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Option pricing with time-changed Lévy processes

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Abstract

In this paper, we introduce two new six-parameter processes based on time-changing tempered stable distributions and develop an option pricing model based on these processes. This model provides a good fit to observed option prices. To demonstrate the advantages of the new processes, we conduct two empirical studies to compare their performance to other processes that have been used in the literature.

Keywords: Option pricing, stochastic volatility, stochastic-time change, Lévy processes, tempered stable distributions

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I. Introduction

Since the ground-breaking work of Black and Scholes (1973) and Merton (1973) – typically referred to as the Black-Scholes model – option pricing has been based on the assumption that asset returns are normally distributed. However, not only has the normal distribution assumption been rejected by numerous empirical studies, it is a well-documented fact that asset returns exhibit asymmetry and heavy tails. There are two ways that have been proposed in the literature to deal with non-normality. The first is to include stochastic volatility and allow the variance of the normal distribution to change over time.[1] The second approach uses jumps in the return model.[2]

In the model we propose in this paper, we combine these two approaches. We use two probability distributions from the family of tempered stable distributions which are capable of capturing both asymmetry and heavy tails and apply a stochastic-time change to them. The application of the first distribution – the classical tempered stable (CTS) distribution[3] – to option pricing was studied by Carr *et al.* (2002). Carr *et al.* (2003) further developed this model and used a stochastic-time change to include stochastic volatility. Although the work of Carr *et al.* (2003) is as a milestone in the literature on option pricing with tempered stable distributions, the use of a stochastic-time change to derive a more realistic price process is not conceptually new. Clark (1973) applies the concept of stochastic-time change to Brownian motions in order to obtain more realistic speculative prices. Later, Ikeda and Watanabe (1981) offer insights in stochastic-time changes from the perspective of stochastic differential equations. Two centuries later, Barndorff-Nielsen and Shephard (2003) studied non-Gaussion stochastic volatility models. Carr and Wu (2004) extended the approach further by providing an efficient way to include the correlation between the stock price process and the stochastic-time change. Huang and Wu (2004) conducted a specification analysis of different option pricing models and concluded that the best pricing model is one based on a process with a high-frequency jump component and diffusion component, with one time change applied to the jump component and one time change applied to the diffusion component. The second

[1] The model by Heston (1993) is the most well-known model using this approach.
[2] This approach was first introduced by Merton (1976).
[3] This distribution is also known as the CGMY distribution.

distribution – the rapidly decreasing tempered stable (RDTS) distribution – was introduced and studied by Kim *et al.* (2010) and Rachev *et al.* (2011).

The contribution of this paper is twofold. First, we apply the techniques of Carr *et al.* (2003) to the RDTS process and present a simple way to reduce the number of parameters from seven to six. Second, we conduct an empirical study to illustrate that the reduction of the number of parameters does not influence the performance of our models. We do not follow the approaches of Carr and Wu (2004) and Huang and Wu (2004) in this work because we want to provide an option pricing model with as few parameters and components as possible.

Providing an option pricing model with as few parameters as possible is desirable for two reasons. First, from a practitioner's point of view, less parameters are desirable because model calibration can be done faster and more efficiently. Second, from a theoretical perspective, by introducing a stochastic-time change we are walking the thin line between capturing the observed information content correctly and over-fitting our model. A reduction of the number of parameters reduces the risk of over-fitting our model.

The remainder of this paper is organized as follows. In Section II, we briefly review the tempered stable distributions of interest and some important formulas from option pricing. After this introductory section, in Section III, we introduce stochastic volatility using a continuous-time change. We empirically evaluate the performance of our proposed models in Section IV and offer some concluding remarks in Section V.

II. Tempered Stable Processes and Option Pricing

In this section, we first introduce the risk-neutral stock price process as a mean-corrected ordinary exponential of a Lévy process and discuss the merits of the RDTS process afterwards.

Let r denote the risk-free interest rate and assume that the dividend paid by S_t is zero. The risk-neutral stock price process is given by

$$S_t := S_0 \frac{e^{rt+X_t}}{\mathbb{E}[e^{X_t}]} = S_0 \exp((r+\omega)t + X_t), \tag{1}$$

where ω is given by the equation $e^{-\omega t} = \phi_{X_t}(-i)$ and X is the return process. We consider two types of Lévy processes as return process: the CTS process and the RDTS process. We state without proof the characteristic functions of the two definitions for the Lévy processes:

CTS process: $\phi_{X_t}(u) = \exp(tC\Gamma(-\alpha)((\lambda_- - iu)^\alpha - \lambda_-^\alpha + (\lambda_+ + iu)^\alpha - \lambda_+^\alpha))$,

where $\Gamma(x) := \int_0^\infty t^{x-1}e^{-t}dt$ denotes the gamma function.

RDTS process: $\phi_{X_t}(u; \alpha, C, \lambda_+, \lambda_-, m) = \exp(ium + C(G(iu; \alpha, \lambda_+) + G(-iu; \alpha, \lambda_-)))$,

where

$G(x; \alpha, \lambda) = 2^{-\frac{\alpha}{2}-1}\lambda^\alpha\Gamma\left(-\frac{\alpha}{2}\right)\left(M\left(-\frac{\alpha}{2}, \frac{1}{2}; \frac{x^2}{2\lambda^2}\right) - 1\right) + 2^{-\frac{\alpha}{2}-\frac{1}{2}}\lambda^{\alpha-1}x\Gamma\left(\frac{1-\alpha}{2}\left(M\left(\frac{1-\alpha}{2}, \frac{3}{2}; \frac{x^2}{2\lambda^2}\right) - 1\right)\right)$,

Γ is the gamma function and M is the confluent hypergeometric function.[4]

It is also possible to add a drift term to the process X, but based on the empirical results of Carr *et al.* (2002), we do not extend our model with an additional drift term.

We next define the expCTS and expRDTS processes as follows. Let S_t be defined as in Equation (1) with return process X. If X is a CTS process, S_t is called an exponential CTS (expCTS) process. If X is a RDTS process, S_t is called exponential RDTS (expRDTS) process.

An efficient way for pricing European contingent claims is through the characteristic function. Lewis (2001) derived the following pricing formula for European call options. If the stock price S_t is given as $S_t = S_0\frac{e^{rt+X_t}}{\mathbb{E}[e^{X_t}]}$, where X is a stochastic process and ϕ_{X_t} denotes the characteristic function of X_t, then the value of an European call option is:

$$C_t = \frac{K^{1+\rho}e^{-r(T-t)}}{\pi S_t^\rho}\mathrm{Re}\left(\int_0^\infty e^{-iu\log K/S_t}\frac{e^{(T-t)\log\phi_{X_1}(u+i\rho)}}{(\rho - iu)(1+\rho-iu)}du\right), \tag{2}$$

where $\rho < -1$ such that $\phi_{X_1}(u + i\rho) < \infty$ for all $u \in \mathbb{R}$. A similar result holds for European put options.

Now we demonstrate why under certain conditions the RDTS model is superior to the CTS model. Consider a European power call option, which is a European call option with terminal payoff $\max(S_T^n - K, 0)$, for $n \in \mathbb{N}$ fixed. We recall that $\phi_{X_t}(u + i\rho) < \infty$ holds for $-\lambda_+ < \rho < \lambda_-$ if X is a CTS process and for $\rho \in \mathbb{R}$ if X is a RDTS process. We start calculating the expected final payoff to illustrate the problem.

[4]The work of Carr *et al.* (2002) and Bianchi *et al.* (2011) offer a formal introduction of these processes over the Lévy measure.

$$
\begin{aligned}
\mathbb{E}[(S_T^n - K)^+] &= \int_{-\infty}^{\infty} (S_0^n e^{nx} - K)^+ f_{X_T}(x)dx \\
&= \int_{\log(K/S_0^n)}^{\infty} (e^{nx+n\log(S_0)} - K) f_{X_T}(x)dx \\
&= \int_{\log(K/S_0^n)}^{\infty} (e^{nx+n\log(S_0)} - K) \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(u+i\rho)x} \phi_{X_T}(u+i\rho)dudx \\
&\overset{Fubini}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{\int_{\log(K/S_0^n)}^{\infty} (e^{nx+n\log(S_0)} - K) e^{-i(u+i\rho)x} dx}_{=(*)} \, \phi_{X_T}(u+i\rho)du
\end{aligned}
$$

$$
\begin{aligned}
(*) &= \int_{\log(K/S_0^n)}^{\infty} (e^{nx+n\log(S_0)} - K) e^{-izx} dx, \;\; z = u + i\rho \\
&= S_0^n \underbrace{\int_{\log(K/S_0^n)}^{\infty} e^{nx-izx} dx}_{=(**)} - K \int_{\log(K/S_0^n)}^{\infty} e^{-izx} dx \\
&= S_0 \left[\frac{e^{(n-iz)x}}{1-iz} \right]_{\log(K/S_0)}^{\infty} - K \left[\frac{e^{-izx}}{-iz} \right]_{\log(K/S_0)}^{\infty}
\end{aligned}
$$

Next, we conclude that $\rho < -n$ is necessary to price European power call options with power n :

$$
\frac{e^{(n-iz)x}}{1-iz} = \frac{e^{nx-i(u+i\rho)x}}{1-i(u+i\rho)} = \frac{1}{1+\rho-iu} e^{(n+\rho)x} e^{-iux}
$$

and

$$
e^{(n+\rho)x} \overset{n\to\infty}{\to} 0 \text{ if and only if } \rho < -n.
$$

Since $-\lambda_+ < \rho < \lambda_-$ is required for the CTS model, it is not possible to price European power call options in a CTS model with $n > \lambda_+$. In this case, the RDTS is a good alternative.

III. Including Stochastic Volatility

Three alternative ways for modeling stochastic volatility have been proposed in the literature – regime-switching models, time-series models for the volatility, and stochastic process to model the stochastic volatility. The principal advantage of the third model class is that it can be implemented efficiently, thereby enabling model calibration.

In this paper, we explore the introduction of stochastic volatility by allowing the variance of the distributions to vary over time by introducing a stochastic-time change. This concept uses a stochastic process to model stochastic volatility and, as mentioned earlier, has been subject of researchers since the early 1970s. Carr *et al.* (2003) apply this technique using the CTS distribution. Here, we first apply the technique of stochastic-time change as introduced in Carr *et al.* (2003) to the RDTS distribution and then, by explaining the intuition behind the stochastic-time change, we reduce the number of parameters from seven to six. In the next section, we show empirically that this reduction has no negative effect on the ability of the model to capture the market's information content.

Although there is a large class of stochastic processes capable of serving as stochastic-time change, we restrict our considerations to the Cox-Ingersoll-Ross (CIR) process.[5] A process $(y_t)_{t\in T}$ is referred to as a CIR process or a square-root process if the dynamics of y are given by the following stochastic differential equation: $dy_t = \kappa(\eta - y_t)dt + \lambda\sqrt{y_t}dW_t,\ y_0 = \tilde{y}$, with κ, η, λ, and $\tilde{y} \in \mathbb{R}$.

We choose the CIR process as intrinsic time because of two financial insights this process provides. First, the CIR process is mean reverting, meaning that it fluctuates around a fixed mean determined by the parameter η. This is desirable because we want the stochastic-time change to fluctuate around a fixed average − increasing in turbulent times and falling below average in quiet times. The second desirable property of the CIR process is that it is a strictly positive process. From a modelling perspective, a negative time change would not make sense. We will fix the parameter η and reduce the number of variables of the CIR process from four to three. We justify the evidence for this step later. We next define

$$Y_t := \int_0^t y(s)ds, \tag{3}$$

where y_t is a CIR process. We will refer to this random variable as the 'economic clock'.

If we let Y_t be the random variable given by Equation (3), the characteristic function of Y_t is known in closed form:[6]

$\phi(u, t, y(0); \kappa, \eta, \lambda) = A(t, u)e^{B(t,u)y(0)}$,

[5] This process was formally introduced by Cox *et al.* (1985).
[6] See Cox *et al.* (1985) for further details.

with $\gamma = \sqrt{\kappa^2 - 2\lambda^2 iu}$,
$A(t,u) = \frac{\exp(\frac{\kappa^2 \eta t}{\lambda^2})}{\cosh(\frac{\gamma t}{2}) + \frac{\kappa}{\gamma}\sinh(\frac{\gamma t}{2})^{2\kappa\eta/\lambda^2}}$, and
$B(t,u) = \frac{2iu}{\kappa + \gamma\coth(\frac{\gamma t}{2})}$.

Time–changed processes of interest

We next introduce the formal definition of a time-changed Lévy processes. Let $X = (X_t)_{t\in\mathcal{T}}$ denote a Lévy process and $Y = (Y_t)_{t\in\mathcal{T}}$ as defined in Equation (3) with X and Y being independent. Then the process

$$Z_t := X_{Y_t} \tag{4}$$

is called a time-changed Lévy process. If X is a CTS process, Z is a time-changed CTS (TCCTS) process. If X is a RDTS process, Z is a time-changed RDTS (TCRDTS) process.

If the parameter η of the stochastic-time change is fixed to $\eta = 1$, we use the notation TCCTSη if X is a CTS process, and TCRDTSη if X is a RDTS process. As above, we introduce the expTCCTS, the expTCCTSη, the expTCRDTS, and the expTCRDTSη processes as the mean-corrected ordinary exponential of the TCCTS, the TCCTSη, the TCRDTS, and the TCRDTSη processes, respectively.

Theoretical evidence for these processes

Our objective is to provide an option pricing model with as few parameters as possible, therefore, we assume X and Y to be independent. Another beneficial effect of this assumption is that the characteristic function of such a time-changed process is known in closed form:[7]
$\phi_Z(u,t) = \phi_Y(-i\psi_X(u),t)$, where ψ_X is the characteristic exponent of X.

One drawback of these processes is that by introducing stochastic volatility, we will have four more parameters and hence model calibration becomes more difficult and the common black box optimization algorithms provided by several numerical software packages no longer

[7]We refer to Carr *et al.* (2003, p. 353) for a proof of this result.

provide reliable results. Therefore, we offer a way to reduce the number of free parameters in the time change from four to three, by fixing $\eta = 1$. As we will see in the next section, this parameter reduction has no major effect on the performance of our model.[8]

IV. Empirical Investigation

To analyze the performance of our models, we performed two empirical studies. The first compares the performance of the time-changed CTS model to the time-changed RDTS model for options with different maturities. The purpose of this study is twofold. First, beyond its theoretical superiority, we want to empirically test if the time-changed RDTS model is superior to the time-changed CTS model. Second, we want to measure the potential adverse influence of fixing the parameter η in the different models.

Since Carr *et al.* (2003) already explained that a time change is necessary to obtain good calibration results for options with different maturities, we did not compare the performance of the time-changed models to the models without time change in the first study. We conduct the second study to compare the performance of the time-changed models to the models without time change on options with the same maturities.

Calibration to options with different maturities

Our first empirical study is similar to the investigation by Carr *et al.* (2003). They obtained market prices of out-of-the-money (OTM) options on the S&P 500 index with maturities between one month and one year for the second Wednesday of each month of the year 2000. With these data, they calibrated their proposed models and concluded that the time-changed CTS model is the best time-changed model for option pricing. To be more precise, they did not evaluate the same time-changed CTS process as we do. They considered a more general version of this process, with different values for C and α for the positive and the negative parts of the Lévy measure.

Instead of OTM option prices, we collected both out-of-the-money and in-the-money

[8] Beside the empirical power of this approach, this approach can also be justified by the intuitive argument that fixing the parameter η to 1 means that the economic time should fluctuate around the business time, but in "normal" market periods we assume that business time and economic time are equal.

(ITM) call option prices on the S&P 500 index with maturities between one month and one year and strikes between 80% and 120% of the current spot level from OptionMetrics's Ivy DB in the Wharton Research Data Services. Our reason for choosing these call options instead of OTM options is twofold. First, options with strike prices close to the current spot level are more liquid than options with strikes far from the current spot level. Therefore, in our opinion, the observed prices of these near-the-money options are a better representation of the information content of the market. Second, empirical studies based on OTM options typically do not consider deep OTM options. For instance, Huang and Wu (2004) in conducting an empirical study using OTM options choose OTM options with strikes between 67.11% and 119.89% of the current spot level.[9] Therefore, the moneyness level in our approach does not substantially differ from using OTM options.

We use the closing prices of European call options traded on the Chicago Board Option Exchange (CBOE) between January 1 and September 30, 2008. We restrict our study sample to options with maturities between four weeks and one year, applying the following three filters to the dataset. First, we remove options with no positive difference between bid and ask prices and options where the bid price is not strictly positive. To apply the next two filters, we consider options with the same maturity for each date. For the second filter, if there are less than 10 options with the same maturity, we remove them from the sample. Third, we remove options where the price difference is less than 0.05% of the spot level of the S&P 500 index.

In order to obtain comparable sample data, we do not consider dates where less than two or more than five maturities are remaining. We further restrict our study sample to the dates where between 150 and 200 options are remaining. This leaves 62 sample dates and a total number of 11,903 options. We performed the calibration by applying the fast Fourier pricing scheme proposed above using least squares estimation.We used the Matlab command lsqcurvefit to calibrate the model.[10]

[9]They stated that moneyness of options they considered, defined as $k = \log(K/S)$, ranges between 0.1814 and 0.3988.

[10]We used the Matlab command $lsqcurvefit$ to calibrate the model.

To assess the goodness of fit, we use the average percentage error (APE), defined as:

$$APE := \frac{\sum_{k=1}^{n}(|MarketPrice - ModelPrice|)}{\sum_{k=1}^{n}(|MarketPrice|)}.$$

There are two principal findings of the empirical analysis. First, as illustrated in panel (a) of Figure 1, the expTCCTSη process is better able to fit observed option prices than the expTCRDTSη process in most cases. As we can see in Figure 1 (a), there are some sample days where the expTCRDTSη process performs better than the expTCCTSη process. However, the problem with the expTCRDTSη process is that there are sample days where it exhibits a very poor performance. For instance, between 0 and 10 days, the APE of the expTCRDTSη is almost twice that of the APE of the expCTSη. The same result holds when comparing the expTCCTS and the expTCRDTS models, as illustrated in Figure 1 (b). Second, comparing panel (a) and (b) of Figure 1 leads to the conjecture that fixing the parameter η has no major negative effect on the performance of the option pricing models. The two panels in Figure 2 provide further evidence to support this conjecture. First, Figure 2 (a) motivates fixing the parameter η from an empirical perspective. Although at a first glance the estimates of η for the two processes seem to differ significantly, the parameter is between 1 and 1.3 for all sample days. Figure 2 (b) shows the difference between the APE for the processes with fixed and variable η. The average difference between the time-changed model with fixed and variable η is 0.3% for the CTS models and 0.015% for the RDTS models.

In fact, Figure 2 (b) indicates that the APE for the processes with fixed η is smaller than the APE of the other processes in some cases. This result seems odd upon initial examination because fixing η is only a special case of the time-changed tempered stable processes with variable η. But the optimization problem underlying the model calibration is rather complex and the black box algorithms used for the calibration cannot guarantee that the result is a *global* minimum.[11] This result emphasizes the need for an option pricing model with as few parameters as possible. Figure 2 (a) shows that the parameter η does not substantially differ from one in the calibration of the expTCCTS and the expTCRDTS process.

[11]We used the Matlab command $lsqcurvefit$ to calibrate the model.

Calibration to options with same maturities

Our second study is closely related to the study of Rachev *et al.* (2011).[12] They calibrated different tempered stable option pricing models to observed option prices on the S&P 500 index on August 6, 2008. We apply the same filters to our observed option prices as they did. We consider call options with prices between \$5 and \$180, and between 80% and 120% of the current spot level. The spot price of the S&P 500 that day was 1289.19. We also use the same dividend and risk-free interest rate as they did, which are 2.03501% and 1.6%, respectively. We fit the expCTS, expTCCTSη, expRDTS and expRDTSη models to ATM call options on the S&P 500 index with the same maturities. We calculate the implied volatility of the market options and the implied volatility of our model prices by solving the equation $C_{BS}(\sigma) = C_{Market}$, where C_{BS} is the option price calculated with the Black-Scholes formula as a function of σ and C_{Market} is the observed option price.

Figure 3 illustrates our results for options with 10 days to maturity and options with 136 days to maturity. The case of 10 days to maturity shows the superiority of the expTCRDTSη process over the other processes. While the other three processes produce a volatility smile, the expTCRDTSη process is the only process able to capture the volatility skew of the market correctly. The case of 136 days to maturity shows that all processes provide similar results for longer maturities. Table 1 summarizes our estimation results for options with the same maturities.

V. Conclusion

We present four time-changed Lévy processes for option pricing and embed the theory of time-changed Lévy processes from Carr *et al.* (2003) in the option pricing theory with tempered stable distributions. The assumption that the economic time should fluctuate around business time lead to two new six-parameter processes. Empirically, we find that these models provide a good fit to observed option prices and provide almost identical results as the seven-parameter benchmark models. Among all models, the time-changed CTS model with fixed η performed best for options with different maturities, while the time-changed RDTS

[12]See section7.5.2 of their book.

model with fixed η was superior to the other models for options with the same maturity.

We provide two reasons why practitioners should employ time-changed Lévy processes. First, our empirical study suggests that it is possible to provide a good fit for at-the-money call options with different maturities. Second, we reduced the number of model parameters without observing a negative influence on performance.

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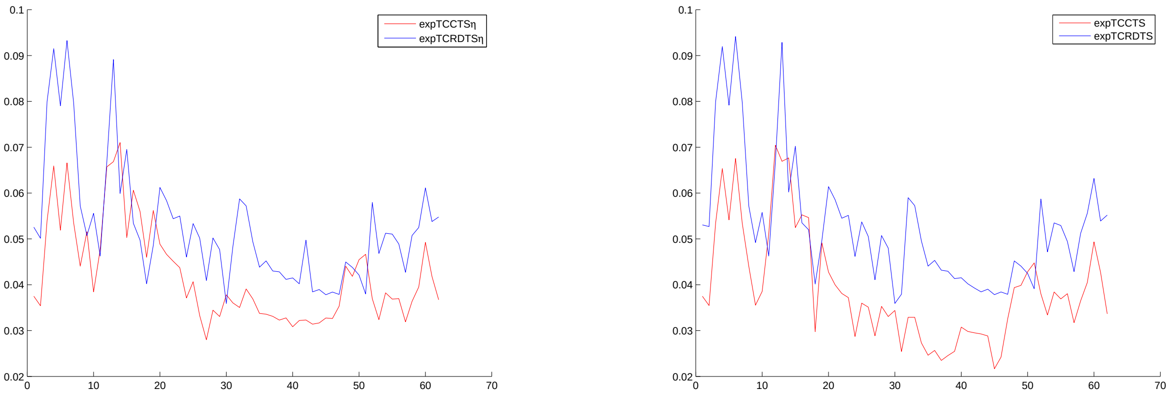
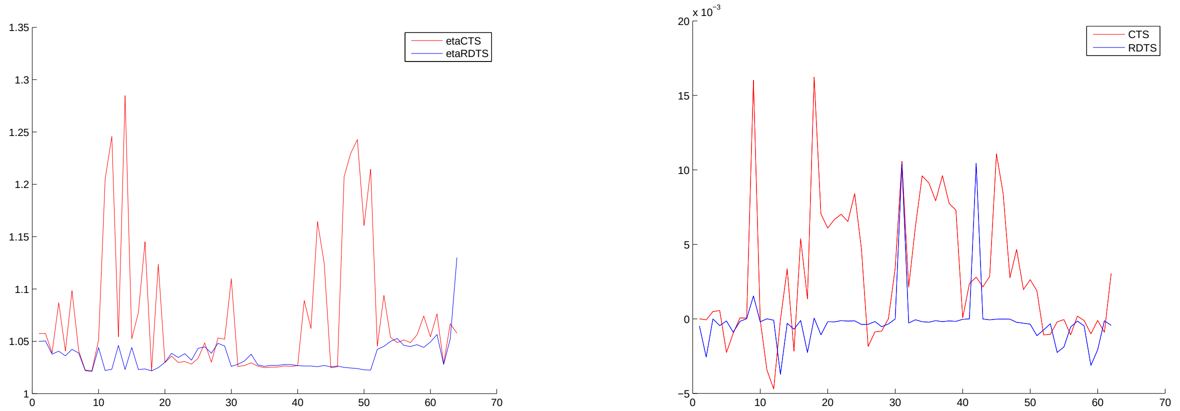


(a) Estimation errors for the time-changed models with fixed η.

(b) Estimation errors for the time-changed models with variable η.

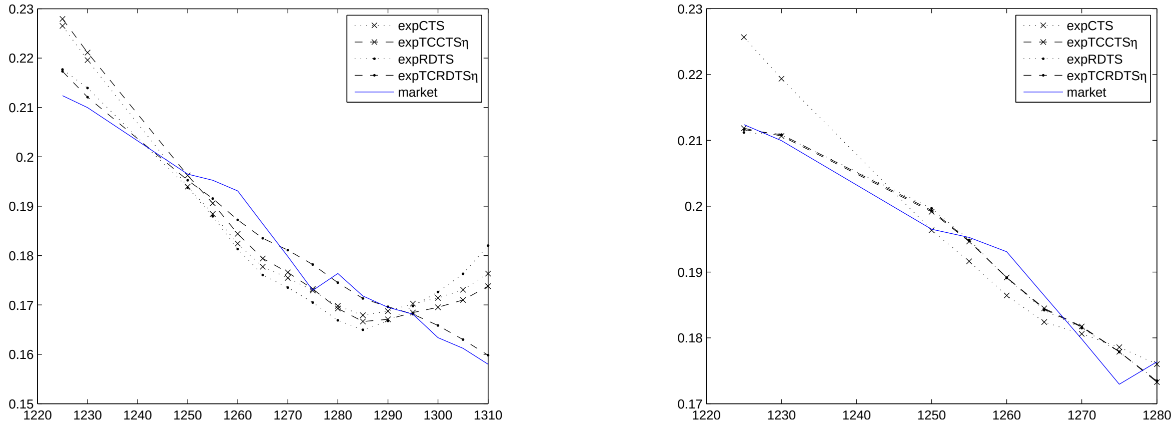
Fig. 1: Illustration of the estimation errors for the different processes.



(a) Calibration results for the parameter η for every sample point.

(b) Difference between the estimation error of the models with fixed and variable η.

Fig. 2: Analysis of the influence of the parameter η to the calibration results.



(a) Days to maturity:10

(b) Days to maturity:136

Fig. 3: Illustration of the implied volatility calculated from market data (August 6, 2008) and the implied volatility calculated from the calibrated option pricing model.

τ	No. of Options	Model	α	C	λ_+	λ_-	y_0	κ	λ	APE
10	15	expCTS	0.89	0.61	52.40	12.41	–	–	–	1.72%
		expTCCTSη	0.55	–	256.95	61.18	18.98	0.80	1.22	1.52%
		expRDTS	1.07	0.19	100.00	3.77	–	–	–	2.10%
		expTCRDTSη	0.58	–	48.96	9.00	15.26	9.93	1.02	0.56%
136	9	expCTS	0.90	0.69	63.08	14.11	–	–	–	0.65%
		expTCCTSη	0.51	–	45.63	192.90	15.08	0.48	1.30	0.41%
		expRDTS	0.53	1.73	73.61	10.72	–	–	–	0.42%
		expTCRDTSη	0.71	–	59.67	97.19	9.81	1.10	1.65	0.41%

τ:Days to maturity

Table 1: Results for the calibration of the parameters for options with the same maturity