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Bid Regulations in a Multi-unit Uniform Price Auction

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Abstract

This paper examines the effect of bid regulations on the range of potential equilibrium prices in a multi-unit uniform price auction with heterogenous bidders. General bid caps destroy equilibria with prices above the bid cap and create new equilibria with prices way below the cap. A cap only for larger firms does not guarantee market prices below that cap. A sufficiently high bid floor only for smaller firms destroys some or all pure strategy equilibria despite their prices being above the bid floor. With a general bid floor this happens only with considerably higher bid floors.

Keywords: Multi-unit Auctions, Heterogenous Bidders, Bid Regulation.
JEL-Classification: D43, D44, L12, L13, L51

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1 Introduction

This study aims at deriving in a very simple framework with perfect information, how different sorts of bid regulations change the possible range of equilibrium prices in a multi-unit uniform price auction with heterogenous bidders. Such auctions are very common in electricity markets[1], but are also used in, for example, treasury auctions (see e.g. Brenner *et al.* (2009)), emission permit auctions (see e.g. Betz *et al.* (2010)) and even in order to place IPOs on financial markets (see e.g. Degeorge *et al.* (2010)). This paper analyzes the effects of general and selective bid caps and bid floors on an auction where multiple units of a homogenous product are traded at a uniform price for all successful bidders independent of their specific bid.

Bid caps, bid floors and reserve prices exist in a couple of multi unit uniform price auctions. They are currently intensely discussed for markets in which pollution permits are traded in order to reduce uncertainty with regard to their price and the resulting opportunity costs of investments in abatement (see e.g. Wood and Jotzo (2011) and Hasegawa and Salant (2015)). The analysis here is, however, inspired by the New York Installed Capacity Market (NYICAP) for electricity generating capacity. It considers bid caps and bid floors in a procurement setting and in a certain environment.[2] The paper models general as well as selective bid floors and general as well as selective bid caps. Selective as opposed to general refers to not all bidders having to obey them. In the NYICAP inspired setting bidders differ in the capacity they can sell. Selective bid caps apply only to firms with large capacities, whereas selective bid floors only apply to firms with small capacities.[3]

Note that a bid cap in a procurement auction is equivalent to a reserve price in an auction where the auctioneer does not buy, as in our setting, but

[1]The Electricity Pool in England and Wales before the reform in 2001, the Nord Pool in Scandinavia, the Spanish wholesale market, as well as the NYISO and the ERCOT market in the US are still or were organized as multi-unit uniform price auctions. See Bergman *et al.* (1999), Crampes and Fabra (2005), Newbery (2005), Hortaçsu and Puller (2008) and Zhang (2009).

[2]The downward sloping linear demand for capacity in the NYICAP is known to the bidders.

[3]Note that the NYICAP market has general and selectively more stringent bid caps for large capacity owners, but only selective and no general bid floors. The selective bid floors apply to new entrants which are most likely but not necessarily small in capacities. For the details of the NYICAP market's regulation see the New York ISO's homepage `www.nyiso.com`

sells something as in the typical theoretical auction design. In an optimal standard single unit auction design with heterogenous bidders the auctioneer should discriminate his or her reservation prices according to the bidders' types and potentially distort the reservation prices above his or her true valuation (see Myerson (1981) and for qualifications of his results Levin and Smith (1996) and Jehiel and Lamy (2015)). In addition Bresky (2013) shows under which conditions reserve prices do not only increase the auctioneer's expected revenues in a simple multi-unit uniform price auction with ex ante symmetric bidders, but also overall welfare. The investigated discriminatory application of bid caps are here, however, not part of an optimal auction design.

The approach here resembles more the one of Kotowski (2015) who shows for a single unit auction with ex ante homogenous bidders that assigning a higher reservation price to one group of bidders than to the remaining bidders might benefit that group due to the externality that the rest of the bidders bid at a lower level due to their lower reservation price. The selective bid cap, analyzed here, potentially also creates such an externality for the large capacity bidders to whom the cap applies in the analyzed multi-unit procurement setting. The reason is that, similar to the single unit auction, it can change the bidding behaviour of those bidders to whom the selective bid cap does not apply.

The analysis of selective bid caps relates to the literature on handicaps in single unit procurement auctions which emphasizes that the targeted favoured group might not necessarily benefit from another group's handicap (see e.g. Kirkegaard (2013) and Mares and Swinkels (2014)). The same can be true in this study's otherwise pretty different multi-unit procurement setting where the handicapped group with a lower bid cap might, despite a reduction in the procurement price, finally benefit from its handicap. The handicap might induce another unconstrained firm to overbid the constrained firms such that the former highest bidder procures now a larger quantity at a lower price but still generates higher profits.

This theoretical study is based on a very simple model of a multi-unit uniform price auction without asymmetric information, first introduced by von der Fehr and Harbord (1993).[4]. The particular version of the model used here is a linear version of the more general one, presented in Moreno

[4]Schwenen (2015) shows that the NYICAP data from 2006-2008 are in line with the conclusions of such an auction model

and Ubeda (2006) and Ubeda (2004). To my knowledge bid regulations have never been theoretically analyzed in this framework.[5]

There is, however, an experimental literature which investigates the effect of automated mitigation procedures and/or soft price caps on the prices in stylized electricity markets (see, e.g. Kiesling and Wilson (2007) who also consider their effect on investment decisions, Vossler *et al.* (2009) and Shawan *et al.* (2011)) and the effect of general price caps on the bidding behavior of firms and their investment decisions (see Le Coq *et al.* (2016)). The stylized electricity markets in these experimental papers resemble the model of multi-unit auctions in von der Fehr and Harbord (1993) with ex ante symmetric bidders and an inelastic stochastic demand. Vossler *et al.* (2009) has also an additional treatment with a price elastic demand. The considered bid or price regulations are, however, different in these papers.

Kiesling and Wilson (2007) and Vossler *et al.* (2009) consider soft price caps where the auctioneer treats all price bids below each unit's relevant soft price cap as in a uniform price auction, but those above as in a discriminatory price auction. These mechanisms are obviously very different from either selective or general bid caps analyzed here exclusively in the framework of uniform price auctions. Shawan *et al.* (2011) assume a soft price cap, which relates to the average historical successful bids for the specific generation unit in case of congestion in the experimental network. In their case the auctioneer calculates the fictitious uniform price in case no firm had exceeded its soft bid cap. If the uniform price derived from the actual bids exceeds this fictitious price by a certain threshold, then each bid above the relevant soft price cap is substituted by its soft price cap. This is a bit closer to the analyzed situation here with selective bid caps. However, the bid caps here are not dynamic and do not allow the bidders to manipulate future caps via their own current bidding behavior like in Shawan *et al.* (2011).

The experiments in Le Coq *et al.* (2016) show in line with the predictions from theory that prices are significantly higher with a pivotal bidder than without. The market price is, however, more often identical with the lower price cap than with the higher price cap which is not in line with theory. The latter might, however, be due to the also larger capacity investment incentive with higher price caps. In the analyzed model here, due to the

[5]de Frutos and Fabra (2012) investigate, however, the effect of regulated forward positions on the bidding behaviour of competing firms and the equilibrium price in a very similar framework than the one here.

price elastic demand, equilibrium prices can differ from either the price cap or the marginal costs. Whether equilibrium prices can, however, deviate from the marginal cost level, also depends on whether there exists at least one firm with a pivotal status.[6] Contrary to the experimental setting in Le Coq *et al.* (2016) the analysis does only consider exogenous capacities and does not allow for investments.

The main results here are that without any bid regulations the possible equilibrium prices do not only depend on the total capacity in the market relative to the demand parameter, but also on the distribution of that capacity among the bidders. Under usual circumstances general bid caps do not only prevent bids and therefore equilibrium prices above the bid cap, but they also allow equilibrium prices far below the bid cap to exist which otherwise would not be sustainable. Thus, the auctioneer can potentially reduce market power in the market and procure higher quantities via the introduction of general bid caps. Only if total capacities are either very small or very large there is no effect. If bid caps only apply to firms with relatively large capacities their effect might be weakened if some intermediately sized firms are not restricted and can still set their monopoly price on the residual demand above the bid cap. Sufficiently high general price floors applied to all firms destroy pure strategy equilibria at the lower end of the equilibrium price range, even if their equilibrium price exceeds the price floor. This implies potentially more market power of the bidders, more expensive procurement and smaller procured quantities. With general bid floors the only remaining pure strategy equilibrium might be the one where all firms bid at the floor. If this happens, the auctioneer avoids at least also the market equilibria with the highest market power and the lowest procured quantity. With selective bid floors this is not an option. If, in this case, the floors are sufficiently high (but below the highest equilibrium price without price floors) pure strategy equilibria seize to exist.

Section 2 introduces the main assumptions and identifies the characteristics of the market outcome without any bid regulation. Section 3 focusses on the market outcome changes if the system operator either introduces general bid caps or bid caps only for firms with relatively large capacities, or if the system operator introduces general price floors or price floors only for firms with relatively small capacities. In section five I discuss whether the firms with a stricter bid cap or a higher bid floor than the other firms necessarily

[6] Here we call such a firm potentially price setting or marginal.

suffer or can potentially also gain from such a discrimination compared to the other firms in the market. The final conclusions in section 5 focus on which changes one can expect to observe on the NYICAP market due to the changes of the bid regulation in 2008 and what might have motivated these changes. In addition the section also discusses whether the findings of this study are also relevant for other markets which are organized as multi-unit uniform price auctions.

2 The Model without Bid Regulations

2.1 Model Assumptions

Consider a market with a set of $N = \{1, 2, \ldots, n\}$ active firms with $n > 2$. Each firm $i \in N$ owns a certain amount of capacity K_i that it potentially can supply on the capacity market. For now it is assumed that supplying parts of the capacity or all of it on the market does not cause any costs. Assume that the firms are indexed such that $K_i \geq K_{i+1}$. In addition define $\bar{K} = \sum_{i=1}^{n} K_i$ as the total capacity available, and $\bar{K}_{-i} = \bar{K} - K_i$ as the total capacity available if firm i does not supply its capacity K_i. Demand $D(p)$ for capacity is linear in the market price p with

$$D(p) = \alpha - \beta p \text{ and } \alpha, \beta > 0. \tag{1}$$

Each firm can submit a price bid $b_i \geq 0$ at or above which it is willing to supply its total capacity K_i to the market.[7] The auctioneer sorts all bids according to the demanded price in an ascending order and forms an aggregate supply function. The equilibrium price is the price at which the supply function equals the ex ante publicly known demand.[8] All firms which bid their capacity at a price below the equilibrium price sell their total capacity. Those, which bid above, sell nothing, whereas the marginal firm(s) that bids(bid) the equilibrium price might be rationed in order to balance supply and demand.[9] The vector of all bids $(b_1, \ldots, b_n)$ needs to be a Nash

[7]The Model is the same as in Moreno and Ubeda (2006) and Ubeda (2004) only with a more specified linear demand and, for now, without any possibility to invest in capacity.

[8]Note that the market clearing price p does not need to be identical with any of the bids if market clearing happens between two existing bids or if the highest bid is insufficient to balance supply and demand.

[9]In the case of multiple marginal bidders we assume that they are rationed according to their relative share in the total capacity bid at the marginal price by the marginal bidders.

equilibrium in order to determine an equilibrium price in the auction.

Note that if the firms could split their total capacity into $l \geq 1$ discrete pieces for which they could demand different minimum prices to supply them to the market, this would not change the potential equilibrium prices.[10]

2.2 The Equilibria of the Unregulated Model

When characterizing the potential market equilibria which depend on the capacities held by the firms, it is worthwhile as in Moreno and Ubeda (2006) to split the firms according to their capacities in those who can potentially be price setting or marginal in an equilibrium and those who cannot. For the start define the two sets of firms

$$Q = \{j \in N | \bar{K}_{-j} < \alpha\} \text{ and } O = \{j \in N | \bar{K}_{-j} \geq \alpha\}. \tag{2}$$

Proposition 1 *The firms $j \in O$ can only set the marginal price in a Nash equilibrium if $Q = \emptyset$. Then the equilibrium price is $p = b_j = 0$. If $Q \neq \emptyset$, then the price in any pure strategy Nash equilibrium needs to be strictly larger than the marginal costs of zero.*

Proof. Suppose $j \in O$ and $b_j > 0$ is the equilibrium price. This is only possible if the bids of all other firms $i \in N$, $i \neq j$, satisfy $b_i \geq b_j$. However, then any firm $i \neq j$ has an incentive to undercut firm j slightly in a Bertrand fashion in order to increase its profit. If, on the other hand, $j \in O$ and $b_j = 0$, then $p = b_j = 0$ cannot be an equilibrium price, if there are still firms $i \in Q$. These firms always have an incentive by unilaterally bidding $\alpha > b_i > b_j = 0$ to increase the price to b_i and, thus, increase their profit. This argument holds also true, if the low bidding firm with $b_j = 0$ belongs to the set Q instead. ■

Not surprisingly, firms, whose capacity is so small that it is never needed to supply total demand, even at a price of zero ($j \in O$), can never be price setters in equilibrium as long as other firms have capacities large enough to be necessary to satisfy demand at a price of zero.

[10] See also Fabra *et al.* (2006) who prove this in their Lemma 2 for 2 competing firms, Schwenen (2015) who makes an argument for $n > 2$ firms and the explanation in Appendix A.

Let us assume for the rest of the analysis that the firms' capacities are such that $Q \neq \emptyset$. This implies

$$\bar{K} \geq K_j > \bar{K} - \alpha \text{ for some } j \in N. \tag{3}$$

The residual demand of firm $j \in Q$

$$D^r(p, \bar{K}_{-j}) = \max\{\alpha - \bar{K}_{-j} - \beta p, 0\}$$

is the demand left to firm j if all other firms in the market offer their total capacities and demand is efficiently rationed. For firm j's residual demand firm j's monopoly price is

$$p_j = \arg\max\{pD^r(p, \bar{K}_{-j})\} = \frac{\alpha - \bar{K}_{-j}}{2\beta}. \tag{4}$$

The price p_j is only feasible as an equilibrium price if $D^r(p_j, \bar{K}_{-j}) < K_j$ or

$$K_j > \alpha - \bar{K}. \tag{5}$$

If for none of the firms $j \in Q$ condition (5) holds, the only feasible equilibrium price is the minimum market clearing price

$$\bar{p} = \left\{p | D(p) = \bar{K}\right\} = \frac{\alpha - \bar{K}}{\beta}. \tag{6}$$

The following proposition characterizes the potential Nash equilibria in the multi-unit uniform price auction.

Proposition 2 *If $Q \neq \emptyset$ then, depending on the capacities of the firms $j \in Q$, we can distinguish between two different cases.*

(i) If $K_j \leq \alpha - \bar{K}$ for all $j \in Q$, there are infinitely many equilibria in pure strategies where each firm $j \in N$ bids $b_j \leq \bar{p}$. The equilibrium price is $p = \bar{p}$ regardless of the firms bids and all firms $j \in N$ sell their total capacity K_j.

(ii) If $K_j > \alpha - \bar{K}$ for some $j \in Q$, there are multiple equilibria in pure strategies where one of the firms $j \in P$ with

$$P = \{j \in Q | K_j \geq \frac{(\alpha - \bar{K})^2 + K_1^2}{2K_1}\} \tag{7}$$

bids $b_j = p_j$ as defined in (4) and sells the part of its capacity necessary to satisfy the residual demand $D^r(p_j, \bar{K}_{-j})$. All other firms $i \in N \setminus j$ bid $b_i \leq \underline{b}_j$ with

$$\underline{b}_j = \frac{(\alpha - \bar{K} + K_j)^2}{4\beta K_j} < p_j = b_j \tag{8}$$

and sell their total capacity K_i. The equilibrium price is identical with the bid $b_j = p_j = p$ of the highest bidding firm j.

Proof. See the argument in Appendix B.1. ■

Proposition 2 implies that, if all firms $j \in Q$ have a rather small capacity with $K_j \leq \alpha - \bar{K}$, then the equilibrium price is always determined by the balance of total available capacity and the demand for capacity and is given by $\bar{p}$ as defined in (6).[11] If some firms $j \in Q$ are larger with $K_j > \alpha - \bar{K}$, the firm with the largest capacity, firm 1, then necessarily belongs to P as defined in (7), meaning it could potentially be a price setting firm with $b_1 = p_1 = p$. However, whether firm 1 is the only possible price setting firm, and therefore p_1 the only equilibrium outcome, depends on the capacities of the other firms $j \in Q$. Given $K_1 > \alpha - \bar{K}$, and therefore also $1 \in P$, two different situations characterized in figure 1 and 2 can occur.

Suppose $\bar{K} < \alpha$ as in figure 1, then all firms $j \in N$ belong to Q and $O = \emptyset$. Note that if $0 < \bar{K} < \frac{\alpha}{2}$, meaning total capacity is smaller than the capacity that a not capacity constrained monopolist would offer, we necessarily have $\alpha - \bar{K} > \frac{\alpha}{2} > K_1$. Neither firm 1 nor any other firm j with $K_j < K_1$ can be an element in P and be price setting in equilibrium. This situation is sketched on the left-hand side of figure 1. With $\bar{K} < \alpha$ and $K_1 \geq K_j$ for all $j \neq 1$ only those combinations of K_1 and K_j can occur which lie to the east of the upward sloping and to the west of the downward sloping dashed 45^o-degree line. Thus, $P = \emptyset$ and the equilibrium price is necessarily the minimum market clearing price $p = \bar{p}$.

Suppose $\frac{\alpha}{2} < \bar{K} < \alpha$, meaning total capacity exceeds the monopoly supply of an unconstrained monopolist but falls short of the supply in an unconstrained competitive market. Then firm 1 can be a price setter in equilibrium, if $K_1 > \alpha - \bar{K}$, implying $1 \in P$. Whether other firms $j \in Q$ can also be price setters (and elements of P) depends on their capacity relative

[11] In Figure 1 this is the case as soon as $K_1 \leq \alpha - \bar{K}$ which necessarily implies $K_j \leq K_1 \leq \alpha$ for all $j \in Q$.

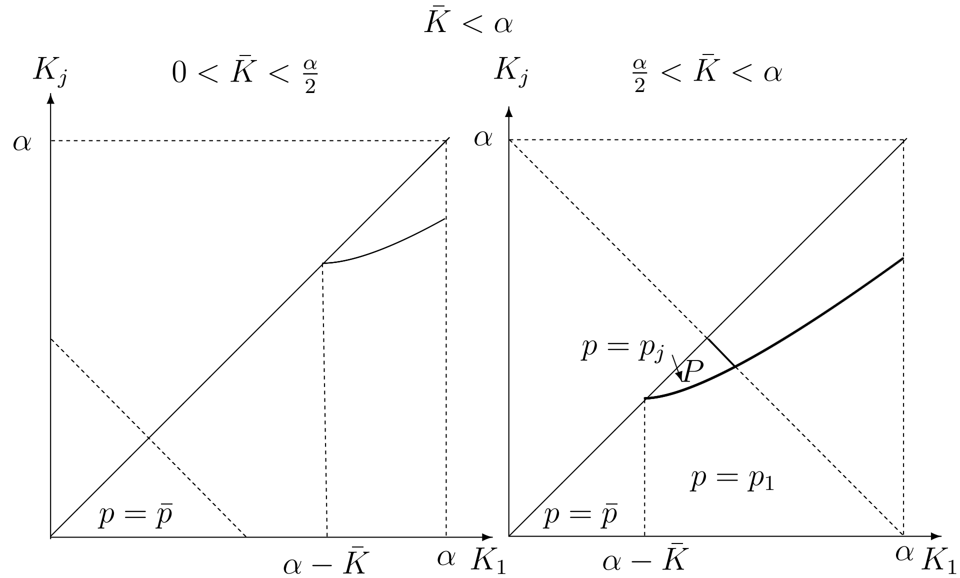

Figure 1: *Characterization of Potentially Price Setting Firms with* $\bar{K} < \alpha$

to the capacity of firm 1. If their smaller capacity is relatively close to firm 1's, such that $j \in P$, they can also be price setting firms and their monopoly prices on their respective residual demand $p = p_j$ can also be market equilibria. On the right-hand side in figure 1 all potentially price setting firms j with $K_j \leq K_1$ must have a capacity K_j above the bold curve, but since $\bar{K} < \alpha$ also implies $K_1 + K_j \leq \bar{K} < \alpha$ these firms capacities cannot be to the east of the dashed downward sloping 45^o-line.

Now suppose $\alpha \leq \bar{K} < 2\alpha$ (total capacity exceeds the supply in an unconstrained perfectly competitive market) as in figure 2, then not all firms are necessarily an element of Q. If $Q = \emptyset$ this implies a relatively even distribution of all capacities and that proposition 1 applies with the equilibrium price being $p = 0$ as in an unconstrained perfectly competitive market. However, if $Q \neq \emptyset$, then firm $1 \in Q$ and firm $1 \in P$ necessarily holds. Thus, firm 1 can be a price setter with $p = p_1$. In addition all those firms $j \in Q$ with a capacity $K_j \leq K_1$ such that it exceeds the bold curved line on the left-hand side of figure 2 can also be price setters which would result in an equilibrium price of $p = p_j$.

Note that as soon as $K_1 > \alpha$ holds, no other firm $j \neq 1$ can be an element of Q and therefore P because necessarily $K_{-j} > \alpha$. The equilibrium price can only be either firm 1's monopoly price on its residual demand $p = p_1$, if

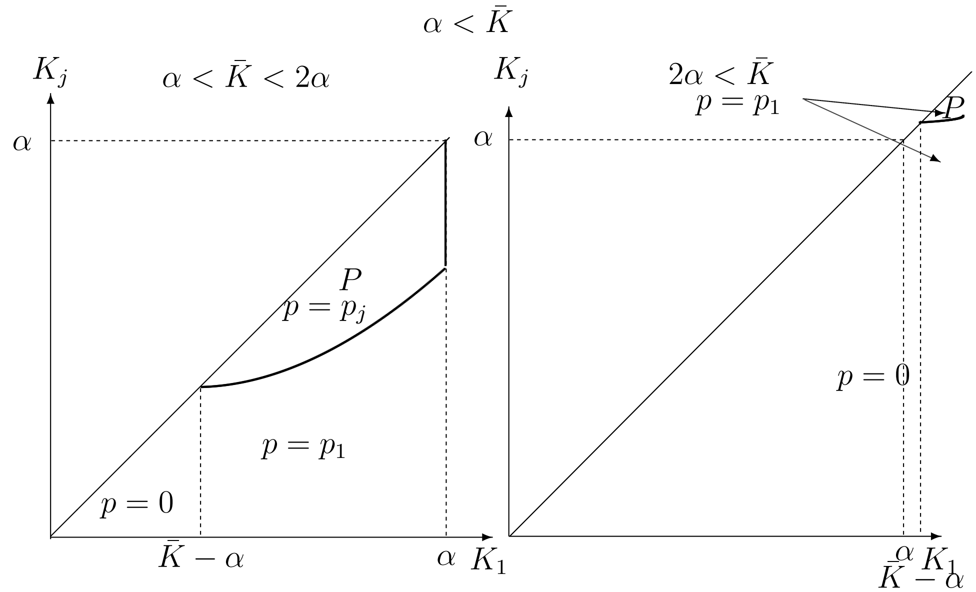

Figure 2: *Characterization of Potentially Price Setting Firms with* $\alpha \leq \bar{K}$

$K_1 > \bar{K} - \alpha$, or the perfectly competitive price in a non-constrained market $p = 0$, if $K_1 \leq \bar{K} - \alpha$ and proposition 1 applies. These are the only two possible prices for $\bar{K} > 2\alpha$, the case illustrated on the right-hand side of figure 2.

Obviously, the potential market prices and highest potential bids in the market depend, on the one hand, on the level of the total capacity but, on the other hand, also on how unequally distributed the capacities of the different generators are.[12] The following corollary characterizes the upper and lower limit of the market price and results mainly from proposition 1, the definition of P in equation (7) of proposition 2 and the definition of the market price, given in equation (4).

Corollary 1 *The market prices depend on the total capacity and on the inequality of the individual firms' capacities in the following way.*

(i) For $0 \leq \bar{K} < \frac{\alpha}{2}$ *the market price is unique and satisfies*

$$p = \frac{\alpha - \bar{K}}{\beta} = \bar{p}.$$

[12]This resembles the result of de Frutos and Fabra (2012) who show in a similar setting that the equilibrium prices do not only depend on the total regulated forward positions of all the firms in the market but also how they are distributed among all the firms.

(ii) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ the market price is either unique and satisfies

$$p = \frac{\alpha - \bar{K}}{\beta} = \bar{p} \text{ if } K_1 < \alpha - \bar{K} \text{ holds,}$$

$$\text{or otherwise } p \in \left[\frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}, \frac{\alpha - \bar{K} + K_1}{2\beta}\right].$$

(iii) For $\alpha \leq \bar{K} < 2\alpha$ the market price is either unique and satisfies

$$p = 0 \text{ if } K_1 < \bar{K} - \alpha \text{ holds,}$$

$$\text{or } p \in \left[\frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}, \frac{\alpha - \bar{K} + K_1}{2\beta}\right] \text{ if } \bar{K} - \alpha \leq K_1 < \alpha \text{ holds,}$$

$$\text{or otherwise it is again unique with } p = \frac{\alpha - \bar{K} + K_1}{2\beta} = p_1.$$

(iv) For $2\alpha \leq \bar{K}$ the market price is always unique and

$$p = 0 \text{ if } K_1 < \bar{K} - \alpha \text{ holds and}$$

$$p = \frac{\alpha - \bar{K} + K_1}{2\beta} = p_1 \text{ otherwise.}$$

Corollary 2 characterizes the non-marginal bids and follows from applying proposition 1 and the definition of P, given in equation (7), to the upper limit of the non-marginal bids, given in equation (8) of proposition 2.

Corollary 2 *The non-marginal bids in equilibrium need to satisfy the following conditions.*

(i) For $0 \leq \bar{K} < \frac{\alpha}{2}$ all bids can be non-marginal and need to satisfy

$$b_j \leq \frac{\alpha - \bar{K}}{\beta} \text{ for all } j \in N.$$

(ii) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ again all bids need to satisfy

$$b_j \leq \frac{\alpha - \bar{K}}{\beta} \text{ if } K_1 < \alpha - \bar{K} \text{ holds.}$$

If $\alpha - \bar{K} \leq K_1$ holds instead, the non-marginal bids need to satisfy

$$b_i \leq \underline{b} \text{ with } \underline{b} \in \left[\frac{(\alpha - \bar{K} + K_1)^4}{8\beta K_1[(\alpha - \bar{K})^2 + K_1^2]}, \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}\right].$$

(iii) *For* $\alpha \leq \bar{K} < 2\alpha$ *all bids are marginal and must satisfy*

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$

If $\bar{K} - \alpha \leq K_1 < \alpha$ *holds instead, the non-marginal bids need to satisfy*

$$b_i \leq \underline{b}_j \text{ with } \underline{b}_j \in \left[\frac{(\alpha - \bar{K} + K_1)^4}{8\beta K_1[(\alpha - \bar{K})^2 + K_1^2]}, \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}\right]$$

or, if $\alpha \leq K_1$ *holds, they need to satisfy*

$$b_i \leq \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1} = \underline{b}_1.$$

(iv) *For* $2\alpha \leq \bar{K}$ *again all bids are marginal and must satisfy*

$$b_j = 0 \text{ for } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$

If $K_1 \geq \bar{K} - \alpha$ *holds instead, the non-marginal bids need to satisfy*

$$b_i \leq \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1} = \underline{b}_1.$$

3 The Effect of Bid Regulations on Market Outcomes

This section analyzes how different types of bid regulations change the market outcomes. First it focusses on maximum bids in the form of bid caps and later on bid floors or minimum bids. In both cases the analysis starts with either general bid caps or general bid floors which apply to all firms in the market. Afterwards the focus shifts to selective bid regulations, meaning bid caps only for firms with large capacities and bid floors only for firms with low capacities.[13]

[13] The NYICAP market which inspired this study has general and selectively more stringent bid caps, but only selective and no general bid floors.

3.1 The Effect of Bid Caps

3.1.1 General Bid Caps

A general bid cap can only have an effect if it forces at least some firms to change their bidding behaviour. From proposition 1 we know that if $Q = \emptyset$ then all firms $j \in O = N$ bid $b_j = 0$ and the equilibrium price is $p = 0$. No bid cap that forces all firms $j \in N$ to bid $b_j \leq \hat{b}$ with $\hat{b} \geq 0$ can change this bidding behaviour. This is obviously different if $Q \neq \emptyset$ and proposition 2 applies without a bid cap.

Suppose $K_j \leq \alpha - \bar{K}$ for all firms $j \in Q$. Then the bid cap might change the bidding behaviour if $\hat{b} < \bar{p}$ holds. Note, however, that the price $\bar{p}$, defined in equation (6), balances total capacity with total demand. Thus, the bid cap would here, nevertheless, not change the market price, because total supply and total demand cannot be balanced at any price $p \leq \hat{b} < \bar{p}$. The auctioneer needs to elevate the market price from the potentially highest bid $\hat{b}$ to $p = \bar{p}$ to balance total supply and demand. The same is true if some firms $j \in Q$ have a capacity with $K_j > \alpha - \bar{K}$, but the bid cap is still set such that $\hat{b} < \bar{p}$, then all firms bid at or below the bid cap $\hat{b}$, but the final market price is again $p = \bar{p}$.

Proposition 3 *If $Q = \emptyset$ and all firms $j \in N$ also satisfy $j \in O$, then a bid cap $\hat{b} \geq 0$ which only allows the firms to bid a price $b_j \leq \hat{b}$ for their total capacity, does not change the firms' bidding behaviour. All firms $j \in N$ still bid $b_j = 0$ and, thus, the market price is still $p = b_j = 0$. If $Q \neq \emptyset$ and if the bid cap is below the market clearing price ($\hat{b} \leq \bar{p}$), the bid cap, despite potentially reducing the bid of some bidders $j \in Q$, does again not change the market price of $p = \bar{p}$ as defined in (6).*

Proof. See the arguments above.■

Suppose now $K_j > \alpha - \bar{K}$ for some $j \in Q$, then a higher bid cap with $\hat{b} > \bar{p}$ does not only potentially change the bidding behaviour, but also the market outcome. For having an effect on the bidding behaviour, and on the market price, the bid cap needs to either constrain the optimal bid of the marginal bidder, or the bid cap needs to change the set of potentially marginal bidders P as given in equation (7) in proposition 2.

Proposition 4 *If $Q \neq \emptyset$, if some firms $j \in Q$ have a capacity $K_j > \alpha - \bar{K}$ and if the bid cap exceeds the market clearing price ($\hat{b} > \bar{p}$), then the bid cap only has an impact on the bidding behavior of the firms, if it constrains the potential bids of some firms $j \in Q$, meaning $p_j > \hat{b} \Leftrightarrow K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ holds.*

(i) If $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ holds for all firms $j \in Q$, then there is a set of Nash equilibria where any of the firms $j \in Q$ with $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ bids $b_j = \hat{b}$ and all other firms $i \in N \setminus j$ set $b_i \leq \underline{\hat{b}}_j$ with

$$\underline{\hat{b}}_j = \frac{\hat{b}(\alpha + K_j - \bar{K} - \beta\hat{b})}{K_j} < \underline{b}_j, \tag{9}$$

and with $\underline{b}_j$ defined in equation (8). The market price is $p = \hat{b}$.

(ii) If only for some firms $j \in Q$ the condition $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ holds, the equilibria described in (i) still exist, but there might be other Nash equilibria. In these additional equilibria one of the other firms $j \in Q$ with $\alpha - \bar{K} \leq K_j \leq 2\beta\hat{b} - (\alpha - \bar{K})$ bids its monopoly price on the residual demand $b_j = p_j = p < \hat{b}$ and all the other firms $i \in N \setminus j$ bid $b_i \leq \underline{b}_j$. These equilibria exist if $j \in \hat{P}$ with

$$\hat{P} = \left\{ j \in Q | K_j \geq \frac{2\beta\hat{b}(\alpha - \beta\hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K})}{K_1} \right\}. \tag{10}$$

If for all firms $j \in Q$ the condition $K_j \leq 2\beta\hat{b} - (\alpha - \bar{K})$ holds, the bid cap has no effect on any firm's bidding behaviour in equilibrium and proposition 2 still applies.

Proof. See the the proof in Appendix C.■

Note that the threshold that the capacity of an unconstrained firm j (with $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$) needs to exceed in order to be potentially price setting in case (ii) ($j \in \hat{P}$) is lower than the threshold that defined whether firm j could be a price setter without the bid cap ($j \in P$ defined in proposition 2). This results from other firms $i \in Q$ with $K_i > K_j$ and $p_i > \hat{b}$ finding it now less attractive to overbid firm j because they are constrained to the bid cap $\hat{b}$ and can no longer bid the monopoly price on their residual demand. Therefore firms $j \in Q$ with smaller capacities than before can be price setters in an equilibrium and, if they are, smaller equilibrium prices prevail than without the bid cap.

With $K_1 > 2\beta\hat{b} - (\alpha - \bar{K})$ and $p_1 > \hat{b} > \bar{p}$ firm 1 with the largest capacity is constrained by the bid cap, but this also implies that $K_1 > \alpha - \bar{K}$. The latter excludes the case that $0 < \bar{K} < \frac{\alpha}{2}$ (or case (i) as well as part of case (ii) from corollary 1). Corollary 3 characterizes the market prices in this case.

Corollary 3 *If $Q \neq \emptyset$ and $K_1 \leq 2\beta\hat{b} - (\alpha - \bar{K})$ holds, then a bid cap $\hat{b} > \bar{p}$ does not change the prices in the market, characterized in corollary 1. If, however, $Q \neq \emptyset$ and $K_1 > 2\beta\hat{b} - (\alpha - \bar{K})$ hold, then a bid cap $\hat{b} > \bar{p}$ changes the market prices in the following way.*

(i) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ the market prices satisfy now

$$p \in \left[\hat{b}\frac{\alpha - \beta\hat{b} + K_1 - \bar{K}}{K_1}, \hat{b}\right].$$

(ii) For $\alpha \leq \bar{K} < 2\alpha$ the market price is unique and satisfies

$$p = 0 \text{ if } K_1 < \bar{K} - \alpha \text{ holds.}$$

$$\text{If } \bar{K} - \alpha \leq K_1 < \alpha \text{ holds instead, } p \in \left[\hat{b}\frac{\alpha - \beta\hat{b} + K_1 - \bar{K}}{K_1}, \hat{b}\right] \text{ and,}$$

$$\text{if } \alpha \leq K_1, \text{ the unique market price is } p = \hat{b}.$$

(iii) For $2\alpha \leq \bar{K}$ the market price is always unique and satisfies

$$p = 0 \text{ if } K_1 < \bar{K} - \alpha \text{ holds and}$$

$$p = \hat{b} \text{ otherwise.}$$

So, the bid cap does not only reduce the upper limit of the possible equilibrium prices, but also reduces the lower limit, even if the lower limit does not exceed the bid cap. This is due to the fact that now smaller firms j can be price setting (with $j \notin P$, but $j \in \hat{P}$), and smaller firms set lower prices because they have a lower residual demand.

Since the threshold for non-marginal bids $\underline{\hat{b}}_j$, defined in equation (9), increases in the capacity K_j of the price setting firm j, and now smaller firms can be price setting, the bid cap also affects the non-marginal bids, again without them necessarily exceeding the bid cap.

Corollary 4 *Given $Q \neq \emptyset$ and $K_1 > 2\beta\hat{b} - (\alpha - \bar{K})$ hold, then a bid cap $\hat{b} > \bar{p}$ should influence the non-marginal bids in the following way.*

(i) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ the non-marginal bids need to satisfy

$$b_i \leq \underline{\hat{b}}_j \text{ with } \underline{\hat{b}}_j \in \left[\frac{\beta^2\hat{b}[2(\alpha - \bar{K} - \beta\hat{b}) + K_1]}{2\beta\hat{b}(\alpha - \beta\hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K})}, \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta\hat{b})}{K_1}\right].$$

(ii) For $\alpha \leq \bar{K} < 2\alpha$ the bids must satisfy

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$

If, in this case, $\bar{K} - \alpha \leq K_1 < \alpha$ holds instead, the non-marginal bids need to satisfy

$$b_i \leq \underline{\hat{b}}_j \text{ with } \underline{\hat{b}}_j \in \left[\frac{\beta^2\hat{b}[2(\alpha - \bar{K} - \beta\hat{b}) + K_1]}{2\beta\hat{b}(\alpha - \beta\hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K})}, \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta\hat{b})}{K_1}\right],$$

or, if $\alpha \leq K_1$,

$$b_i \leq \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta\hat{b})}{K_1} = \underline{\hat{b}}_1.$$

(iii) For $2\alpha \leq \bar{K}$ the bids must satisfy

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$

If, in this case, $\bar{K} - \alpha \leq K_1$ holds instead, the non-marginal bids need to satisfy

$$b_i \leq \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta\hat{b})}{K_1} = \underline{\hat{b}}_1.$$

3.1.2 Selective Bid Caps

What happens if the bid cap is only selectively applied to firms with $K_j > \hat{K} > 0$ instead of to all firms participating in the auction?[14] Note that, if either the bid cap $\hat{b}$ applies to all potentially price setting firms $j \in Q$ because $\hat{K} < \bar{K} - \alpha$, or if it applies to all firms $j \in Q$ for which the related bid cap is potentially binding, meaning $\hat{K} \leq 2\beta\hat{b} - (\alpha - \bar{K})$,[15] then nothing changes compared to a general bid cap characterized in proposition 3 and 4 as well as in corollary 3 and 4.

Proposition 5 *Suppose $Q \neq \emptyset$ and that a bid cap, exceeding the market clearing price ($\hat{b} > \bar{p}$), is imposed only on firms $j \in N$ with a capacity $K_j > \hat{K}$.*

(i) This does not change the market equilibrium compared to a general bid cap characterized in proposition 4, if either $\hat{K} < 2\beta\hat{b} + \bar{K} - \alpha$ or if all the firms $j \in Q$ have a capacity $K_j > \hat{K}$.

(ii) If $K_1 < 2\beta\hat{b} - (\alpha - \bar{K})$ holds, then the selective bid cap does not have any influence on the equilibrium behaviour of any of the firms and the market equilibria are still the same as in proposition 2 without any bid cap.

Proof. For (i) see the argument above. No matter whether bid caps are selective or applied to all firms, they need to constrain the optimal behaviour of at least one firm to have an effect. Condition (ii) is the same as (iii) in proposition 4 and ensures that even the largest firm with the highest monopoly price on its residual demand is not constrained by the bid cap. ■

Changes compared to a general bid cap can only occur if $\hat{K} > 2\beta\hat{b} - (\alpha - \bar{K})$ holds, meaning that the cap does not apply to all potentially constrained firms with $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$. Firms with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ can, on the one hand, still bid their monopoly price on the residual demand,

[14] Note that in the NYICAP market $\hat{K}$ is determined such that $\hat{K} = \bar{K} - K^p > \bar{K} - \alpha$ with $K^p \in (0, \alpha)$ being the reference quantity. The firms with $K_j > \hat{K} = \bar{K} - K^p$ cannot bid their capacity at a higher price than the reference price that satisfies $K^p = \alpha - \beta\hat{b} \Leftrightarrow \hat{b} = \frac{\alpha - \hat{K}}{\beta}$.

[15] All firms j with $K_j \geq 2\beta\hat{b} - (\alpha - \bar{K})$ have a monopoly price on their residual demand which exceeds the bid cap, $p_j \geq \hat{b}$, and are potentially constrained by the bid cap.

although $p_j > \hat{b}$ holds, and, on the other hand, their optimal overbidding strategy, in case a firm with a smaller capacity places the highest bid, is still $b_j = p_j > \hat{b}$. The following proposition takes only these truly selective bid caps into account.

Proposition 6 *Suppose $Q \neq \emptyset$, that a bid cap, exceeding the market clearing price ($\hat{b} > \bar{p}$), is imposed only on firms $j \in N$ with a capacity $K_j > \hat{K}$, that there are firms $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ and that at least the largest firm 1 is constrained by the selective bid cap, meaning $K_1 > \hat{K} > 2\beta\hat{b} - (\alpha - \bar{K})$.*

(i) The equilibria described in case (i) of proposition 4 with a constrained firm $j \in Q$ with $K_j > \hat{K}$ bidding $\hat{b}$ and all the other firms $i \in N \setminus j$ bidding $b_i \leq \underline{\hat{b}}_j$, defined in (9), only exist, if at the same point in time there is no unconstrained firm i with capacity $K_i \in (2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \bar{K} - \alpha)}, \hat{K})$.

(ii) Additional equilibria exist, if there are unconstrained firms $j \in Q$, the capacity of which satisfies $2\beta\hat{b} + \bar{K} - \alpha < K_j < \hat{K}$ and

$$K_j \geq \frac{K_i^2 + (\alpha - \bar{K})^2}{2K_i} \text{ for all } i \text{ with } K_i \in (K_j, \hat{K}) \tag{11}$$

being the capacity of any other larger selectively non-constrained firm i. In these equilibria with $p = p_j$ one of these firms j bids its monopoly price on its residual demand $p_j > \hat{b}$ and all other firms $i \in N \setminus j$ bid $b_i \leq \underline{b}_j$, defined in (8).

(iii) Equilibria with an unconstrained firm $j \in Q$ with $K_j \leq 2\beta\hat{b} + \bar{K} - \alpha$ bidding its monopoly price on its residual demand $p_j \leq \hat{b}$ and all other firms $i \in N \setminus j$ bidding $b_i \leq \underline{b}_j$ can still exist, if firm j's capacity satisfies condition (11) and $K_j \in \hat{P}$ as defined in (10) of proposition 4.

Proof. See Appendix D. ■

Proposition 6 has three major implications for the potential equilibrium. First, equilibria where the firms j with the largest capacities, $K_j > \hat{K}$, bid the bid cap do no longer always exist. If there are selectively non-constrained firms $i \in Q$ with capacity $K_i \in (2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \bar{K} - \alpha)}, \hat{K})$, relatively close to the threshold $\hat{K}$, these firms have an incentive to overbid any firm $j \in Q$ bidding $b_j = \hat{b}$.

Second, prices above the bid cap can occur in equilibrium, if there are firms $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$. They can be price setters in equilibrium with $p = p_j > \hat{b}$, if their capacity K_j is sufficiently large compared to the largest selectively non-constrained firm (see condition (11)). Note that condition (11) is not a pure subset of P defined in (7) of proposition 2 because it potentially also includes firms $j \notin P$ with smaller capacities, because the largest selectively non-constrained firm has a smaller capacity than the largest firm 1 ($K_i < \hat{K} < K_1$).

Third, prices below the bid cap might only occur in equilibrium if, for some firms $j \in Q$ with $K_j \leq 2\beta\hat{b} + \bar{K} - \alpha$, both conditions, (11) and $K_j \in \hat{P}$, defined in equation 10 of proposition 4, are satisfied. These two conditions ensure that neither the larger constrained firms $i \in Q$ with $K_i > \hat{K}$, nor the larger non constrained firms with $K_i \leq \hat{K}$ want to overbid firm j. Which of the two constraints is binding depends on the level of the bid cap $\hat{b}$ and the critical capacity level $\hat{K}$ above which firms are constrained.

Corollary 5 resembles corollary 1 and 3 and identifies the boundaries for the equilibrium prices.

Corollary 5 *Given $Q \neq \emptyset$ and some firms $j \in Q$ have a capacity $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ then a bid cap $\hat{b} > \bar{p}$, only applied to firms $j \in Q$ with $K_j > \hat{K} > 2\beta\hat{b} - (\alpha - \bar{K})$, results in the same observed market prices as in corollary 3, if there are no firms j with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$. If, however, such firms exist and if at least $K_1 > \hat{K}$ holds, the boundaries for the market price change in the following way.*

(i) For $\frac{\alpha}{2} \leq \bar{K} < 2\alpha$ the market price satisfies

$$p \in \left[\max\left\{\hat{b}\frac{\alpha - \beta\hat{b} + K_1 - \bar{K}}{K_1}, \frac{(\alpha - \bar{K} + \hat{K})^2}{4\beta\hat{K}}\right\}, \frac{\alpha - \bar{K} + \hat{K}}{2\beta}\right].$$

(ii) For $2\alpha \leq \bar{K}$ the market price satisfies

$$p \in \left[\max\left\{\hat{b}\frac{\alpha - \beta\hat{b} + K_1 - \bar{K}}{K_1}, \frac{(\alpha - \bar{K} + \hat{K})^2}{4\beta\hat{K}}\right\}, \frac{\alpha - \bar{K} + \hat{K}}{2\beta}\right]$$

$$\textit{if } \bar{K}-\alpha \leq \hat{K} < \alpha \textit{holds, and is unique with } p = \frac{\alpha - \bar{K} + \hat{K}}{2\beta} \textit{ if } \hat{K} > \alpha.$$

If $K_1 \leq \hat{K}$ holds, then corollary 1 still applies.

As soon as $\hat{b} < \frac{\alpha - \bar{K} + \hat{K}}{2\beta} < \frac{\alpha - \bar{K} + K_1}{2\beta}$ holds, a market price above the bid cap is possible in the market. The price is strictly below the potential upper level in the case without any bid cap described in corollary 1.

Under the restrictions of corollary 5 that a firm j exists with $K_1 > \hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ and $\hat{b} > \bar{p}$, the case $K_1 < \bar{K} - \alpha$ is implicitly excluded, meaning the unconstrained perfectly competitive price $p = 0$ can never be an equilibrium outcome. In addition $p = \frac{\alpha - \bar{K} + \hat{K}}{2\beta}$ can only be a unique equilibrium outcome if total capacity satisfies $\bar{K} \geq 2\alpha$. Otherwise the largest firm's capacity K_1 and the just not constrained firm's capacity $\hat{K}$ cannot both exceed α.

3.2 The Effect of Bid Floors

Bid caps in a procurement setting potentially reduce the market power of the bidding firms. Therefore it is easy to understand why an auctioneer might be interested in implementing them.[16] Since bid floors have intuitively the opposite effect, it is not clear on first glance, why an auctioneer might find them attractive.

In the NYICAP market the ISO's motivation is to secure a sufficiently high incentive to invest in new capacity. The ISO hopes that additional electricity generating capacity reduces the prices in another market, the market for electricity. In the same way consideration outside of the auction at hand must motivate bid caps in a traditional selling auction design which are equivalent to bid floors in the procurement setting.[17]

[16]They are equivalent to a price floor in a traditional selling auction where they potentially reduce bid shading.

[17]Caps in pollution permit auctions are currently not very high on the political agenda. However those who oppose the introduction of bid floors usually argue that higher pollution prices do not necessarily incentivize polluting industries to invest in abatement, but to relocate to other jurisdictions where pollution is cheaper. Relocations mean local job and growth losses. This argument could, of course, also motivate the introduction of price caps in markets for pollution permits, if pollution prices are considered to be too high.

3.2.1 A General Bid Floor

A general bid floor which is set at $b^f > 0$ forces all firms $j \in N$ to bid a price for their capacity which exceeds the bid floor ($b_j \geq b^f$). Obviously, this destroys the equilibrium that we have identified in proposition 1 for the case of $Q = \emptyset$ in which all firms $j \in N$ bid $b_j = 0$ and the equilibrium price would be $p = 0$. However, this is also the case where capacity is so abundant that even the largest capacity is not needed to supply the potentially largest demand. It is difficult to imagine that in such a situation the auctioneer sees it fit to artificially increase the capacity's price in order to, for example, stimulate investment in it.

Now consider the case with scarcity but without market power where $K_j \leq \alpha - \bar{K}$ holds for all $j \in Q$ and the market clearing price $\bar{p}$ is the equilibrium price (see proposition 2) without bid floors. Introducing one with $0 < b^f \leq \bar{p}$ might destroy some of the equilibria where some firms j bid $b_j < b^f$ without it, but the outcome in equilibrium is unchanged. The equilibrium price is still the market clearing price $p = \bar{p}$ set by the auctioneer to balance demand with the totally available supply. Of course, if the bid floor exceeds the market clearing price, $b^f > \bar{p}$, the market can never be balanced and all equilibria described in proposition 2 (i) no longer exist. Again, this does not seem especially relevant.[18]

Proposition 7 *If $Q \neq \emptyset$ and $K_j \leq \alpha - \bar{K}$ for all $j \in Q$, the level of the general bid floor b^f determines whether there are infinitely many equilibria in pure strategies or none at all. If the bid floor is smaller than the market clearing price ($0 < b^f \leq \bar{p}$), all firms $j \in N$ bid between these two prices ($b^f \leq b_j \leq \bar{p}$) and the market equilibrium is $p = \bar{p}$ as in case (i) of proposition 2 without a bid floor. If the bid floor exceeds the market clearing price ($b^f \leq \bar{p}$) the auctioneer can no longer clear the market and an equilibrium no longer exists.*

Proof. See the argument above. ■

What changes if at least the largest capacity owner can have market power, $K_1 > \alpha - \bar{K}$? Again, then some of the equilibria described in proposition 2 (ii) with the price setting firm $j \in P$ bidding above the price floor $b_j = p_j > b^f$ and with all non-price setting firms $i \in N \setminus j$ setting $b_i < b^f < \underline{b}_j < p_j$ do

[18] The auctioneer could set such a price. No firm would sell its total capacity and none of them had an incentive to invest. The price floor could not stimulate extra investments.

no longer exist. However, this does not undermine p_j as a market outcome, as long as there are still equilibria with firm $j \in P$ bidding $b_j = p_j$ and all other firms $i \in N \setminus j$ bidding $b^f \leq b_i \leq \underline{b}_j < p_j$.

These equilibria might, however, cease to exist, if the price floor exceeds the threshold $\underline{b}_j$ below which the firms $i \in N \setminus j$ need to bid in order to prevent the price setting firm j from underbidding them in equilibrium without a price floor. If all firms $i \neq j$ bid the price floor $b^f \geq \underline{b}_j$ instead, the price setting firm j cannot underbid any of the other firms $i \in N \setminus j$. Thus, an alternative equilibrium might exist in which firm $j \in P$ with $\underline{b}_j < b^f$ bids its monopoly price on its residual demand $b_j = p_j$ and all other firms $i \in N \setminus j$ bid now $b_i = b^f$. For this to be true, first, none of the firms $i \in N \setminus j$ may have an incentive to overbid firm j, and, second, firm j may not have an incentive to match the other bidders by setting $b_j = b^f$. Note that the set P, defined in equation (7)) is constructed such, that the first condition holds true if $j \in P$.[19] The following proposition clarifies when the second condition also holds true and the alternative equilibrium exists.

Proposition 8 *If $Q \neq \emptyset$ and $K_j > \alpha - \bar{K}$ for some $j \in Q$, the type of equilibrium which exist with a general bid floor $b^f > 0$ again depends on its level.*

(i) If $b^f \leq \underline{b}_j$ holds for all firms $j \in P$, there are still infinitely many equilibria as in proposition 2 (ii). In these equilibria one firm $j \in P$ bids its monopoly price on its residual demand and all other firms $i \in N \setminus j$ bid between the bid floor b^f and the threshold $\underline{b}_j$, meaning $b^f \leq b_i \leq \underline{b}_j < p_j$. The equilibrium price is $b_j = p_j = p$.

(ii) If $\underline{b}_j < b^f \leq c_j$ holds for some firms $j \in P$ with

$$\bar{c}_j \equiv \frac{\alpha}{2\beta} - \frac{\sqrt{(\bar{K} - K_j)K_j(\bar{K}K_j - (\alpha - \bar{K})^2)}}{2\beta K_j} < p_j,$$

then there still can be multiple equilibria. In these equilibria one firm $j \in P$ bids its monopoly price on its residual demand which is the equilibrium price $b_j = p_j = p$. The other firms $i \in N \setminus j$ bid either $b_i \in [b^f, \underline{b}_j]$ or, if the bid floor exceeds the respective price setting firm's threshold $\underline{b}_j$, they bid the bid floor, $\underline{b}_j < b_i = b^f \leq \bar{c}_j$. Any firm $j \in P$, for which $b^f > \bar{c}_j$ holds, can no longer be price setting and its monopoly price p_j on its residual demand can no longer be the equilibrium price.

[19] See the proof of proposition 2 in B.1.

(iii) If $\bar{c}_j < b^f < p_j$ for all firms $j \in P$, then there is a unique equilibrium in which all firms $i \in N$ bid $b_i = b^f$.

Proof. See the arguments above for the destruction of some of the pure strategy equilibria from proposition 2 (ii). Note that

$$\frac{\partial \underline{b}_j}{\partial K_j} = \frac{K_j^2 - (\alpha - \bar{K})^2}{4\beta K_j^2} > 0,$$

meaning the threshold $\underline{b}_j$ increases in the capacity of the price setting firm $j \in P$. That implies that none of the equilibria described in proposition 2 (ii) continue to exist if the price floor exceeds the largest firm's threshold, $b^f > \underline{b}_1$. For the conditions which ensure that the alternative pure strategy equilibria exist, see Appendix E. ■

Since the alternative threshold $\bar{c}_j$ also increases in the price setting firm's capacity K_j, increasing the bid floor b^f destroys first the equilibria with relatively low equilibrium prices. However, in the extreme case where the bid floor exceeds even the largest firm's threshold, $b_f > \bar{c}_1$, all market equilibria with market power disappear and, if the equilibrium was before $p = p_1$, the equilibrium price can be nevertheless be reduced if the price floor satisfies $\bar{c}_1 < b^f < p_1$.

3.2.2 A Selective Bid Floor

Now consider selective bid floors that force only some firms $j \in N$ with $K_n < K_j < \breve{K} < K_1$ to bid $b_j \geq b^f > 0$. Selective bid floors have the same effect as general ones for all equilibria with a price setting firm j for which $\underline{b}_j > b^f$. However, the difference is that with general bid floors a firm $j \in P$ with $p_j > \bar{c}_j > b^f > \underline{b}_j$ can still bid $b_j = p_j$ and be price setting with everybody else bidding $b_i = b^f < p_j$ in equilibrium, as described in proposition 8. These equilibria do no longer exist, if the price setting firm j's capacity is such ($K_j > \breve{K}$) that it is not constrained to bid at or above b_f. Therefore it has an incentive to underbid b^f above which the constrained firms $c \in N \setminus j$ with $K_c < \breve{K}$ are forced to bid.

Thus selective bid floors destroy the alternative equilibria described in proposition 8(ii) if the price setter j is not constrained by the floor. In the extreme case where $b^f > \underline{b}_1$, and all price setting firms $j \in P$ are non-constrained with $K_j \geq \breve{K}$ the selective bid floor destroys all equilibria, because all firms bidding b_f is no longer an equilibrium. In this case any potential price setter $j \in P$ wants to underbid the constrained firms c with $b_c = b^f > \underline{b}_j$. The following proposition characterizes the equilibria with a selective bid floor.

Proposition 9 *If $Q \neq \emptyset$ and $K_j > \alpha - \bar{K}$ for some $j \in Q$, the selective bid floor which forces only those firms j with $K_j < \breve{K} < K_1$ to bid $b_j \geq b^f > 0$ changes the type of equilibrium which exist relatively to proposition 8.*

(i) If $b^f \leq \underline{b}_j$ holds for all firms $j \in P$, there are still infinitely many equilibria as in proposition 2 (ii). In these equilibria one firm $j \in P$ bids its monopoly price on its residual demand and all constrained firms $c \in N \setminus j$ with $K_c < \breve{K}$ bid between the bid floor b^f and the threshold $\underline{b}_j$, meaning $b^f \leq b_c \leq \underline{b}_j < p_j$, whereas all other non-constrained firms $i \in N \setminus j$ with $K_i \geq \breve{K}$ bid $0 \leq b_i \leq \underline{b}_j$. The equilibrium price is $b_j = p_j = p$.

(ii) If $\underline{b}_j < b^f \leq c_j$ holds for some firms $j \in P$ with

$$\bar{c}_j \equiv \frac{\alpha}{2\beta} - \frac{\sqrt{(\bar{K} - K_j)K_j(\bar{K}K_j - (\alpha - \bar{K})^2)}}{2\beta K_j} < p_j \text{ and } K_j < \breve{K}$$

for all these firms, then there still can be multiple equilibria. In these equilibria one firm $j \in P$ bids its monopoly price on its residual demand which is the equilibrium price $b_j = p_j = p$. The other firms $c \in N \setminus j$ with $K_c < \breve{K}$ bid either $b_c \in [b^f, \underline{b}_j]$ or, if the bid floor exceeds the respective price setting firm's threshold $\underline{b}_j$, they bid the bid floor with $\underline{b}_j < b_c = b^f < \bar{c}_j$. All other non-constrained firms $i \in N \setminus j$ bid $b_i \in [0, \underline{b}_j]$ if the price setter j is non-constrained ($K_j \geq \breve{K}$) or $b_i \in [0, b^f]$ if it is constrained ($K_j < \breve{K}$). Any firm $j \in P$, for which either $b^f > \bar{c}_j$ holds, or for which $\underline{b}_j < b_f$ and $K_j > \breve{K}$ holds, can no longer be price setting and its monopoly price p_j on its residual demand can no longer be the equilibrium price.

(iii) If $\underline{b}_1 < b^f < p_1$ holds and all potentially price setting firms $j \in P$ are non-constrained with $K_j \geq \breve{K}$, then there is no longer any equilibrium in pure strategies.

Proof. See the arguments above. ■

Compared to a general bid floor potentially even more low price equilibria disappear than if no price floors were implemented.

4 Winners and Losers of the Bid Regulation

Here we discuss which firms lose and which firms gain from the introduction of (potentially selective) bid caps or bid floors. Obviously firms can win or gain if the regulations change the market outcome. This means that a bid cap prevents a market equilibrium with a high price or the bid floor prevents an equilibrium with a low price.

Whether firms lose or gain depends in case of a price cap also on whether the firms play still the same role in the new equilibrium as before. If, for example, after the introduction of a general bid cap the same firm j as before bids the highest price, but now $\hat{b}$ instead of p_j, then all firms lose. The marginal bidder is no longer able to set a price that maximizes its profit on the residual demand for capacity, but is forced to bid lower and therefore achieves a lower suboptimal profit, despite selling a larger capacity. The inframarginal bidders still sell their total capacity, but also suffer from the lower price.

Now suppose that, due to the introduction of a general bid cap, firm j that used to bid $p_j > \hat{b}$ is no longer the highest bidder but substituted by another marginal bidder, firm i which is bidding either $\hat{b}$ or $p_i < \hat{b}$. In these new equilibria the former marginal firm j gains compared to the situation without the cap, as long as the new marginal firm is not a low capacity firm that only became marginal due to the bid cap and would have been overbid by firm j without the bid cap.

Without a bid cap all potentially marginal firms (all firms $j \in P$ as defined in equation (7) from proposition 2) prefer the equilibria where another firm $i \in P$ bids highest and becomes marginal instead of them even if the equilibrium price might be lower because inframarginal firms sell more. Thus, if the introduction of the bid cap triggers a role shift such that it is now no longer firm j but another firm i which bid highest then firm j can benefit. For this to happen firm i may not be one of the firms that would never be marginal without a cap because it would be overbid by firm j and all other firms with a higher capacity than firm j. All inframarginal firms lose due to the lower price and, in case of the newly marginal firm i, also due to the lower quantity.

Selective bid caps do not make a big difference. They matter in terms of gains and losses, if the equilibrium price is reduced. The pattern is the same as with general bid caps. All Firms lose if the marginal firm stays the same as without the cap. If a role shift occurs the formerly marginal firm is potentially the only one which gains. Losses might however be lower than with the same general bid cap, if the equilibrium price reduction is lower because the equilibrium price still exceeds the bid cap.

No matter whether the bid cap is general or selective, it either has no effect on the equilibria or it reduces the industry's revenues and the auctioneer's procurement costs.[20] Thus, the incentive to invest in capacity or not to divest capacity is either not affected or reduced. Price floors have the opposite effect. Either they do not have an effect or they increase the industry's revenues and the auctioneer's procurement costs because they prevent some low price equilibria.

So if firm j was the marginal bidder in an equilibrium now prevented by the price floor it will always benefit together with all inframarginal firms which stay inframarginal because the price increases and the marginal firm sells more. If this happens there is always a role shift, meaning firm j can no longer be the highest bidder and the role of the marginal firm must now be played by another firm $i \in P$. Firm i is then the only firm which loses due to the price floor and the price increase because it sells less of its capacity.

5 Conclusions

Our theoretical analysis suggests that the reduction of selective bid caps in the NYICAP in 2008 should have reduced the upper limit of the range of observed equilibrium prices but not necessarily to the level of the selective bid caps. The same should hold for the inframarginal bids. At the same point in time the newly introduced selective price floors should have increased the lower level of the observed equilibrium prices in the auction, even if the lower limits of the observed equilibrium prices without the price floors were way

[20]Since the marginal firm maximizes always its profits on the residual demand instead of on the total demand, the equilibrium price is always below the price that maximizes the industry's revenues from the mechanism. A further reduction in price due to the price cap always means that total revenues from the mechanism in the industry decrease further.

above the introduced price floors. Trivially there will be more inframarginal bids above the selective price floor.

So, both changes together should have compressed the observed equilibrium prices from both sides and, chosen adequately, should have reduced the deviations from the targeted reference price. Whether the expected revenues from the capacity market were increased or reduced depends on the chosen bid caps and bid floors, but taken together it is not clear from the outset whether the incentive to invest in new electricity capacity increased or decreased. The only effect of the reform could have been that the regulators more often than not hit the reference price.

Of course our model can also be applied to other multi-unit uniform price auctions. However, in reality reservation prices are rarely used in treasury auctions although there is evidence for underpricing (see Kremer and Nyborg (2004) and Keloharyu *et al.* (2005)) and it seems clear not only from the analysis here that the prices of treasury bonds could be increased by using them.[21] Betz *et al.* (2010) report that bid caps are relatively common in emission permit auctions. Our analysis shows that regulation authorities can use a reserve price to ensure a sufficiently high price.

A drawback of our analysis is, of course, that there is no asymmetric information as, for example, in Ausubel *et al.* (2014). So, it would be interesting to analyze bid caps and bid floors in a setting where, for example, only the firm itself knows, how much capacity it can supply to the market. Then the auction should also serve the purpose to inform the auctioneer about the availability of capacity and too stringent attempts by the auctioneer to keep the price within limits could easily be counter-productive in pursuing this purpose.

[21] Some bidders in the US treasury auction are, however, restricted in the quantity that they can buy.

Appendix

A Explanation for Why Splitting a Firm's Total Capacity Does not Change the Outcome of the Auction

To see this, remember that the demand is certain and common knowledge. Then, if a firm's bid for its total capacity is optimal, given the other firms' bids, this firm never gains from withholding part of its capacity at this bid and ask for a higher (or lower) price on the withheld capacity.

With a higher price bid on only a portion of the capacity, the firm either sells the same at the same price, if it is still underbidding the marginal firm or, if it is bidding above the marginal firm or it is increasing its own marginal bid, it can potentially increase the auction price at the cost of selling less than before. However, if increasing the price (and losing volume) increased the firms surplus, it could have achieved the same higher surplus if it had bid the higher price already on the total capacity before (if itself stays marginal) or if it had increased the price to a level slightly below the now newly marginal firm. In both cases this contradicts the assumption that the original bid on the total capacity was, to begin with, a best response to all the other bids in the market.

Similarly, lowering the price on a portion of the capacity can also not be beneficial. If the original bid on the total capacity is not marginal, lowering the price on a portion, either changes nothing, because the bid was already before below the marginal bid or, if it was the marginal bid or above it, it increases potentially the volume at a cost of a lower price. Again if the latter is beneficial, the firm should have bid lower on its total capacity already at the start and the original bid cannot have been a best response.[22]

[22] Of course these arguments would no longer hold if the demand were stochastic at the time of the firm's bid or if the different units of a firm's capacity had different supply costs.

B Proof of Proposition 2

B.1 Characterization of the Nash Equilibria for Less Capacity Constrained Firms

Assume $Q \neq \emptyset$ and consider first the case with $K_j > \alpha - \bar{K}$ for some $j \in Q$. This implies that these firms can potentially bid their monopoly price on the residual demand p_j and be price setting with $b_j = p_j = p$ because it is feasible. Note that $p_j > p_i$ if $K_j > K_i$. Firm j bidding $b_j = p_j = p$ and all other firms bidding lower can only be a Nash equilibrium if no low bidding firm wants to overbid and firm j does not want to underbid the second highest bid. Let us first consider overbidding by other firms. If $K_i < K_j$ then optimally overbidding $b_j = p_j$ implies $b_i = p_j + \varepsilon$ with $\varepsilon \to 0$ and a profit of $p_j D^r(p_j, K_{-i})$ because $p_i < p_j$. One can show that for all $i > j$ the profit from overbidding is smaller than the profit from either underbidding or matching $b_j = p_j$ if $K_j > \alpha - \bar{K}$. Now suppose $K_i > K_j > \alpha - \bar{K}$ then the optimal overbidding strategy for firm i would be to bid $b_i = p_i$. However, firm i only wants to overbid firm j with $b_j = p_j$ if the profit from doing so is higher than matching or underbidding firm j. Note that matching is always weakly dominated by undercutting. So firm i only has an incentive to overbid if

$$p_i D^r(p_i, K_{-i}) > p_j K_i \Leftrightarrow K_j < \frac{(\alpha - \bar{K})^2 + K_i^2}{2K_i}.$$

This implies that $b_j = p_j = p$ and all other firms bidding lower can only be an equilibrium strategy if the condition above holds for none of the firms with $K_i > K_j$. Since the right hand side of this inequality increases in K_i for $K_i > K_j > \alpha - \bar{K}$, this is ensured if it holds for the firm with the highest capacity K_1, meaning (7) holds. Now let us check under which condition underbidding the second lowest bid by firm j is more profitable for firm j. Suppose $b_i < b_j = p_j$ is the second highest bid. Firm j is only tempted to undercut if

$$b_i K_j > p_j D^r(p_j, K_{-j}) \Leftrightarrow b_i > \frac{(\alpha - \bar{K} + K_j)^2}{4\beta K_j}.$$

Thus, if (8) holds firm j does not have an incentive to undercut and the equilibria characterised in proposition 2 do exist for $K_j > \alpha - \bar{K}$.

B.2 Characterization of the Nash Equilibria for More Capacity Constrained Firms

Assume again that $Q \neq \emptyset$, but consider now the case with $K_j \leq \alpha - \bar{K}$ for all $j \in Q$. Then for all the potentially price setting firms $j \in Q$ the monopoly price on their specific residual demand is smaller than the price that balances total capacity with total demand, meaning $p_j \leq \bar{p}$. Suppose now that firm $i \in N$ bids the highest price $b_i \geq b_j$ for all $j \in N$ with $j \neq i$ for its capacity K_i. As long as $b_i \leq \bar{p}$ the equilibrium price would be $\bar{p}$ because the auctioneer needs to increase the price from b_i to $\bar{p}$ in order to balance supply and demand and firm i would sell all its capacity K_i at the price $p = \bar{p}$. None of the other firms $j \neq i$ would like to overbid firm i. For all $\bar{p} \geq b_j > b_i$ the auction price would not change and any firm j would still sell its total capacity at $p = \bar{p}$, whereas firm j would lose profits as soon as it sets $b_j > \bar{p} \geq b_i$ because then firm j would generate its monopoly profit on its residual demand which is due to $\bar{p} > p_j$ monotonously decreasing for all $p = b_j > \bar{p} > p_j$. In addition firm i with $b_i \leq \bar{p}$ has neither an incentive to underbid any of its competitors nor to increase its bid $b_i > \bar{p}$. In the first case nothing would change for firm i. It would still sell its total capacity at a price of $p = \bar{p}$. In the second case firm i would also decrease its profits because of the same argument made before for an overbidding firm j. Thus, there are infinitely many equilibria in pure strategies where all the firms $j \in N$ bid $b_j \leq \bar{p}$.

C Proof of Proposition 4

Assume again that $Q \neq \emptyset$ holds and that $K_j < \alpha - \bar{K}$ holds for some $j \in Q$. Assume in addition that the bid cap can potentially have an influence on the market price because $\hat{b} > \frac{\alpha - \bar{K}}{\beta} = \bar{p}$. Those firms j for which the monopoly price on their residual demand exceeds the bid cap $p_j > \hat{b} \Leftrightarrow K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ can at most bid $\hat{b}$ and they are willing to do so as long as

$$\hat{b} D^r(\hat{b}, K_{-j}) \geq b_i K_j \text{ for all } i \in N \setminus j.$$

This implies that all the other firms $i \in N \setminus j$ bid such that $b_i \leq \hat{\underline{b}}$ where $\hat{\underline{b}}$ is defined in equation (9) in proposition 4 and $\underline{b}$ in equation (8) in proposition 2.

Note that overbidding is not an option for all firms $i \in N \setminus j$ and matching the bid $b_j = \hat{b}$ would clearly reduce their profits. Thus, for all firms $j \in Q$

with $K_j > 2b\hat{b} - (a - \bar{K})$ bidding $\hat{b}$ and all other firms $i \in N \setminus j$ bidding $b_i \leq \underline{\hat{b}}$ are Nash equilibria. Now assume that the firm $j \in Q$ with $K_j \leq 2\beta\hat{b} - (\alpha - \bar{K})$ sets its monopoly price p_j on the residual demand. As long as there are no other firms $i \in N \setminus j$ for which the bid cap could potentially bind, meaning $K_i \leq 2\beta\hat{b} - (\alpha - \bar{K})$ for all $i \in N \setminus j$ the bid cap has no impact and proposition 2 still applies. However if there are other firms i with $K_i > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ then firm j bidding p_j and all other firms $i \in N \setminus j$ bidding $b_i < p_j$ can only be part of a Nash equilibrium if none of the firms with $K_i > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ has an incentive to overbid with $b_i = \hat{b}$ meaning

$$\hat{b}D^r(\hat{b}, K_{-i}) \leq p_j K_i \Leftrightarrow K_j \geq \frac{2\beta\hat{b}(\alpha - \beta\hat{b} + K_i - \bar{K}) - K_i(\alpha - \bar{K})}{K_i}.$$

When taking into account that the condition above needs to hold for all firms with $K_i > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ and that the right-hand side of the condition increases in K_i it becomes obvious that it must be satisfied for the largest potentially restrained bidder $K_i = K_1 > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ for the Nash equilibrium with $b_j = p_j$ and $b_i \leq \underline{b}$ to exist for $K_i > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ which implies (10), given in proposition 4.

D Proof of proposition 6

If $\hat{K} < 2\beta\hat{b} + \bar{K} - \alpha$ then the bid constraint $\hat{b}$ does not constrain a firm with capacity $K_i = \hat{K}$ because its monopoly price on the residual demand p_i does not exceed $\hat{b}$ but potentially some of the firms $j \in Q$ with $K_j > \hat{K}$. On the other hand if all the firms $j \in Q$ have a capacity $K_j > \hat{K}$ all of them are potentially constrained. In both cases the selective bid cap has the same effect as if it would not have been tied to the condition that $K_j \geq \hat{K}$ and proposition 4 would still apply.

If there are, however, firms $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ and at least $K_1 > \hat{K}$ then these firms j are not constrained by the bid cap, although they would be if $\hat{b}$ would not have been applied only selectively. These firms can still bid $b_j = p_j$ whereas at least firm 1 is constrained and can only bid $b_1 \leq \hat{b} < p_1$.

Of course all the firms $j \in Q$ with $K_j > \hat{K}$ can still be price setting in equilibrium if

$$\hat{b}D^r(\hat{b}, \bar{K}_{-j}) \geq b_i K_j \text{ for all } i \in N \setminus j \Leftrightarrow b_i \leq \underline{\hat{b}}_j$$

with $\underline{\hat{b}}_j$ defined in (9) and none of the firms $i \in Q$ with $K_i \in (2\beta\hat{b}+\bar{K}-\alpha, \hat{K})$ wants to overbid firm j by bidding $b_i = p_i > \hat{b}$.[23] The latter holds true if

$$\hat{b}K_i \geq p_i D^r(p_i, K_{-i}) \quad \Leftrightarrow \quad K_i \leq 2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \bar{K} - \alpha)} \quad (12)$$
$$\text{for all } i \text{ with } K_i \in (2\beta\hat{b} + \bar{K} - \alpha, \hat{K}).$$

Thus if (12) holds, the equilibria where any of the firms j with $K_j > \hat{K}$ is bidding $\hat{b}$ and all the other firms $i \in N \setminus j$ bid $b_i \leq \underline{\hat{b}}_j$ still exists. Note that with $\hat{K} \leq 2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \bar{K} - \alpha)}$ condition (12) is necessarily satisfied.

In addition the unconstrained firms j with $K_j \in (2\beta\hat{b} - (\alpha - \bar{K}), \hat{K})$ could bid $b_j = p_j > \hat{b}$ and be price setting. This could only be an equilibrium if firm j could not generate more profits by undercutting another firm i's bid b_i, meaning all other firms need to bid $b_i \leq \underline{b}_j$ which is defined in (8). At the same point in time no other firm i with $K_i \in (K_j, \hat{K})$, should be tempted to overbid firm j with $b_i = p_i$.[24] Thus, $b_j = p_j$ for firm $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ and $b_i \leq \underline{b}_j$ for all $i \in N \setminus j$ can only be an equilibrium if

$$p_j K_i \geq p_i D^r(p_i, K_{-i}) \Leftrightarrow K_j \geq \frac{K_i^2 + (\alpha - \bar{K})^2}{2K_i} \text{ for all } K_i \in (K_j, \hat{K}). \quad (13)$$

Note that the latter inequality is increasing on the right-hand side in K_i and is necessarily satisfied if $K_j > \frac{\hat{K}^2 + (\alpha - \bar{K})^2}{2\hat{K}}$.

Also firms $j \in Q$ with $\hat{K} > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ can potentially be price setting and bid $b_j = p_j \leq \hat{b}$ in equilibrium. In this case the other firms $i \in N \setminus j$ need to bid again $b_i \leq \underline{b}_j < p_j$. However, now all firms $i \in Q$ with $K_1 \geq K_i > K_j$ could potentially overbid firm j with either $b_i = \hat{b}$ if $K_i \in [\hat{K}, K_1]$ or with $b_i = p_i$ if $K_i \in (K_j, \hat{K})$. In an equilibrium they should not have an incentive to do so. The latter implies not only that(13) needs to hold but also

$$p_j K_i \geq \hat{b} D^r(\hat{b}, K_{-i})$$

[23]Note that the firms $i \in Q$ with $K_i \notin (2\beta\hat{b} + \bar{K} - \alpha, \hat{K})$ do not want to overbid $b_j = \hat{b}$ because $p_i < \hat{b}$.

[24]Note that the firms i with $K_i \in [\hat{K}, K_1]$ cannot overbid firm j's bid $b_j = p_j$ and, as before no firm with $K_i < K_j$ will ever be tempted to overbid.

$$\Leftrightarrow K_j \geq \frac{2\beta\hat{b}(\alpha - \beta\hat{b} + K_i - \bar{K}) - K_i(\alpha - \bar{K})}{K_i} \text{ for all } K_i \in [\hat{K}, K_1].$$

This restriction is again increasing in K_i and is therefore equivalent with $K_j \in \hat{P}$ as defined in (10).

E Existence of the Alternative Equilibria with General Price Floors

Note that if all firms bid at the bid floor each firm achieves a profit of

$$b^f \frac{K_j(\alpha - \beta b^f)}{\bar{K}}.$$

If firm $j \in P$ bids its monopoly price on its residual demand, instead, and all other firms i stick to the price floor, $b_i = b^f$ firm j's profit is

$$\frac{(\alpha - \bar{K} + K_j)^2}{4\beta}.$$

The former exceeds the latter as long as $b^f \leq \bar{c}_j$ holds, meaning that the price setting firm j does not want to match the other firms' bid at the bid floor. The firms that bid at the bid floor do not want to overbid firm j as long as $j \in P$. Note that $\bar{c}_j > \underline{b}_j$ and that it is also increasing in K_j for the potentially relevant range of $\max\{\alpha - \bar{K}, \bar{K} - \alpha\} < K_j < \bar{K}$. As soon as the bid floor exceeds the threshold c_1, implying $p_1 > b^f > \bar{c}_1$, even firm 1 with the largest profit from bidding its monopoly price on its residual demand prefers to bid at the price floor instead. Since no longer any firm wants to overbid if all firms j set $b_j = b^f$ and no firm can underbid, this is an equilibrium in pure strategies. Other bidding equilibria in pure strategies with $b_j = b > b^f$ for all $j \in N$ cannot exist because then each firm has an incentive to underbid in order to sell its total capacity instead of a rationed share.

References

Ausubel, L. M., P. Cramton, M. Pycia, M. Rostek and M. Weretka (2014), 'Demand Reductions and Inefficiency in Multi-unit Auctions'. *The Review of Economic Studies*, 81, pp. 1366–1400.

Bergman, L., G. Brunekreeft, C. Doyle, N.-H. M. von der Fehr, D. M. Newbery, M. Pollitt and P. Règibeau (eds.) (1999), *A European Market for Electricity?*, London and Stockholm: CEPR and SNS.

Betz, R., S. Seifert, P. Cramton and S. Kerr (2010), 'Auctioning Greenhouse Gas Emission Permits in Australia'. *The Australian Journal of Agricultural and Resource Economics*, 54, pp. 219–238.

Brenner, M., D. Galai and O. Sade (2009), 'Sovereign Debt Auctions: Uniform or Discriminatory'. *Journal of Monetary Economics*, 56, pp. 267–274.

Bresky, M. (2013), 'Revenue and Efficiency in Multi-unit Uniform-price Auctions'. *Games and Economic Behavior*, 82, pp. 205–217.

Crampes, C. and N. Fabra (2005), 'The Spanish Electricity Industry: Plus ça change...'. *The Energy Journal*, 26(European Energy Liberalisation Special Issue), pp. 127–153.

Degeorge, F., F. Derrien and K. L. Womack (2010), 'Auctioned IPOs: The US Evidence'. *Journal of Financial Economics*, 98, pp. 177–194.

Fabra, N., N.-H. von der Fehr and D. Harbord (2006), 'Designing Electricity Auctions'. *The Rand Journal of Economics*, 37(Spring), pp. 23–46.

von der Fehr, N.-H. and D. C. Harbord (1993), 'Spot Market Competition in the UK Electricity Industry'. *The Economic Journal*, 103, pp. 531–546.

de Frutos, M. and N. Fabra (2012), 'How to Allocate Forward Contracts: The Case of Electricity Markets'. *European Economic Review*, 56, pp. 451–469.

Hasegawa, M. and S. Salant (2015), 'The Dynamic of Pollution Permits'. *Annual Review of Resource Economics*, 7, pp. 61–79.

Hortaçsu, A. and S. L. Puller (2008), 'Understanding Strategic Bidding in Multi-unit Auctions: A Case Study of the Texas Electricity Spot Market'. *The Rand Journal of Economics*, 39, pp. 86–114.

Jehiel, P. and L. Lamy (2015), 'On Discriminations in Auctions with Endogenous Entry'. *The American Economic Review*, 105, pp. 2595–2643.

Keloharyu, M., K. G. Nyborg and K. Rydqvist (2005), 'Strategic Behavior and Underpricing in Uniform Price Auctions: Evidence from Finnish Treasury Auctions'. *The Journal of Finance*, 60, pp. 1865–1902.

Kiesling, L. and B. J. Wilson (2007), 'An Experimental Analysis of the Effect of Automated Mitigation Procedures on Investment and Prices in Wholesale Electricity Markets'. *Journal of Regulatory Economics*, 31, pp. 313–334.

Kirkegaard, R. (2013), 'Handicaps in Incomplete Information All-pay Auctions with a Diverse Set of Bidders'. *European Economic Review*, 64, pp. 98–110.

Kotowski, M. H. (2015), 'On Asymmetric Reserve Prices', mimeo, John F. Kennedy School of Government, Harvard University.

Kremer, I. and K. G. Nyborg (2004), 'Underpricing and Market Power in Uniform Price Auctions'. *Review of Financial Studies*, 17, pp. 849–877.

Le Coq, C., H. Orzen and S. Schwenen (2016), 'Pricing and Capacity Provision in Electricity Market: An Experimental Study'. Working Paper 37, Stockholm Institute of Transition Economics, Stockholm School of Economics, Stockholm.

Levin, D. and J. L. Smith (1996), 'Optimal Reservation Prices in Auctions'. *The Economic Journal*, 106, pp. 1271–1283.

Mares, V. and J. M. Swinkels (2014), 'On the Analysis of Asymmetric First Price Auctions'. *Journal of Economic Theory*, 152, pp. 1–40.

Moreno, D. and L. Ubeda (2006), 'Capacity Precommitment and Price Competition Yield the Cournot Outcome'. *Games and Economic Behavior*, 56, pp. 323–332.

Myerson, R. B. (1981), 'Optimal Auction Design'. *Mathematics of Operations Research*, 6, pp. 58–73.

Newbery, D. (2005), 'Electricity Liberalisation in Britain: The Quest for a Satisfactory Wholesale Market.' *The Energy Journal*, 26(European Energy Liberalisation Special Issue), pp. 43–70.

Schwenen, S. (2015), 'Strategic Bidding in Multi-unit Auctions with Capacity Constrained Bidders: The New York Capacity Market'. *The Rand Journal of Economics*, 46, pp. 730–750.

Shawan, D. L., K. D. Messer, W. D. Schulze and R. E. Schuler (2011), 'An Experimental Test of Automatic Mitigation of Wholesale Electricity Prices'. *International Journal of Industrial Organization*, 29, pp. 46–53.

Ubeda, L. (2004), 'Capacity and Market Design: Discriminatory versus Uniform Auctions', mimeo, Universidad Carlos III de Madrid and Purdue University.

Vossler, C. A., T. D. Mount, R. J. Thomas and R. D. Zimmermann (2009), 'An Experimental Investigation of Soft Price Caps in Uniform Price Auction Markets for Wholesale Electicity'. *Journal of Regulatory Economics*, 36, pp. 44–59.

Wood, P. J. and F. Jotzo (2011), 'Price Floors for Emission Trading'. *Energy Policy*, 39, pp. 1746–1753.

Zhang, N. (2009), 'Generators' Bidding Behavior in the NYISO Day-ahead Wholesale Electricity Market'. *Energy Economics*, 31, pp. 897–913.