to assign a credit rating to it. Ratings have been assigned to CPDOs by major rating agencies by comparing the default probability or the expected loss of the structure to thresholds which are typically adjusted versions of bond default probabilities [5, 25]. These ratings follow similar procedures adopted for CDO tranches [9] and share many of their drawbacks. As will become clear in the sequel, we do not condone the use of such 'ratings' as an appropriate metric for the risk of a complex product such as a CPDO. However, given their widespread use, we will compute sample ratings in various examples and examine their properties in the case of CPDOs.

Separate ratings are assigned to the coupons and the principal note of a CPDO. In the sequel we will focus on the approach based on default probabilities.

The rating on the coupon note is based on the probability of the CPDO cashing out. This probability can be found by Monte Carlo simulations and is translated into a rating according to the rating thresholds, an example of which is given in table 1. Both cash out scenarios and scenarios in which the CPDO survives until expiry but is unable to repay par in full will result in default on principal note and the probability of this is likewise found by Monte Carlo simulations. Thresholds in table 1 are used to translate the probability of default on principal into a rating.

Major rating agencies [5, 25, 23] have analyzed CPDOs using high-dimensional

%	**AAA**	**AA+**	**AA**	**AA-**	**A+**	**A**	**A-**
10 year PD	0.73	1.01	1.49	1.88	2.29	2.72	3.56
	BBB+	**BBB**	**BBB-**	**BB+**	**BB**	**BB-**	**B+**
10 year PD	4.78	7.10	12.31	14.63	19.94	26.18	32.76

Table 1: Standard & Poor's CDO rating thresholds in terms of default probabilities. Source: [19].

models for co-movements of ratings and spreads for all names in the reference portfolio. In such models the defaults in the underlying index are generated through a detailed modeling of rating migrations of the underlying names, and the index spread is modeled as a stochastic process depending on the average rating of names in the index. Such detailed joint modeling of rating and spread movements is not accessible to entities other than rating agencies due to lack of historical data on ratings. We will argue below that in fact such a complex framework may not be necessary: the main features of CPDOs can be captured with a low-dimensional model, which can be more readily estimated, simulated and analyzed.

3 Top-down modeling of CPDOs

The above considerations show that the risk and performance of a CPDO strategy mainly depend on

- the behaviour of the index default swap spread
- the number of defaults/the total loss in the reference portfolio
- index roll effects

CPDO cashflows do not depend directly on features such as individual name ratings, the identity of the defaulting entities, the spreads of individual names, etc. This suggests that the risk of CPDOs can be parsimoniously modeled by describing defaults at the portfolio level using a top-down model.

We consider an arbitrage-free market model represented by a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$, where P denotes the real-world probability of market scenarios (statistical measure). We consider as numeraire the zero-coupon bond $B(t,T)$ and denote by $Q \sim P$ the forward measure associated with this numeraire [15]. The spot yield curve $s \mapsto R(t,s)$ at date t is defined by

$$B(t,s) = \exp[-(s-t)R(t,s)],$$

and the LIBOR rates at date t are given by $L(t,s) = \frac{\frac{1}{B(t,s)}-1}{s-t}$. In the examples we shall use a flat term structure $B(t,s) = e^{-r(s-t)}$ but this is by no means necessary.

Denote by N_t the number of defaults in the underlying portfolio up to time $t \leq T$; $(N_t)_{t\in[0,T]}$ is a point process. As we shall see below, we need to model the dynamics of N_t under P *and* Q. The dynamics will be described by specifying an *intensity* for N_t under each probability measure.

The T^I-year index default swap spread is determined such that the risk-neutral expected value of the default leg of the swap is equal to the expected value of the premium leg. Denote by $(L_t)_{t\in[0,T]}$ the loss process. Assuming a constant recovery level R across all names in the underlying index, we have $L_t = \frac{(1-R)}{N^I} N_t$. The default leg of the index default swap is a stream of payments that cover the portfolio losses as they occur. At time $t \leq T^I$ the cumulative discounted losses are given by

$$\begin{aligned}
D_t &= E^Q\left[\int_t^{T^I} B(t,s)dL_s \Big| \mathcal{F}_t\right] \\
&= B(t,T^I)E^Q\left[L_{T^I}|\mathcal{F}_t\right] - L_t - \int_t^{T^I} \overbrace{\frac{\partial}{\partial s}B(t,s)}^{-R(t,s)B(t,s)}\; E^Q\left[L_s|\mathcal{F}_t\right] ds \\
&= \frac{(1-R)}{N^I}\left(B(t,T^I)E^Q\left[N_{T^I}|\mathcal{F}_t\right] - N_t + \int_t^{T^I} R(t,s)B(t,s)E^Q\left[N_s|\mathcal{F}_t\right] ds\right).
\end{aligned}$$

The value of the premium leg at time t as a function of the index default swap spread S is

$$\begin{aligned}
P_t(S) &= S \sum_{t_l \in \mathbf{CD}\cap[t,T^I]} B(t,t_l)\Delta(t_l)\left(1 - \frac{E^Q\left[N_{t_l}|\mathcal{F}_t\right]}{N^I}\right) \\
&= S\, D_t^{\text{swap}}.
\end{aligned}$$

Finally, the swap spread contracted at time t for a swap expiring at T^I is

$$S(t,T^I) = \frac{\frac{(1-R)}{N^I}\left(B(t,T^I)E^Q\left[N_{T^I}|\mathcal{F}_t\right] - N_t + \int_t^{T^I} R(t,s)B(t,s)E^Q\left[N_s|\mathcal{F}_t\right] ds\right)}{D_t^{\text{swap}}}. \quad (4)$$

3.1 Modeling default risk

The main ingredient to the model is the dynamics of the number of defaults N_t. We propose here to use a reduced-form approach for modeling N_t: the occurrence of defaults is specified via the aggregate default intensity, $(\lambda_t)_{t\in[0,T]}$, defined as the conditional probability per unit time of a default in the portfolio. This intensity-based approach has been used in the recent literature to model portfolio credit risk [8, 17]. A special case is the Cox process framework: conditionally on some underlying market factor $(X_t)_{t\in[0,T]}$, N_t follows an inhomogeneous Poisson process with intensity $(\lambda(X_t))_{t\in[0,T]}$.

The choice of dynamics for the risk-neutral default intensity determines the slope of the term structure of credit spreads. This influences the CPDO performance via the profit/loss from liquidation of swap contracts on roll dates, since at these dates the CPDO manager buys back protection of a $(T^I - \frac{1}{2})$-tenor swap, protection that was initially sold with a T^I-year tenor. An upward (downward) sloping term structure will on average imply a profit (loss) on roll dates. Empirically, we typically observe an upward sloping term structure.

It is crucial to be able to compute the default swap spread in an efficient manner in the simulations and cash flow computations. As noted above, the expression for the swap spread requires computation of the expected number of defaults and/or the survival probabilities efficiently. These computations, especially the computation of the T^I-year swap spread, will be made tractable by choosing affine processes for the default intensity under Q.

Under an equivalent probability measure $P \sim Q$, the point process N_t will in general have a different intensity process [4, Theorem VI.2.] of the form $\lambda_t^Q = \vartheta_t \lambda_t^P$, where ϑ is a strictly positive predictable process which characterizes the risk premium for the uncertainty associated with the timing of defaults. For simplicity, we assume that the statistical default intensity is proportional to the risk neutral intensity: $\lambda_t^P = \frac{1}{\vartheta}\lambda_t^Q$ where ϑ is the risk premium.

3.2 Default intensity

We model the default events via the default intensity λ_t, defined as the $\mathcal{F}_t$-intensity of the default process N_t, where $\mathcal{F}_t$ designates the market history up to t, including observations of past defaults. Intuitively, the default intensity λ_t is the conditional probability per unit time of the next default event, given past market history:

$$\lambda_t = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P\big(N_{t+\Delta t} = N_{t-} + 1 | \mathcal{F}_t\big).$$

This naturally leads to a default intensity which jumps at default dates. Thereby the default process becomes self-affecting in that one default may have spill-over effects on other names and trigger a cluster of defaults.

Example 3.1 (Markovian defaults) *A simple way to model the impact of past defaults on the default rate is to model the default intensity as a function of the total number of defaults:*

$$\lambda_t = f(t, N_t).$$

This leads to a Markov process for N_t which is easy to simulate and in which loss distributions and other quantities may be computed by solving a system of linear ordinary differential equations. [8] show that the intensity function f implied from market prices of CDO tranches exhibit a strong, non-monotone, dependence of the default intensity on the number of defaults. However this model is too simple for our purpose since it leads to piecewise-deterministic spread dynamics between default dates, whereas the CPDO is sensitive to spread volatility.

Closed-form expressions for the swap spread may be readily obtained by assuming that the risk neutral default intensity (λ_t^Q) is an affine jump-diffusion:

$$d\lambda_t^Q = \mu(\lambda_t^Q)dt + \sigma(\lambda_t^Q)dW_t + \eta dZ_t, \tag{5}$$

where the coefficients $\mu(\cdot)$, $\sigma^2(\cdot)$ and the intensity of the jump process Z are affine functions of λ_t^Q. Transform methods can be applied to give an explicit expression for the swap spread as done in [14]. To compute $E^Q[N_s|\mathcal{F}_t]$ for $s \in]t, T^I]$ consider the 2-dimensional process $Y_t = (\lambda_t^Q, N_t)'$. Y is of the general affine form (5) and the drift function can be written $K_0 + K_1 Y_t$, the volatility $H_0 + H_1 Y_t$ and the jump intensity of the 2-dimensional jump process $\Lambda_0 + \Lambda_1 Y_t$. Define the Laplace transform $\theta : \mathbb{C}^2 \to \mathbb{C}$ of ν by

$$\theta(c) = \int_{\mathbb{R}^2} e^{c \cdot z} d\nu(z).$$

In affine models [13] the conditional expectation of the number of defaults can be expressed as an affine function of the state variable

$$E^Q[N_s|\mathcal{F}_t] = A(t) + B(t) Y_t. \tag{6}$$

for $v = (0 \quad 1)'$. $A : [0, s] \to \mathbb{R}$ and $B : [0, s] \to \mathbb{R}^2$ are determined by the following differential equations

$$\begin{aligned} \partial_t B(t) &= -K_1' B(t) - \Lambda_1 \nabla\theta(0) \cdot \eta B(t) && (7) \\ \partial_t A(t) &= -K_0 \cdot B(t) - \Lambda_0 \nabla\theta(0) \cdot \eta B(t) && (8) \end{aligned}$$

with terminal conditions $A(s) = 0$ and $B(s) = v$ and where $\nabla\theta$ denotes the gradient of θ. These expressions can in turn be used to compute (6). In special cases (7)–(8) can be solved analytically, providing an analytic expression for $E^Q[N_s|\mathcal{F}_t]$ and thereby for the swap spread.

Example 3.2 (Self-exciting defaults) *An example of a self-exciting default process is given by the model of [17] where the default intensity jumps up by a magnitude proportional to the loss at defaults and follows a diffusive process between default times. The intensity process is given by*

$$d\lambda_t^Q = \kappa\big(\theta - \lambda_t^Q\big)dt + \sigma\sqrt{\lambda_t^Q} dW_t + \eta dL_t, \qquad \lambda_0^Q > 0 \tag{9}$$

where L denotes the loss process. The intensity of the default counting process N is thus updated at each default and undergoes a jump. Since this default intensity follows a CIR-process (13) between defaults, $2\kappa\theta \geq \sigma^2$ is required to ensure $\lambda_t^Q > 0$ almost surely.

Between default events the intensity reverts back to its long term level θ exponentially in mean at a rate $\kappa \geq 0$ with diffusive fluctuations driven by a Brownian motion. The default counting process N is self-exciting because the intensity of N_t increases at each default event. This property captures the feedback effects (contagion) of defaults observed in the credit market.

The process (9) belongs to the class of affine processes, where the expected number of defaults is given in closed form: For $B(t) = (B_1(t), B_2(t))'$, $B_2(t) = 1$, where

$$\begin{aligned} B_1(t) &= -\frac{1}{\kappa + \eta\left(\frac{1-R}{N^I}\right)} \left(e^{-\left(\kappa+\eta\left(\frac{1-R}{N^I}\right)\right)(T^I - t)} - 1\right) && (10) \\ A(t) &= \frac{\kappa\theta}{\left(\kappa + \eta\left(\frac{1-R}{N^I}\right)\right)^2} \left(e^{-\left(\kappa+\eta\left(\frac{1-R}{N^I}\right)\right)(T^I - t)} - 1\right) + \frac{\kappa\theta}{\kappa + \eta\left(\frac{1-R}{N^I}\right)}(T^I - t), && (11) \end{aligned}$$

we have $E^Q[N_{T^I}|\mathcal{G}_t] = A(t) + B_1(t)\lambda_t^Q + N_t$*, which gives an analytic expression for the* T^I*-year swap spread.*

$\diamond$

3.3 Cox process framework

A special class of default intensity models is where the default counting process N is specified as a Cox process [21]. Let X be a Markov process on a probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ designating a risk factor and define $\mathcal{G}_t = \sigma\{X_s | s \leq t\}$. We model the default intensity by $\lambda_t := \lambda(X_t)$ where $\lambda : \mathbb{R}^n \to \mathbb{R}_+$ is a non-negative function. Assume that

$$\Lambda_t := \int_0^t \lambda_s ds < \infty \quad \text{almost surely} \qquad t \in [0, T].$$

Let $\tilde{N}$ be a standard unit rate Poisson process independent of $\mathcal{G}_t$. A Cox process N with intensity (λ_t) can be constructed as $N_t := \tilde{N}_{\Lambda_t}$. It is straightforward to check that $N_t - \int_0^t \lambda_s ds$ is a $\mathcal{F}_t$-martingale and thereby (λ_t) is a $\mathcal{F}_t$-intensity for N_t. Default times can then be simulated/generated successively as

$$\tau_i = \inf\left\{ t > \tau_{i-1} \Big| \int_{\tau_{i-1}}^t \lambda_s ds \geq E_i \right\}, \tag{12}$$

where $(E_i)_{i=1,..,N^I}$ is a sequence of independent, identically distributed standard exponential random variables.

The main advantage of specifying the default intensity as a Cox process is the simple method for generating default events as given (12). One restriction, though, implied by Cox specification is that the default intensity process is not affected by the occurrence of default events. This leads to an underestimation of default clustering effects [11] due to the fact that in the Cox framework the hazard rate λ_t depends only on the history of the factor process X but not on the default itself.

Example 3.3 (A Cox process with CIR hazard rate) *Let the risk-neutral hazard rate (i.e. under Q) be defined by the CIR dynamics:*

$$d\lambda_t^Q = \kappa\big(\theta - \lambda_t^Q\big)dt + \sigma\sqrt{\lambda_t^Q}dW_t, \qquad \lambda_0^Q > 0. \tag{13}$$

This model leads to a mean-reverting and non-negative short term spread if $2\kappa\theta \geq \sigma^2$*. This is a special case of Example 3.2 with* $\eta = 0$ *and here (10)–(11) reduce to*

$$\begin{aligned} B_1(t) &= -\frac{1}{\kappa}\big(e^{-\kappa(T^I - t)} - 1\big) \\ A(t) &= \int_t^{T^I} \kappa\theta B_1(s)ds = \frac{\theta}{\kappa}\left(e^{-\kappa(T^I - t)} - 1\right) + \theta(T^I - t). \end{aligned}$$

$\diamond$

Another choice of hazard rate is the exponential OU process:

Example 3.4 (Exponential Ornstein-Uhlenbeck process) *In this model the hazard rate is assumed to follow*

$$\frac{d\lambda_t^Q}{\lambda_t^Q} = \alpha\big(\beta - \ln\lambda_t^Q\big)dt + \xi dW_t. \tag{14}$$

This process is mean-reverting and non-negative, which are desirable qualities for a hazard rate process, it has a log-normal distribution and is stationary for long time horizons. The exponential OU process produces heavier tails in the distribution of the default intensity than the CIR model, for which increments follow a χ^2-distribution. The process (14) is not affine and the index default swap spread needs to be computed via quadrature.

◇

3.4 Modeling the index roll

When modeling the default intensity of the underlying index, it is crucial to take the semi–annual rolling of the index into account: the replacement of downgraded names results in a negative jump in the default intensity and thereby also in the swap spread on average implying a loss when liquidating swap contracts. In the long run, rolling the index also has a positive effect as the portfolio default risk and thereby the portfolio loss is lowered.
We consider two possible approaches for modeling the roll over effect.

The simplest model is to include a constant proportional jump size in the default intensity on each roll date. However, empirical observations show variation in the jump sizes, so extending this setup to allow for two possible jump sizes $h_1, h_2 \in [0,1]$, not necessarily taken with equal probability, is more realistic.

To model in more detail the index roll, we assume that the index is homogeneous, such that all individual name default intensities $(\Lambda^1, ..., \Lambda^{N^I})$ are independent with identical distribution denoted F. For a given roll date $T_j \in \mathbf{RD}$ let $(\bar{\Lambda}^1_{T_j}, ..., \bar{\Lambda}^{N^I}_{T_j})$ be a realization of N^I independent F distributed variables. Rolling the index corresponds to removing a number of the highest realizations and replacing these by new independent draws from the F distribution. The roll over effect is then given as the difference between the average intensity before and after the roll. This setup requires an assessment of the average number of names removed on each roll date and of the individual default intensity distribution.

Assuming that the term structure is upward sloping, the effect from rolling down the credit curve will counteract the effect from rolling over the index. Empirically these two effects are more or less observed to offset each other. In the following, the first mentioned approach for modeling the roll over effect will be taken, and the jump sizes $\{h_1, h_2\}$ are chosen such as on average to cancel the roll down effect implied by the dynamics of the default intensity. Note that, the proportional jump in the default intensity does not affect the calculation of the index spread, since this references the current index, not the rolling index.

4 Performance and risk analysis

We analyze the performance of the CPDO strategy by Monte Carlo simulations. The aim is not only to assess the rating based on the default and cash out probabilities but also to study other risk measures such as the loss distribution and the expected shortfall. Further, we wish to identify key parameters and study the dependence of the CPDO performance on these parameters.

4.1 Simulation results

We model the risk neutral intensity λ_t^Q by a CIR process with jumps at default events (9) given in example 3.2. We will study the performance of CPDOs in two credit market configurations. The first corresponds to the historical credit environment during the period 2004–2007 and is based on the study of [1], who estimate the parameters in (9) using data for the spread of the CDX index. Since the CPDO considered here is written on both ITRAXX and CDX, we multiply the estimated default intensity of [1] by two, thereby implicitly assuming that the spread of ITRAXX has properties similar to CDX. This leads to a risk neutral intensity specified by the parameters

$$\theta = \lambda_0^Q = 1.7 \qquad \kappa = 0.35 \qquad \sigma = 0.75\sqrt{2} = 1.061 \quad \text{and} \quad \eta = 0.8.$$

[1] find that the risk premium $\vartheta_t = \frac{\lambda_t^Q}{\lambda_t^P}$ for correlated default risk may fluctuate widely, typically in the range 10–30. However, ϑ increased to much higher levels during the market turmoil in late 2007 and was very close to zero during the benign credit environment in 2005–2006. Yet, to maintain a parsimonious model, we assume a constant risk premium at $\vartheta = 20$. Let $r = 0.05$ and $R = 0.4$. For the roll over effect, we choose a fairly low relative jump size $h_1 = 0.05$ in most scenarios (95%) and occasional (5%) large downward jumps of $h_2 = 0.2$ on roll dates.

We simulate the path of the risk neutral intensity piecewise between arrivals of default events according an Euler discretization scheme as the i'th inter-arrival intensity follows a CIR-process started at $\lambda_{\tau_{i-1}}^Q$. The next default event τ_i is generated according to (12), and since the loss process L jumps at the default event, so does the intensity: $\lambda_{\tau_i}^Q = \lambda_{\tau_i-}^Q(1 + \eta\frac{1-R}{N^I})$. After the jump, the default intensity again follows a CIR-process. Every six months we introduce a negative, proportional jump in the default intensity on index roll dates:

$$\lambda_t^Q = \lambda_{t-}^Q(1 - ph_1 - (1-p)h_2) \quad \text{for} \quad t \in \mathbf{RD},$$

where p is drawn from a Bernoulli distribution with success probability 0.95 and h_1, h_2 are the two possible jump sizes. Then, with the simulated a path for the risk neutral intensity we can calculate the index spread (4) using (6) and (10)–(11). The parameter choice in the historical credit environment corresponds to a spread level around 47 bp and on average 0.7 defaults over the 10 year lifetime of the CPDO.

We examine the case of a CPDO contract paying coupons of 200bp above LIBOR employing a maximum leverage of $M = 15$, $\varepsilon = 0.25$ and cash out threshold $k = 0.1$. The aggressiveness of the target leverage rule (1) is determined by the gearing factor $\beta = 1.7$.

Based on 10 000 simulation runs we find the probability of the CPDO defaulting to be 1.8%. According to Standard & Poor's default probability thresholds given in table 1, this will earn the CPDO principal note a A+ rating. The probability of the CPDO cashing out and thereby defaulting on both coupons and principal note is 0.04% which gives the coupon payments a AAA rating. The expected loss conditional on default occurring, LGD, is 3.5% of notional.

Another useful risk measure is the expected shortfall defined at a given level α by

$$ES_\alpha = E\big[L|L > VaR_\alpha\big] \qquad \text{for} \qquad VaR_\alpha = \inf\big\{l\,|\,P(L > l) < 1 - \alpha\big\},$$

where L denotes the loss of the CPDO. In this credit environment we find $ES_{0.99} = 6.0\%$ of note notional. That is, in the worst 1% of the scenarios, the investor expects to recover more than 90% of the initial investment. If not defaulting the CPDO cashes in after 5.1 years on average. Results are given in table 2.

Market	PD (%)	Cash Out (%)	Rating –	LGD (%)	ES_{99} (%)	Cash In (years)	$E[N_T]$ –
Historical	1.8	0.04	A+	3.5	6.0	5.1	0.69
Stressed	1.2	0.10	AA	9.0	10.5	5.0	1.4

Table 2: Summary of results.

Figure 1 illustrates a typical scenario in the historical credit market. The top left graph shows the portfolio default intensity λ^P, at the top right is the on-the-run index default swap spread $S(t, T^I)$ versus the piecewise constant contracted spread $(\bar{S}_t)$, at the bottom left is the target (m_t) and actual $(\bar{m}_t)$ leverage factors and in the bottom right is the evolution of the CPDO portfolio value (V_t). Note that the spread widening during the years 2–4 and the implied mark-to-market losses result in decreasing CPDO portfolio value. However, the consecutive spread tightening allows the CPDO to cash in after approximately 7 years.

During most of the estimation period 2004–2007 up to the summer of 2007 index spreads were very tight, but increased during the second half of 2007. In 2008 the index spreads increased dramatically. Therefore, we study the CPDO performance in a second market configuration corresponding to a more stressed market environment with higher spread levels. If doubling the risk neutral intensity by employing the parameters

$$\theta = \lambda_0^Q = 3.4 \qquad \kappa = 0.35 \qquad \sigma = 1.5 \quad \text{and} \quad \eta = 1.6,$$

while leaving the risk premium ϑ unchanged, the average index default swap spread is around 95bp and on average 1.4 obligors default over a 10 year period. In a stressed market environment, the maximum allowed leverage is likely to be reduced, so here we set M = 10. The remaining input parameters are left

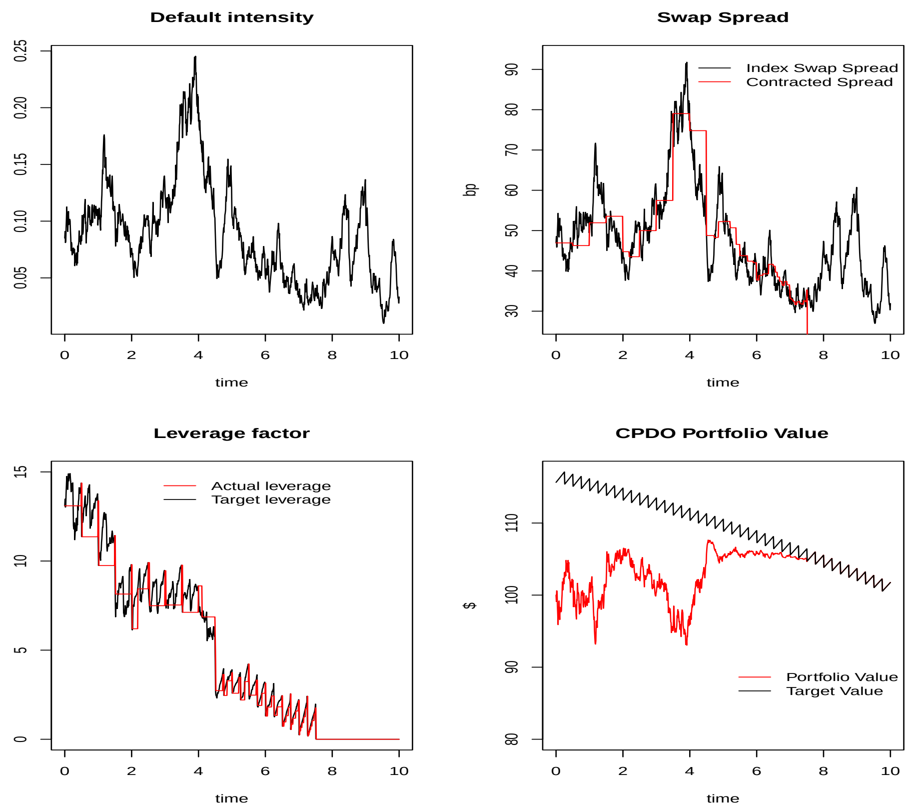

Figure 1: Historical credit market scenario. Top left: Portfolio default intensity λ^P. Top right: On-the-run index default swap spread $S(t, T^I)$ (black line) versus the contracted spread $(\bar{S}_t)$ (red line). Bottom left: Target (m_t) (black line) and actual leverage factor $(\bar{m}_t)$ (red line). Bottom right: CPDO portfolio value (V_t) (red line) compared to the target value (black line).

unchanged. The probability of the CPDO defaulting is 1.2% corresponding to an AA rating, and the cash out probability is 0.10%, which gives the coupons a AAA rating. The expected shortfall is 10.5% of note notional and almost twice as high as the tail loss in the historical credit market. Like in the historical market setting, the CPDO cashes in after 5 years on average.

Figure 2 shows the default intensity, index spread, leverage factors and CPDO portfolio value in a typical scenario in the stressed credit environment. Here, an early spread tightening from 180 bp to 60 bp and the implied mark-to-market gain causes the CPDO to cash in after approximately five years.

4.2 Sensitivity analysis

To assess the various parameters' impact on the CPDO performance we carry out a sensitivity analysis of the dependence of risk measures on model parameters. Simulation results from the two market configurations are found in tables 3–4. Ratings in the tables refer to the principal note and are given according

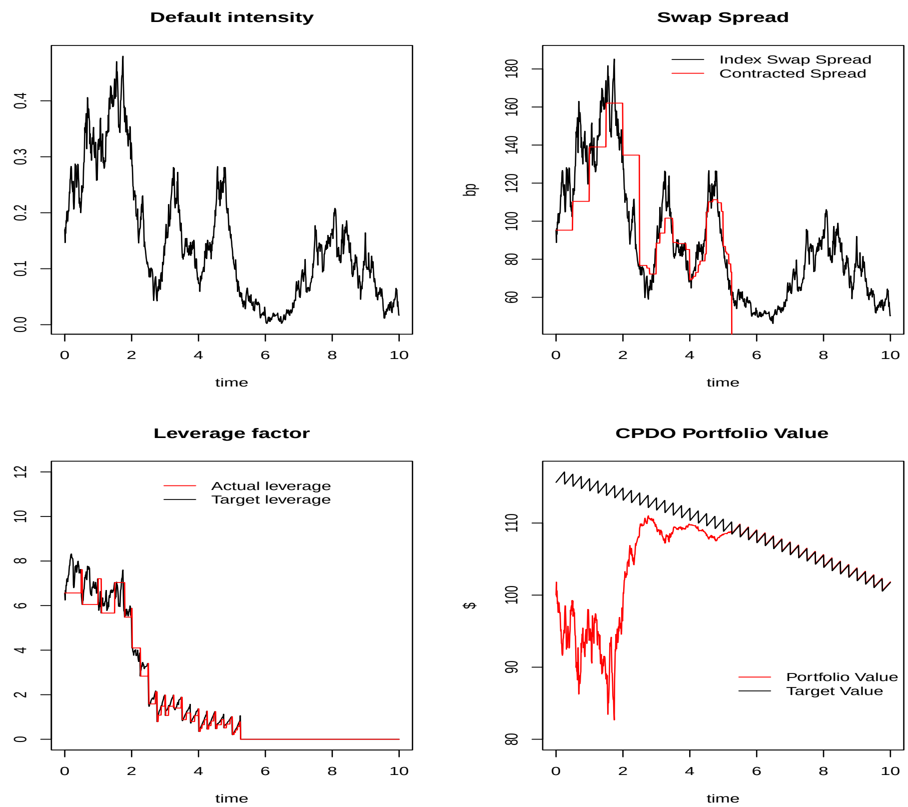

Figure 2: Stressed credit market scenario. Top left: Portfolio default intensity λ^P. Top right: On-the-run index default swap spread $S(t, T^I)$ (black line) versus the contracted spread $(\bar{S}_t)$ (red line). Bottom left: Target (m_t) (black line) and actual leverage factor $(\bar{m}_t)$ (red line). Bottom right: CPDO portfolio value (V_t) (red line) compared to the target value (black line).

to the CDO default matrix of Standard & Poor's [19] given in 1. The main findings are summarized below.

Default intensity

The default risk premium ϑ has a significant influence on the CPDO performance, since ϑ determines the average level of spread income relative to the credit losses incurred. Halving the risk premium from $\vartheta = 20$ to $\vartheta = 10$ doubles the average number of credit events and results in a higher CPDO default probability. The dependence of the CPDO performance on the risk premium is illustrated in figure 3 showing the probability of default and expected 99% shortfall in the historical credit market as a function of the risk premium ϑ based on 10 000 simulations. Not surprisingly, we see downward sloping curves in both cases.

While the average number of defaults in the underlying portfolio have some effect on the CPDO performance, the risk of mark-to-market losses from spread widening is more important. One default causes a reduction of portfolio value

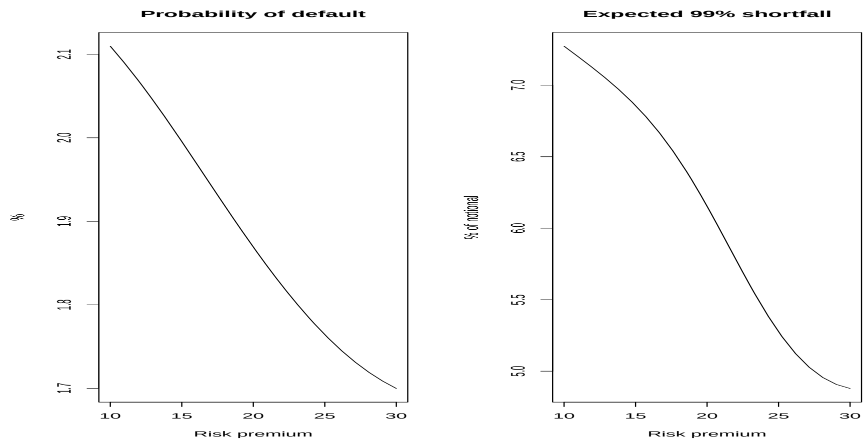

Figure 3: Left: Dependence of CPDO default rate on risk premium ϑ. Right: Dependence of CPDO expected 99% shortfall on risk premium ϑ.

(if leveraged up to 15x) of 3.6%, whereas a 15bp spread widening causes almost 10% reduction of the portfolio value. In our model, the main determinant of possible spread widenings is the mean reversion parameter κ. With a higher mean reversion speed the index spread fluctuates more tightly around its long term mean level which reduces the mark-to-market losses. Therefore a higher κ reduces the probability of default and leads to a lower expected shortfall. This dependence is investigated in figure 4, where the default rate and expected 99% shortfall as a function of the mean reversion speed κ are shown. The minimum level of κ is restricted by the condition $2\kappa\theta \geq \sigma^2$ ensuring that the CIR-process is positive. Notice that the estimates found by [1] only just satisfy this inequality, so there is little room for reducing κ further in this analysis. We see an improved performance for increasing values of κ, both with respect to the CPDO default rate and the expected 99% shortfall.

The parameter underlying the intensity process affecting the CPDO performance the most is the long term mean θ: the higher level of spread income implied by a higher value of θ more than compensates for the increase in credit losses which is also an implication of increasing θ. However, increasing θ also implies a higher probability of the CPDO cashing out and thereby a higher expected shortfall. This is most clearly seen when comparing the historical and stressed credit markets. Also notice, that if the CPDO is issued during a period, where the default intensity is below its long term mean – i.e. if $\lambda_0^Q < \theta$ – then the spread is likely to widen initially, which results in mark-to-market losses. This we see has a large effect on the default and cash out probabilities. The effect of an initial spread tightening ($\lambda_0^Q > \theta$) is less dramatic.

The volatility of the default intensity affecting the volatility of the spread is less important, but we do see that a higher volatility is harmful for the CPDO performance both when it comes to the probabilities of default and cash out as well as the expected shortfall. The reason is that higher volatility leads to more extreme scenarios, which for a CPDO with no upside potential means more

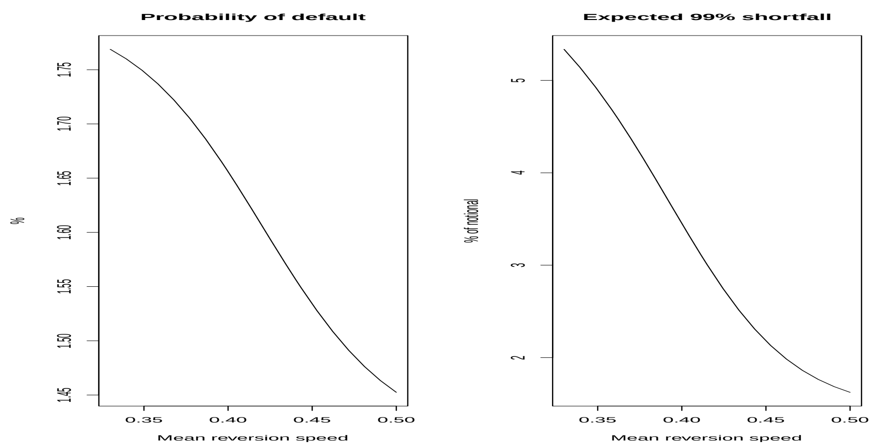

Figure 4: Left: Dependence of CPDO default rate on mean reversion speed κ. Right: Dependence of CPDO expected 99% shortfall on mean reversion speed κ.

extreme loss scenarios and a higher probability of cashing out. Also in this case, the condition $2\theta\kappa \geq \sigma^2$ restricts the analysis of high levels of volatility.

The feedback effect of defaults on the default intensity given in terms of the jump parameter η does not have a clear effect on the CPDO default rate and cash out probability. Since only few defaults happen over the 10 year lifetime of the CPDO, η is of minor importance for the performance. Note that, in the case $\eta = 0$ we recover the CIR-intensity from example 3.3.

We find that the magnitude of the negative jumps in the spread at index roll dates affects the performance significantly: without jumps at roll dates the CPDO performance would improve.

Recovery rate
The level of recovery R upon default in the underlying portfolio affects the outcomes in two ways. First, a higher recovery will increase the CPDO default probability because a higher recovery implies a lower index default swap spread. On the contrary, a high recovery rate will lead to fewer cash out events and lower losses given default.

Interest rates
The level of interest rates is inversely related to the probabilities of default and cash-out: a lower interest rate implies higher probabilities and higher tail losses, which may rise to dramatic levels for very low interest rates. The reason is that a fall in interest rates will result in a higher target value and lead the CPDO strategy to become more aggressive and increase leverage to meet this higher target, thereby increasing its exposure to spread risk. The average cash-in time is also increased as a consequence of lower interest rates.

Leverage strategy
The parameters governing the leverage strategy are key when studying the CPDO performance. Increasing the maximum leverage M will result in lower

default rates at the cost of higher losses. This is exactly the reason for capping the leverage factor, namely to reduce the overall possible loss.

The aggressiveness of the strategy, as described by the gearing factor β, enables to obtain a wide range of CPDO default rates, expected shortfall levels and cash out probabilities. As shown in Figure 6, a low gearing factor reduces the expected shortfall at the cost of higher default probability and vice versa. This is illustrated in figure 5 for both the historical and stressed markets. Another way to view this result is to say that the CPDO manager may attain lower and lower values of default probability but at the price of increasing Expected Shortfall i.e. higher tail risk: thus, if a credit rating based on default probability is the only metric used to assess the strategy, one can always achieve a AAA rating by "pushing the risk far enough into the tails".

A more aggressive strategy also reduces the average cash-in time. Some versions of the CPDO strategy employ in fact a time dependent gearing factor. Alternatively, the risky duration of the liabilities of the CPDO could be used when calculating target leverage instead of the duration of assets. This would result in a less aggressive strategy the first couple of years, but in case the CPDO has not cashed in closer to maturity, the strategy subsequently becomes more aggressive.

If we increase the level of the cash out threshold, k, the default and cash out probabilities as well as the expected shortfall will increase. This happens because a higher cash out threshold eliminates the chance of (partial) recovery the CPDO would have had with a lower k.

Employing a simplified version of the leverage strategy according to which leverage is adjusted only on roll dates lowers the probability of default at the cost of higher tail losses. The reason is that as long as a scenario evolves favourable, i.e. as long as the difference between target and portfolio value is reduced, this simple leverage strategy operates more aggressively because it reduces leverage more rarely than the regular strategy. However, a more aggressive strategy also means higher exposure if spreads suddenly increase and leads to heavier mark-to-market losses.

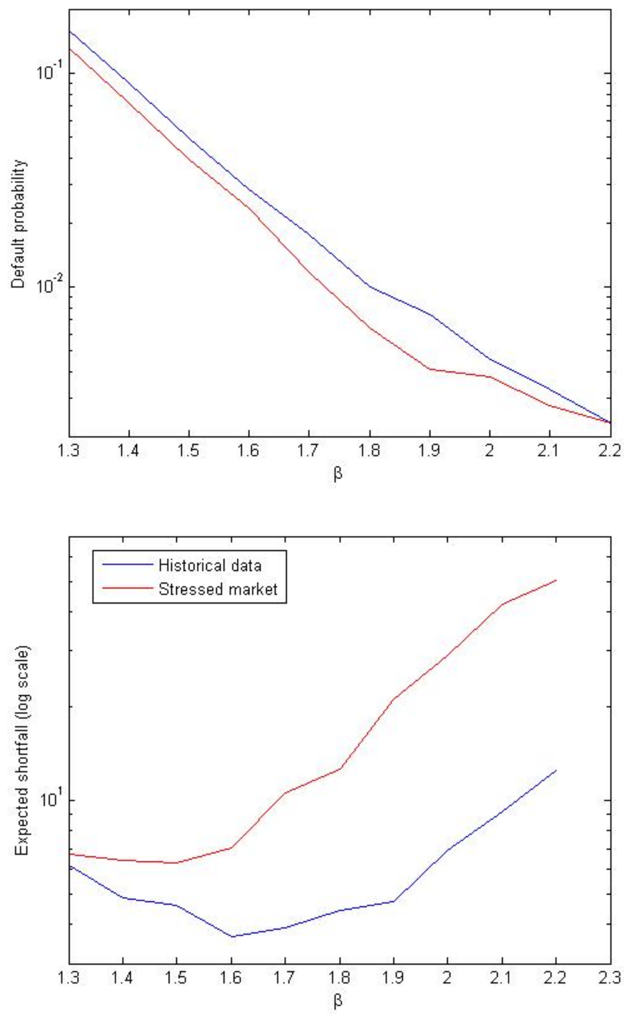

Figure 5: Top: CPDO default rate versus gearing factor β. Bottom: expected shortfall at 99% level versus gearing factor β. The blue curve corresponds to the historical market environment and the red curve to a stressed market.

4.3 CPDO loss distribution

Being a leveraged instrument sensitive to spread risk, a CPDO may generate a loss distribution with highly asymmetric features and using simple statistics such as expected loss may not adequately summarize its risk profile. An example of such a loss distribution is shown in Figure 7 for a stressed credit environment. We see that in most of the default scenarios, only small losses of 0-20% of note notional are incurred. In these cases the CPDO is typically not under distress toward expiry but the employed leverage strategy is not aggressive enough to allow a cash in prior to expiry. The right tail of the loss distribution corresponds to cases where the CPDO cashes out or is very close to cashing out.

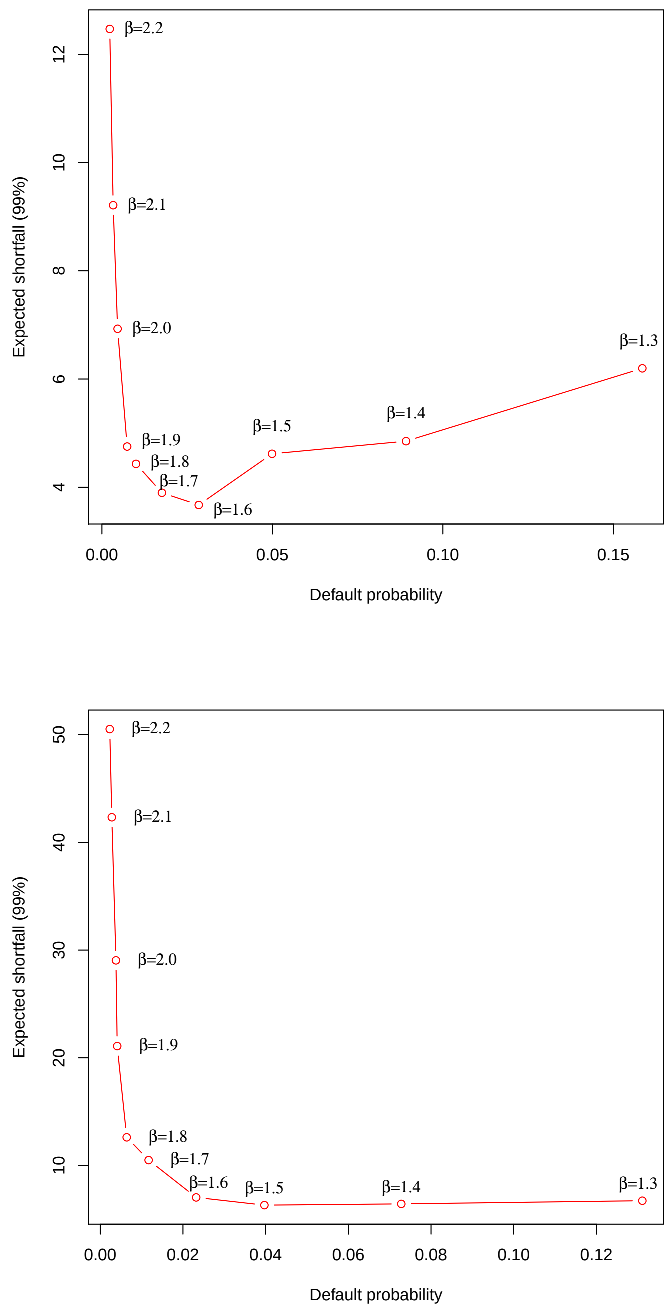

Figure 6: CPDO default rate versus 99% Expected Shortfall, for different gearing factors β. Top: historical environment (pre 2007). Bottom: stressed market environment; note the considerably higher tail risk in the stressed environment.

Historical market	PD (%)	Cash Out (%)	Rating –	LGD (%)	ES_{99} (%)	sd(LGD) –	Cash In (years)	S_0 (bp)	$E[N_T]$ –
standard	1.75	0.04	A+	3.5	6.0	13.2	5.1	47.0	0.69
simple	0.28	0.04	AAA	15.6	15.6	29.7	2.8	47.0	0.68
1% adm. fee	2.06	0.06	A+	3.7	7.2	12.8	5.2	47.0	0.68
$\delta = 0.015$	0.93	0.02	AA+	3.6	3.6	12.7	4.6	47.0	0.67
$\delta = 0.025$	2.97	0.01	A-	2.1	4.9	5.6	5.7	47.0	0.68
$\vartheta = 10$	2.60	0.03	A	3.0	7.0	10.3	5.2	47.0	1.39
$\vartheta = 30$	1.95	0.07	A+	4.6	8.7	16.5	5.1	47.0	0.47
$\sigma = 1.0$	1.82	0.04	A+	3.4	5.9	13.2	5.3	47.0	0.70
$\sigma = 1.09$	1.90	0.05	AA-	3.9	7.2	14.4	5.1	47.0	0.69
$\theta = \lambda_0^Q = 1.61$	1.74	0.03	AA-	3.3	5.5	11.8	5.1	44.4	0.64
$\theta = \lambda_0^Q = 2.0$	1.48	0.01	AA	1.7	2.4	7.1	5.3	55.4	0.80
$\lambda_0^Q = 1.0$	3.27	0.09	A-	4.06	12.0	14.3	5.7	37.7	0.62
$\lambda_0^Q = 2.5$	1.04	0.05	AA	5.7	5.9	19.4	4.8	57.7	0.78
$\kappa = 0.335$	1.94	0.06	A+	3.9	7.4	15.0	5.0	47.0	0.69
$\kappa = 0.5$	1.36	0	AA	0.9	1.2	1.2	6.0	47.0	0.72
$\eta = 0$	1.71	0.02	AA-	2.7	4.4	9.9	5.1	47.1	0.67
$\eta = 1.5$	1.86	0.06	AA-	4.3	7.6	15.5	5.1	46.8	0.69
$R = 0.2$	1.69	0.12	AA-	7.7	12.7	22.5	5.1	62.5	0.67
$R = 0.6$	2.87	0	A-	1.7	3.9	3.0	5.6	31.3	0.69
$r = 0.01$	32.2	0.13	B+	1.1	18.0	5.9	8.0	42.4	0.70
$r = 0.1$	0.06	0.01	AAA	15.1	15.1	34.7	3.4	53.1	0.69
$h_1 = 0$, $h_2 = 0$	0.92	0	AA+	1.0	1.0	1.3	4.9	47.0	0.85
$h_1 = 0.3$, $h_2 = 0.02$	1.56	0.04	AA-	3.8	5.8	14.5	5.0	47.0	0.76
$M = 10$	2.90	0	A-	1.6	3.6	3.1	5.6	47.0	0.70
$M = 20$	1.83	0.11	AA-	6.8	12.2	21.1	5.1	47.0	0.68
$\beta = 1.5$	5.49	0.01	BBB	1.4	4.9	4.2	6.1	47.0	0.69
$\beta = 2$	0.47	0.04	AAA	9.4	9.4	24.4	4.1	47.0	0.69
$k = 0.2$	1.84	0.09	AA-	5.0	9.0	15.9	5.2	47.0	0.68

Table 3: Sensitivity analysis: Historical market setting. The standard parameters are $\theta = \lambda_0^Q = 1.7$, $\kappa = 0.35$, $\sigma = 1.061$, $\eta = 0.8$ and $\vartheta = 20$. The CPDO pays a 200bp spread, has gearing factor $\beta = 1.7$ and maximum leverage of $M = 15$. Note that it is not possible to vary the parameters freely in the sensitivity analysis, since the condition $2\kappa\theta \geq \sigma^2$ should be fulfilled in a CIR model.

4.4 Scenario analysis

The model allows to determine market scenarios that are most harmful for the CPDO performance by studying scenarios in the historical credit market in which the CPDO cashes out. An example of such a scenario is given in figure 8. In this case a more than 100bp spread widening between year 3 and 5 and two defaults between year 3.5 and 4 cause the portfolio value to drop to 10% of notional and the CPDO cashes out at year 4.7.

In general, the main reason causing a CPDO to cash out is continued spread widening. The average cash out time in this credit environment, given that there is a cash out event, is 3.7 years. Since the default intensity is mean reverting, a significant spread widening will at some point be followed by a similar spread tightening, as is also the case in figure 8. If the CPDO survives the period of spread widening it will benefit from the consecutive spread tightening and may thereby regain a large part of the lost capital.

Stressed market	PD (%)	Cash Out (%)	Rating –	LGD (%)	ES_{99} (%)	sd(LGD) –	Cash In (years)	S_0 (bp)	$E[N_T]$ –
standard	1.17	0.10	AA	9.0	10.5	24.8	5.0	95.3	1.38
simple	0.16	0.06	AAA	33.8	33.8	42.7	2.6	95.3	1.37
1% adm. fee	1.24	0.09	AA	7.7	9.5	23.2	5.1	95.3	1.39
$\delta = 0.015$	0.72	0.06	AAA	8.8	8.8	24.8	4.6	95.3	1.4
$\delta = 0.025$	1.99	0.31	A+	15.0	29.5	31.9	5.3	95.3	1.34
$\vartheta = 10$	1.75	0.08	AA-	5.4	9.2	18.3	5.0	95.3	2.75
$\vartheta = 30$	1.12	0.13	AA	11.5	12.5	28.5	4.9	95.3	0.92
$\sigma = 1.4$	1.27	0.06	AA	5.0	6.3	18.5	5.1	95.3	1.39
$\sigma = 1.54$	1.24	0.20	AA	15.3	18.9	32.4	4.9	95.3	1.39
$\theta = \lambda_0^Q = 3.22$	1.15	0.09	AA	8.2	9.4	23.8	4.9	90.1	1.31
$\theta = \lambda_0^Q = 4.0$	0.91	0.03	AA+	3.8	3.8	15.4	5.1	112.8	1.60
$\lambda_0^Q = 2.0$	1.76	0.13	AA-	7.8	13.5	22.8	5.5	76.1	1.21
$\lambda_0^Q = 4.5$	0.73	0.05	AAA	7.1	7.1	22.1	4.7	110.6	1.49
$\kappa = 0.335$	1.27	0.10	AA	8.1	10.2	23.6	4.9	95.3	1.37
$\kappa = 0.5$	0.62	0	AAA	0.8	0.8	1.5	5.8	95.4	1.43
$\eta = 0$	1.32	0.12	AA	9.2	12.1	25.2	4.9	95.8	1.39
$\eta = 2.5$	1.28	0.08	AA	6.7	8.6	21.4	5.0	95.0	1.38
$R = 0.2$	1.19	0.11	AA	9.6	11.4	26.2	5.0	126.8	1.38
$R = 0.6$	1.25	0.01	AA	2.2	2.8	8.1	5.0	63.6	1.39
$r = 0.01$	24.5	0.22	BB-	1.4	25.2	8.7	7.8	86.1	1.38
$r = 0.1$	0.06	0.02	AAA	30.8	30.8	41.3	3.3	107.6	1.37
$h_1 = 0,\ h_2 = 0$	0.36	0.05	AAA	13.0	13.0	31.0	4.7	95.3	1.70
$h_1 = 0.3,\ h_2 = 0.02$	0.69	0.07	AAA	10.3	10.3	27.0	4.9	95.3	1.51
$M = 5$	2.04	0	A+	1.8	3.1	3.1	5.4	95.3	1.39
$M = 15$	1.11	0.08	AA	7.4	8.2	22.7	5.0	95.3	1.38
$\beta = 1.5$	3.90	0.04	BBB+	2.1	6.6	9.1	5.9	95.3	1.37
$\beta = 2$	0.34	0.19	AAA	51.1	51.1	44.7	3.9	95.3	1.38
$k = 0.2$	0.96	0.14	AA+	11.7	11.7	25.7	4.9	95.3	1.36

Table 4: Sensitivity analysis: Stressed credit market. The standard parameters are: $\theta = \lambda_0^Q = 3.4$, $\kappa = 0.35$, $\sigma = 1.5$, $\eta = 1.6$ and $\vartheta = 20$. The CPDO pays a 200bp spread, has gearing factor $\beta = 1.7$ and maximum leverage $M = 10$. Note that it is not possible to vary the parameters freely in the sensitivity analysis, since the condition $2\kappa\theta \geq \sigma^2$ should be fulfilled in a CIR model.

Similarly, one can study the most favorable scenarios for the CPDO. This reveals that initial spread tightening combined with no defaults in the reference portfolio, results in the fastest cash. A fast cash-in scenario is shown in figure 9. In this example, an initial spread tightening from approximately 55bp to 35bp lays the foundation for the CPDO cashing in after 2.8 years.

4.5 Variability of ratings and downgrade probabilities

Given that CPDOs are leveraged and path-dependent instruments, the initial rating of CPDO notes gives only a partial idea of the risk of the instrument. As in the case of CDO tranches studied by [9], a CPDO with initial AAA rating may have a probability of being downgraded which is much higher than a AAA bond. It is therefore interesting to examine the probability of rating downgrades during the lifetime of the CPDO. This can be done by a nested Monte Carlo simulation as suggested by [23].

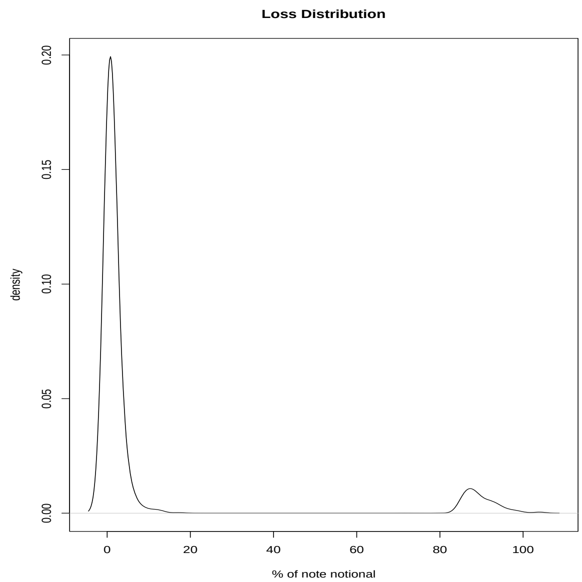

Figure 7: Kernel estimator of loss density in stress scenario (Sec. 4.1) based on 100 000 simulated scenarios.

Suppose that at initiation $t = 0$ the strategy is given rating R_0 corresponding to a T-year probability of default $PD(0,T) \in [L^T, U^T[$, where U^T, L^T are default probability thresholds corresponding to a rating R_0 over a horizon of T years. We want to re-assess the rating R_1 after $T_1 < T$ years by calculating the T_2-year default probability, $PD(T_1, T_2)$ given information available at time T_1, for $T_2 = T - T_1$,. If $PD(T_1, T_2) \notin [L^{T_2}, U^{T_2}[$ the CPDO rating at time T_1 has changed; $R_1 \neq R_0$. In the outer loop of the simulation N_O paths of λ, V, A, $\bar{m}$, etc. up to time T_1 are generated. If the CPDO has cashed out at time T_1 the rating $R_1 = D$ is recorded. For each path that has not cashed out at time T_1, a second Monte Carlo simulation is performed in order to assess the default probability at T_1 and thereby the rating R_1. This is done by simulating N_I paths from time T_1 to expiry T, using the starting values λ_{T_1}, V_{T_1}, A_{T_1}, etc. found in the outer loop. The rating transition probability is estimated by:

$$P(R_1 \neq R_0) \approx \frac{1}{N_O} \sum_{i=1}^{N_O} \mathbf{1}_{\left\{\widehat{PD}_i(T_1,T_2) \notin [L^{T_2}, U^{T_2}[\right\}}, \tag{15}$$

where $\widehat{PD}_i(T_1, T_2)$ denotes the estimated T_2-year probability of default for given values of state variables at T_1 in the i'th outer loop. Now

$$P(\widehat{PD}_i(T_1, T_2) = \frac{j}{N_I}) = P_{\text{bin}}(j; N_I, PD_i(T_1, T_2)),$$

where P_{bin} denotes the binomial point probability in j for N_I trials and success rate equal to the true default probability $PD_i(T_1, T_2)$. Then $\widehat{PD}_i(T_1, T_2) \to$

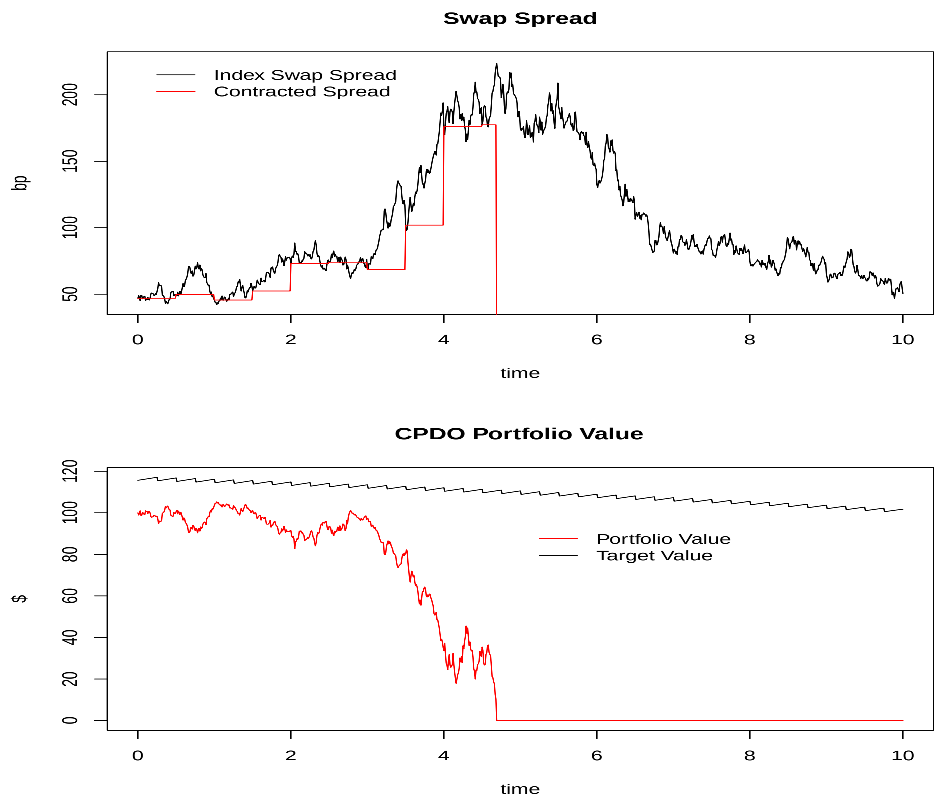

Figure 8: A cash-out scenario in the historical market configuration.

$PD_i(T_1, T_2)$ for $N_I \to \infty$ implying

$$\begin{aligned} E\left[\mathbf{1}_{\left\{\widehat{PD}_i(T_1,T_2)\notin[L^{T_2},U^{T_2}[\right\}}\right] &= \sum_{j=0}^{N_I} \mathbf{1}_{\left\{\frac{j}{N_I}\notin[L^{T_2},U^{T_2}[\right\}} P_{\text{bin}}(j; N_I, PD_i(T_1,T_2)) \\ &\to \mathbf{1}_{\left\{PD_i(T_1,T_2)\notin[L^{T_2},U^{T_2}[\right\}} \qquad \text{for } N_I \to \infty, \end{aligned}$$

i.e. the expected estimated transition indicator converges to the true transition indicator. Hereby it follows, that the estimation in (15) can be performed summing $E\left[\mathbf{1}_{\left\{\widehat{PD}_i(T_1,T_2)\notin[L^{T_2},U^{T_2}[\right\}}\right] = P\left(\widehat{PD}_i(T_1,T_2) \notin [L^{T_2}, U^{T_2}[\right) := p^i_{T_1,T_2}$ over $i = 1, ..., N_O$.

Define the simulated rating transition indicator by

$$y_i = \begin{cases} 1 & \text{if } \widehat{PD}_i(T_1,T_2) \notin [L^{T_2}, U^{T_2}[\\ 0 & \text{otherwise} \end{cases}$$

and let

$$w_i = \begin{cases} y_i & \text{with probability } p^i_{T_1,T_2} \\ 1 - y_i & \text{with probability } 1 - p^i_{T_1,T_2}. \end{cases}$$

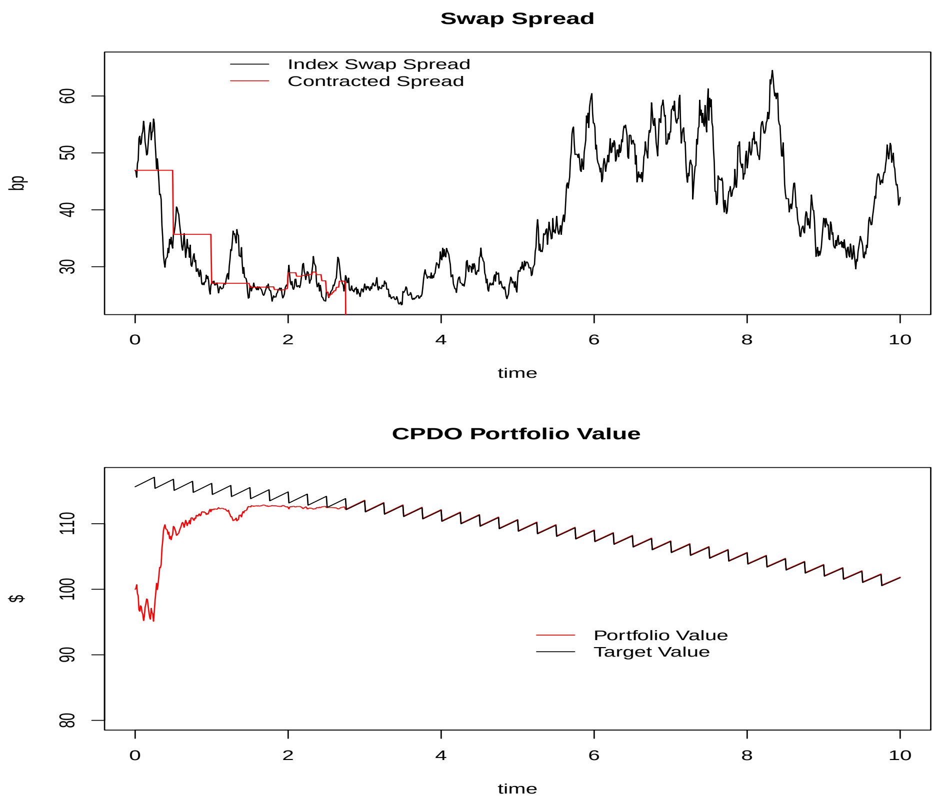

Figure 9: A fast cash-in scenario in the historical market configuration.

Now $w_i y_i + (1 - w_i)(1 - y_i)$ is equal to 1 with probability $p^i_{T_1,T_2}$ and the rating transition probability can then be calculated as

$$P(R_1 \neq R_0) \approx \frac{1}{N_O} \sum_{i=1}^{N_O} (w_i y_i + (1 - w_i)(1 - y_i)) \,.$$

With $N_O = 1\,000$ and $N_I = 10\,000$ the rating transition probabilities are given in table 5. Our results indicate high probabilities for rating downgrades. Since

	$P(R_1 \neq R_0)$	$P(R_1 < R_0)$
$T_1 = 1$	99.3%	7.4%
$T_1 = 5$	99.3%	7.7%

Table 5: Rating transition probabilities.

the average cash-in time is less than 5 years, for a large part of the scenarios the CPDO has already cashed in before year 5, earning the CPDO a AAA rating, which differs from the initial A+ rating obtained in this market configuration. More interestingly, more than 7% of the contracts have been downgraded after one year and this number increases slightly at year 5. Since less than 2% of the contracts end up in default, this is an indication of the CPDO being able to recover even after severe losses. The high rating volatility documented here clearly distinguishes CPDOs from similarly rated investment grade bonds, which are typically expected to maintain their original rating during the lifetime with high probability.

5 Discussion

We have presented a parsimonious model for analyzing the performance and risks of CPDO strategies. We consider a variety of specifications for a one factor top-down model for the index default intensity and show that they allow to study credit ratings, default probabilities, loss distributions and different tail risk measures for the CPDO and capture its risk features in a meaningful yet simple way.

Our results indicate that while coupon notes have a low probability of default, principal notes have typically a much higher probability of default, leading to lower credit ratings under the same market conditions. Also, our scenario analysis identifies a high exposure to credit-spread widening, similar to that observed recently in the market.

Perhaps the most important insight from our study is that CPDOs are less sensitive to default risk than to movements of spreads and behave in this respect more like path-dependent derivatives on the index spread. Our scenario analysis clearly indicates that the worst case scenario for a CPDO manager is that of a sustained period of spread widening. This scenario has precisely happened in the second half of 2007 and in 2008, and has resulted in the forced unwinding of many CPDOs [26] as predicted by our analysis.

In line with the findings of rating agencies, we have found the CPDO structure to be very parameter sensitive. Relatively small changes in certain parameters may result in a jump of several notches in the rating. Accordingly, we conclude that over the lifetime of the CPDO this leads to very high variability of the rating compared to that of AAA-rated bonds. The parameters with respect to which we observed the highest sensitivity are the (long-term) spread level θ setting the spread income generated by the CPDO, the mean reversion speed κ determining the possible spread widening, the risk premium parameter ϑ which governs the discrepancy between market-implied and historical default rates, and the level of interest rates.

Another insight from our analysis is the influence of the aggressiveness of leverage strategy employed. Following a more aggressive leverage rule results in fewer defaults at the cost of higher tail losses. An actively managed CPDO or a time/state dependent gearing factor would therefore possibly result in a better performing CPDO in some scenarios and could be designed to accommodate the risk aversion of the investor. Analyzing risks and performance of such a product would require a subtle specification of actions taken by the manager.

There are straightforward extensions and refinements, e.g. a two factor model for the joint evolution of the default intensity and interest rate or including a stochastic risk premium ϑ. Yet we believe that the top-down model in the basic form introduced here captures the essential risk factors of CPDOs.

It is difficult to compare directly our results with the ratings/default probabilities given by rating agencies since many parameters and contract details enter into these computations. But our model makes it clear that the main factors are the term structure of credit spreads, which determines the roll-down effect, the behaviour of the index spread at each roll date (explicitly modeled here using parameters compatible with historical data) and the dynamics of the spread (mean-reversion, widening/tightening).

More importantly, our analysis shows that within a given rating category a wide range of expected shortfalls may be observed, leading us to conclude that basing the risk analysis of such complex products as CPDOs on ratings or default probabilities alone is not sensible: credit ratings should be complemented by other risk measures such as expected shortfall or other measures of downside risk, in agreement with similar conclusions drawn from studies of CDO tranche ratings [9].

The question of "credit ratings" for such leveraged, path-dependent products such as CPDOs raises various methodological issues. Credit ratings are usually presented to investors as a metric for credit/default risk (as opposed to indicators of "market risk"). However, in the context of structured credit products such as CPDOs, it is clear that the rating will be based on scenario simulations incorporating various market risk factors such as volatility of spreads, the level and volatility of interest rates, etc., blurring the (non-existent?) borderline between credit and market risk and raising questions about the interpretation of such ratings by investors. In fact, as shown in our results, market risk, not default risk, is the main risk of CPDOs.

Thus, our results suggest that for such complex products ratings tend to be misleading and cannot replace a detailed risk analysis. Indeed, some rating agencies refused to rate CPDO deals.

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