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# TRANSFERS AND GIFTS IN OG MODELS: MONETARY STEADY STATES

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# Transfers and gifts in OG models: monetary steady states*

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**Abstract**

*In the present paper the study of the welfare effects of endowment transfers is extended to the set of steady states of a general stationary overlapping generation model. A complete characterization of manipulations by coalitions and transfers which leads to welfare paradoxes is provided.*

**Keywords:** Overlapping-generations models, manipulations, transfer paradox, Leontief effect, steady states, theory of duality.

**JEL-classification:** D51, D91, E32.

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# 1 Introduction

The fact that a country can improve its welfare by giving some of its resources to another country seems surprising. Leontief was the first, in 1936, to point out that such a transfer paradox may occur although transfers such that every country is better off cannot exist due to the first welfare theorem. The conditions associated to the occurrence of the transfer paradox and of manipulations have been studied in detail since then by many authors. In particular Balasko (1978) and Safra (1983) use the theory of duality (see Balasko (1988)) to obtain further results in the case of pure exchange economies. Note that only very few papers investigate these topics in an intertemporal framework. The most relevant study is Galor and Polemarchakis (1987) where it is shown that in a very simple overlapping generation model with only two countries and one produced good pareto and "leontieff" transfers may occur.

In overlapping-generations economies the situation is much more complex than in static general equilibrium economies. Indeed, important differences are the richer structure of the feasible transfers and the existence of two types of stationary equilibria. Furthermore, the fact that the first welfare theorem may fail open the way to pareto transfers.

In the present paper we consider a stationary pure exchange overlapping generations economy with many consumers and commodities. We can interpret this model as one in which there are many infinitely lived countries composed of consumers with finite lives. The model is stationary in the sense that all generations are identical and there is no population growth. With this model we can focus our attention on comparative statics across steady states, i.e. on the effect of transfer schemes on the utility levels reached by the consumers at the steady states. As pointed out by Galor and Polemarchakis (1987), this comparative static analysis, which ignores the utility gains and losses on the transition path, is in the tradition of the literature on the transfer paradox. The approach we follow is an extension of the duality theory developed by Balasko (1978) and Safra (1983). As the present paper will reveal, this extension is not as straightforward as it may be expected, mainly due to the failure of the first welfare theorem and to the complexity of the set of feasible transfers.

In this framework, it is shown that transfer paradoxes may occur at monetary steady states. Manipulations of equilibria by coalitions of countries are shown to occur at nominal steady states. Besides its non-emptiness, the set of transfers leading to welfare paradoxes and manipulations is characterized in detail. In particular, it is shown that if the trade intensity at equilibrium is small, none of the transfer paradoxes may occur at the monetary steady states. It is also shown that at monetary steady states transfers between generations have no impact. This result seems to suggest that trade reforms in the form of redistributions of endowments are usefully followed by some intergenerational transfers. Hence trade policy has to be coordinated with fiscal policy.

The paper is organized as follows: In section 2 the model is introduced and equilibria and steady states are defined; in section 3 transfers and manipulations are defined; in section 4 the analysis at nominal steady states is carried through. Finally, in section 5 some final remarks are offered.

# 2 The model

## Assumptions

A stationary exchange overlapping generations economy is considered. Each generation is composed of $m$ types of consumers which can be considered as to belong to $m$ countries. Each consumer lives for two periods while the country is infinitely lived and stationary in the sense that all generations are identical and there is no population growth. There are $l$ perishable goods per period and there is money. Since the aim is comparative statics between steady states, an economy with no definite beginning or ending is considered.

Let E be such an economy, then $\mathcal{E} = (\omega_i, u_i)_{i=1}^m$, where $\omega_i$ is the initial endowment and $u_i$ is the utility function of consumer $i$. The usual assumptions on utility functions, i.e., strict monotonicity, strict quasi-concavity and boundedness are supposed to be fulfilled. The consumption and the endowments spaces are taken to be $R^{2l}$. This assumption is not standard in the OG literature but is harmless, as is discussed in Ghiglino and Tvede (1995a) and Ghiglino and Tvede (1996).

Consumer $i$ of generation $t$ maximizes his utility function, which depends on his young age and old age consumptions, $(x_t, x_{t+1}) \in \mathbf{R}^{2l}$, subject to his unique budget constraint:

$$\max \quad u_i(x_t, x_{t+1})$$

$$\text{s.t.} \quad p_t \cdot x_t + p_{t+1} \cdot x_{t+1} = p_t \cdot \omega_i^0 + p_{t+1} \cdot \omega_i^1$$

where $(p_t, p_{t+1}) \in \mathbf{R}^{2l}_{++}$ is the price vector. The resolution of this problem leads to the individual demand function $f_i : \mathbf{R}^{2l}_{++} \times \mathbf{R} \rightarrow \mathbf{R}^{2l}$ associated with $(\omega_i, u_i)$. The demand function, $f_i$, has the usual properties such as homogeneity, smoothness and boundedness.

## Equilibria and steady-states

In the present model all generations are identical so aggregate excess demand does not depend explicitly on the particular generation. The excess demand of generation $t$ when young and old is defined by:

$$y(p_t, p_{t+1}, \omega) = \sum_{i=1}^{m} f_i^0(p_t, p_{t+1}, p_t \cdot \omega_i^0 + p_{t+1} \cdot \omega_i^1) - \sum_{i=1}^{m} \omega_i^0$$

$$z(p_t, p_{t+1}, \omega) = \sum_{i=1}^{m} f_i^1(p_t, p_{t+1}, p_t \cdot \omega_i^0 + p_{t+1} \cdot \omega_i^1) - \sum_{i=1}^{m} \omega_i^1$$

where $\omega = (\omega_1, \ldots, \omega_m)$.

Aggregate excess demand also posses the usual properties such as homogeneity, smoothness, boundedness and satisfies the Walras' law:

$$p_t \cdot y(p_t, p_{t+1}, \omega) + p_{t+1} \cdot z(p_t, p_{t+1}, \omega) = 0$$

for all $(p_t, p_{t+1}) \in \mathbf{R}^{2l}_{++}$.

**Definition 1** *An equilibrium is a sequence of prices and endowments* $((p_t)_{t \in \mathbf{Z}}, \omega)$ *such that* $p_t \in \mathbf{R}^{l}_{++}$ *and markets clear:*

$$y(p_t, p_{t+1}, \omega) + z(p_{t-1}, p_t, \omega) = 0$$

*for all* $t \in \mathbf{Z}$.

A **steady state** is an equilibrium for which there exists a price $p \in \mathbf{R}^l_{++}$ and a scalar $\beta \in \mathbf{R}_{++}$ such that $\beta^t p = p_t$ for all $t \in \mathbf{Z}$. Walras' law and market clearing imply that $(\beta - 1)p \cdot z(p, \beta p, \omega) = 0$. Hence, for $\beta \neq 1$ the debt transferred from one generation to the following, $p \cdot z(p, \beta p, \omega)$, is zero. There are two kinds of steady states: *golden rule* or *nominal* if $\beta = 1$ and; *balanced* or *real* if $z(p, \beta p, \omega) = 0$. On the other hand, at golden rule steady states the debt is typically different from zero and on the other hand, at balanced steady states debt is zero. As shown in Kehoe and Levine (1984), all economies have at least one golden rule steady state and one balanced steady state. Of course at golden rule steady states debt may be positive as well as negative. Furthermore, the set of steady states posses some nice properties, as shown in Ghiglino & Tvede (1995b) and in Balasko & Lang (1996).

In the subsequent sections only steady states are considered.

# 3 Transfers and manipulations

In intertemporal economies, as well as in static economies, transfers designate real transfers of goods. The result economy is an economy with a modified distribution of endowments. Clearly only transfers which do not change total resources are feasible so for intertemporal economies transfers of goods between periods are impossible, i.e apples today cannot be made into apples tomorrow. A characteristic of overlapping generations economies is that different generations coexist, therefore transfers within as well as between generations are feasible despite that transfers between periods are impossible. Therefore the redistributions considered here include both situations in which transfers take place within countries and situations in which transfers take place between countries. However, only stationary transfers are considered, i.e. if a transfer between some consumers takes place in period $t$ then the same transfer takes place in every period, from $-\infty$ to $+\infty$. The restriction to stationary transfers is natural in the present framework as long as only comparative statics across steady state is concerned.

Only golden rule steady states are considered therefore it is not important whether consumers receive transfers in the first or the second period of their lifes. Hence, let $t_i \in \mathbf{R}^l$ be the transfer received by consumer $i$. If $(\omega_i^0, \omega_i^1)$ was his endowment before the transfer, then his endowment after the transfer is $(\omega_i^0, \omega_i^1 + t_i)$.

**Definition 2** *Let $t_i^j \in \mathbf{R}^l$ be the transfer from consumer $j$ to consumer $i$ where $i, j \in \{1, \ldots, m\}$. Then a transfer scheme, $\tau$, is defined by the set $\tau = (t_i^j)_{i,j}$, where $t_i^j = -t_j^i$.*

Note that the scheme $\tau$ can be represented by a $(m-1)m$-tuple of vectors in $\mathbf{R}^l$.

Transfers may or may not be beneficial for the consumers involved depending on how prices change with the transfers. There is a transfer paradox at a steady state if a transfer of a positive amount of goods from one consumer to another consumer changes the prices in such a way that the donor is better off and the recipient worst off. A steady state is manipulable if there is a transfer between members of a coalition that changes the prices is such that all members of the coalition are better off.

Since overlapping generations have at least two steady states (one real and one golden rule) it is not clear how transfers change prices. In order to overcome this, a selection of equilibria has to be considered. Let $p(\tau)$ be the prices for the transfer scheme $\tau$, and in order to save notation, let $p = p(0)$.

**Definition 3** *A steady state is* **manipulable** *if there is a coalition $G \subset \{1, \ldots, i, \ldots, m\}$ of at least two members and transfers between the members of $G$ such that:*

$$u_i[(f_i(p, p, p \cdot (\omega_i^0 + \omega_i^1))] \;<\; u_i[f_i(p(\tau), p(\tau), p(\tau) \cdot (\omega_i^0 + \omega_i^1 + t_i))]$$

*for all $i \in G$.*

**Definition 4** *There is a transfer paradox at the steady state $p$ if there exists a non-negative transfer $t_i^j \in \mathbf{R}_+^l$ from consumer $j$ to consumer $i$ such that after the transfer the donor is better off and the recipient worst off, i.e.*

$$u_i[f_i(p, p, p \cdot (\omega_i^0 + \omega_i^1))] \;<\; u_i[f_i(p(\tau), p(\tau), p(\tau) \cdot (\omega_i^0 + \omega_i^1 + t_i^j))]$$

$$u_j[f_j(p, p, p \cdot (\omega_j^0 + \omega_j^1))] \;>\; u_j[f_j(p(\tau), p(\tau), p(\tau) \cdot (\omega_j^0 + \omega_j^1 - t_i^j))]$$

In the sequel only small transfers and smooth selections are considered. The existence of locally smooth selections can be shown for an open and dense set of endowments. Then the occurrence of transfer paradoxes and manipulable steady states can be studied using tools related the implicit function theorem.

# 4 Transfers and steady states

In the sequel of the paper we focus our attention on comparative statics across steady states, i.e. on the effect of transfer schemes on the utility levels reached by the consumers at the steady states. This comparative static analysis ignores the utility gains and losses on the transition path but is in the tradition of the literature on the welfare effects due to international trade.

In the present section the properties of nominal steady states with respect to transfer paradoxes and manipulability are studied. The fact that nominal steady states are Pareto optimal suggests that Pareto improving transfers are impossible. However, it will be proved that manipulations and Leontief transfers do exist.

First, the overlapping generations economy is reduced to a general equilibrium economy. As far as monetary steady states are considered the existence of such a transformation is implicitly assumed in the literature (see for ex. Kehoe and Levine (1984)). We give a more formal statement. Second, the welfare effects of transfers are investigated by applying the duality theory to the set of equilibria of the artificial GE economy. In particular, the existence of manipulations and of transfer paradoxes are obtained as an application of Balasko (1978), Safra (1983) and Safra (1984).

**Lemma 1** [1] *$p$ are prices at a nominal steady state for the overlapping generations economy $(\omega_i, u_i)_{i=1}^m$ if and only if $p$ are prices at an equilibrium for the general equilibrium economy $(\nu_i, v_i)_{i=1}^m$, where $\nu_i = \omega_i^0 + \omega_i^1$ and $v_i(y) = \max_{x^0+x^1=y} u_i(x^0, x^1)$.*

**Proof:** For constant prices $(p, p)$, consumer $i$ of the overlapping generation (OG) economy solves the following problem:

$$\begin{aligned} \max \quad & u_i(x^0, x^1) \\ \text{s.t.} \quad & p \cdot (x^0 + x^1) = p \cdot (\omega_i^0 + \omega_i^1). \end{aligned}$$

[1] Yves Balasko has informed us that he considered a similar transformation in lectures he gave during the winter 94-95 at the University of Geneva. See also Balasko (1997)

For prices $p$, consumer $i$ of the general equilibrium (GE) economy solves the problem:

$$\max_y \quad v_i(y)$$

$$\text{s.t} \quad p \cdot y = p \cdot \nu_i$$

Clearly, the two problems are equivalent in the sense that $(x^{*0}, x^{*1})$ solves the first problem if and only if $y^* = x^{*0} + x^{*1}$ solves the second problem. Indeed, that $v_i(y) = \max_{y=x^0+x^1} u_i(x^0, x^1)$ exists is straightforward and clearly $(x^0, x^1)$ satisfies the budget constraint in the overlapping generations economy if and only if $y = x^0 + x^1$ satisfies the budget constraint in the general equilibrium economy because $\nu_i = \omega_i^0 + \omega_i^1$. Hence the demand function of consumer $i$, $g_i : \mathbf{R}^l_{++} \times \mathbf{R} \to \mathbf{R}^l$ is $g_i(p, w_i) = f_i^0(p, p, w_i) + f_i^1(p, p, w_i)$ so

$$\sum_i f_i^0(p, p, w_i) + f_i^1(p, p, w_i) - \omega_i^0 - \omega_i^1 \;=\; \sum_i g_i(p, w_i) - (\omega_i^0 + \omega_i^1).$$

Therefore, $p$ is the price at a nominal steady state if and only if $p$ is an equilibrium price. That $v$ possesses the usual properties of a utility function can also be checked. In particular, monotonicity can be deduced from the envelope theorem. Strict quasi-concavity is obtained by applying the definition of this property to $v_i(y) = u_i(x^0(y), x^1(y))$ and $v_i(y') = u_i(x^0(y'), x^1(y'))$ where $(x^0(y), x^1(y))$ is the *argmax* in the definition of $v(y)$ and $v(y) \leq v(y')$. Indeed, clearly $v(y) = u(x^0(y), x^1(y)) < u(tx^0(y)+(1-t)x^0(y'), tx^1(y)+(1-t)x^1(y')) < u(x^0(ty+(1-t)y'), x^1(ty+(1-t)y')) = v(ty+(1-t)y')$. Furthermore, since both $f_i^0$ and $f_i^1$ are smooth functions $g_i$ is also smooth. Then, since the demand function is differentiable if and only if the bordered Hessian is non-singular (Theorem 12.1, Handbook of Mathematical Economy, vol. II, Chapter 9), $v_i$ has a non-singular bordered Hessian. Q.E.D.

The proof reveals that the transfer scheme $\tau$ is beneficial for consumer $i$ in the overlapping generations economy if and only if the transfer scheme $\tau$ is beneficial for consumer $i$ in the general equilibrium economy.

Lemma 1 is all what we need to translate all the results concerning GE models to OG models. Note that since the occurrence of the transfer paradox and of manipulable equilibria relies on price variations we need to consider economies with at least two goods, i.e., $l \geq 2$.

Since nominal steady states for overlapping generations economies reduce to equilibria for general equilibrium economies, it is expected that nominal steady states are manipulable for some endowments. The following theorem states that in every economy, specified by the preferences and the total resources, for any coalition $G$ there exists a distribution of initial endowments such that one of the associated nominal steady states is manipulable.

**Theorem 1** *Let $p$ be a nominal steady state of an economy $(u_i)_{i=1}^m, r$ specified by preferences and total resources. Then there exists an open set of endowments such that the nominal steady state $p$ is $G$-manipulable with international transfers, the result being generical.*

**Proof:** Adopting second order perturbations (see Ghiglino & Tvede (1995a) or Ghiglino & Tvede (1996)) the matrix of income effects associated to $g_i(p, w_i)$ is seen to be generically of full rank. Then Theorem 1 follows from Lemma 1 and theorem 6.1 in Safra (1983). As noted before, intergenerational transfers within countries have no effect. Q.E.D.

It is on the other hand possible to identify open sets of endowments such that no manipulation occurs (see Theorem 9.3, Safra (1983)). In the OG economies these sets contain the no-trade nominal steady states (as seen from Ghiglino and Tvede (1995b)). This means that economies close to a nominal steady state and characterized by a small intensity of trade are immune of the "paradoxical" phenomena contained in the above theorem.

**Theorem 2** *Let $(u_i)_{i=1}^m, r$ be an economy specified by the preferences and total resources then there exists an open set of endowments and an open set of transfer schemes such that a transfer paradox occurs at a nominal steady state.*

**Proof:** By inspection of the proofs of Theorem 6.1 and Theorem 7.1 in Safra (1983) it is clear that the existence of a transfer paradox is implied by the multiplicity of equilibria as in the case of manipulations. Then see proof of Theorem 1. Q.E.D.

The results contained in Theorem 1 are obtained for consumption sets without lower bounds. However, by a translation of preferences and endowments these results can be extended to an open set of preferences to positive consumption sets, as shown in Ghiglino and Tvede (1996).

Note that even in the case of positive consumption sets the existence result obtained above holds true for a large set of preferences. In particular, as is the case for the occurrence of fluctuations, a small discount factor is not a sufficient condition to prevent the occurrence of "pathological" phenomena as transfer paradoxes.

An interesting particular case is provided by an economy composed of two countries exchanging two goods. In this case there exists a characterization of economies specified by preferences and distribution of initial endowments giving rise to a transfer paradox:

**Theorem 3** *An economy with two goods per period and two countries, specified by* $(u_i, \omega_i)_{i=1}^2$ *has at least three nominal steady states if and only if there exists a transfer paradox at one of the nominal steady states.*

**Proof:** Apply Lemma 1 then the theorem follows from Balasko (1979). Q.E.D.

Note that unfortunately this characterization does not generalize to economies with more goods and countries. Indeed, for economies with two goods and at least three countries there may exist a transfer paradox even if the economy has only one equilibrium, as seen in Safra (1984).

# 5 Concluding remarks

In static economies there is, at least in the simple two countries two goods case, a relation between the occurrence of a transfer paradox in the neighbourhood of an equilibrium and the tatonnement stability properties of the same equilibrium. In a dynamic framework tatonnement stability is not anymore a clear concept. Instead, a new stability concept appears, the concept of dynamical stability. The relevant question becomes: is there a relation between the occurrence of a transfer paradox in the neighborhood of a steady state and the dynamical stability properties of that steady state? The answer, based on particular examples, seems to be no, but we were unable to find a complete analytical proof of this fact. Note however that, even in static economies there is no relation between tatonnement stability and the occurrence of the transfer paradox as soon as there are more than two countries (Safra (1984)).

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