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## **Social Preferences and Labor Market Policy**

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## Abstract

We find that the main features of labor policy across OECD countries can be explained by a simple general equilibrium search model with risk neutral agents and a government that chooses policy to maximize a social welfare function. In equilibrium, policies are chosen to optimally redistribute income from advantaged to disadvantaged workers. A worker can be disadvantaged in the sense that they may have less ability to acquire and utilize skills in the workplace. The model explains why passive benefits tend to fall and active benefits tend to increase during the course of unemployment spell. The model also explains why countries that appear to pursue equity spend more on both active and passive labor market programs.

## 1 Introduction

This paper studies the impact of different social preferences on the optimal characteristics of labor market policy. It develops a competitive search equilibrium model with a government that pursues a combined goal of maximizing efficiency and equity. Firms make irreversible investments in vacancies. Workers are paid wages and choose to invest in skills. In this environment, the optimal labor policy addresses two social concerns. The first social concern is that some workers have less ability to use and acquire the required skills needed by employers.

We find that our model can explain the main features of labour policy across OECD countries. Moreover, we are able to replicate these features

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about labor policy even though we assume risk neutral agents and a competitive search equilibrium. Therefore, neither borrowing constraints nor wage inefficiencies are important to establishing our results.

A key feature of labor market policy that we seek to explain is the time varying nature of active and passive subsidies. The observed methods of delivering these benefits suggest that the governments are targeting different unemployment groups. On the one hand, passive subsidies target short-term unemployed workers, because these benefits are only of limited duration. On the other hand, active benefits target long-term unemployed workers, because these instruments are generally implemented only after a significant unemployment spell. In other words, over the course of an unemployment spell, the passive benefit given to unemployed workers tends to fall while the active benefit tends to rise.

We are also interested in understanding other facts about labor policies in OECD countries. In particular, our model explains why some countries spend significant resources on both active and passive labor programs while others do not. Consistent with this observation is the fact that high spending countries appear to have better records on income redistribution than low spending countries.

The paper is organized as follows. In section 2, we introduce a simple directed search model with a government that wish to maximize a social welfare function using a number of policy instruments. We begin our analysis by investigating the optimal policy if the government is constrained to use non-time varying policies. We then show that the social problem of low productivity disadvantaged workers leads to a rising active benefit during extended periods of unemployment. Next, we show that the social problem of inefficient home producers leads to a falling passive benefit over the course of unemployment spell. The final section of this paper offers some concluding remarks.

## 2 The model

The workforce consists of a total population of  $N$  infinitely lived workers. The workers are risk neutral with a subjective rate of time preference equal to  $\beta$ . Workers are distinguished in their ability  $i \in \{A, D\}$  to acquire and use employment related skills.

Workers can choose to train,  $h = 1$ , or not,  $h = 0$ . For a worker of type  $i$ , let  $y_h^i$  denote the productivity during employment where  $y_1^i > y_0^i$ . Let the cost of training be  $c_i - g$  units of output per period, where  $g$  is a government subsidy to training. The training decision is modelled as a decision to pursue a career that requires a constant investment in skills (as in any balanced growth path). Therefore, higher productivity is achieved only if the worker pays this cost each period regardless of employment status.

A worker that is unemployed enjoys a psychic benefit to unemployment in addition to the amount consumed. Let  $b_j$  denote this benefit and assume that  $b_{\hat{A}} > b_{\hat{D}}$ .

## 2.1 Search and coordination friction

It is associated with frictions to get workers and jobs coordinated. Firms have free entry and open job vacancies with a resource cost  $k$  per vacancy. The job vacancies are directed towards specific types of labour, search is directed, and each group of workers, distinguished by type and training investment, are in distinct submarkets with independently determined quantities of vacancies. Matching in each submarket is random. Therefore, if  $v$  job vacancies are opened, a job searcher in this submarket is approached by a firm with probability

$$p^i = 1 - e^{-\phi_h^i}, i = A, D,$$

where  $\phi_h^i = v_h^i/s_h^i$ ,  $i = A, D$  is labour market tightness, that is, the ratio of  $v^i$  vacancies to  $s_h^i$  job searchers in the submarket. We assume that all job matches are destroyed with a common exogenous probability,  $\delta$ .

## 2.2 Wage determination

Let  $\Lambda^i$  denote the present value of a match between a worker of human capital level  $h^i$  and a job vacancy. This present value is given by

$$\Lambda_h^i = \frac{y_h^i - (c - g)h^i + r\delta(V_h^i + \Pi_h^i)}{1 - r(1 - \delta)},$$

where  $V^i$  is the present value of a job searcher and  $\Pi^i$  is the expected profit of an unmatched job. Wages are determined by a simple labour auction market (ref: Julien, Kennes and King 2000).<sup>1</sup> Thus the present value  $V^i$  of a job searcher is given by

$$V_h^i = \max\{V^i(u_h^i) + \lambda^i(\phi_h^i)(\Lambda_h^i - V_h^i(u_h^i)), 0\},$$

where  $\lambda_h^i(\phi_h^i) \equiv 1 - e^{-\phi_h^i} - \phi_h^i e^{-\phi_h^i}$  is the probability the worker has multiple offers and  $V^i(u^i)$  is the value of an unemployed worker. The value  $V^i$  is the ‘reserve wage’ of each labour auction. The equilibrium present value of a job vacancy is given by

$$\Pi_h^i = \max\{-k + e^{-\phi_h^i}(\Lambda_h^i - V_h^i(u_h^i)), 0\}, \quad (1)$$

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<sup>1</sup>Specifically, the auction implies that the entire surplus of a match goes to the firm if the worker is matched with only one firm, and the entire surplus goes to the worker if (s)he is matched with two or more firms. An unmatched agent gets zero.

where the free entry of job vacancies ensures that  $\Pi^i = 0$ . A worker that leaves employment by a dislocation shock is a job searcher next period. The value of such a worker is given by

$$V_h^i(u_h^i) = \max\{b - (c - g)h^i + rV_h^i, 0\}, \quad (2)$$

where  $b$  denotes unemployment insurance benefits.

All workers can choose to either train or not. The worker's choice of human capital maximizes the return to a worker that enters the workforce unemployed. Thus

$$h^i = \begin{cases} 1 & V_1^i \geq V_0^i \\ 0 & \text{otherwise} \end{cases}. \quad (3)$$

The values  $V_1^i$  and  $V_0^i$  are each determined by equation (3) for the appropriate value of  $h$ .

### 2.3 The government's problem

Let  $W^A$  and  $W^D$  denote the average per period income of advantaged and disadvantaged workers, respectively. Social welfare is determined by a social welfare function, which takes as its arguments,  $W^A, W^D$  and  $\eta$ . In order to be able to incorporate distributional issues, we assume that the social welfare function has the following functional form:

$$Y = \gamma(\eta W^A + (1 - \eta)W^D) + (1 - \gamma) \min\{W^A, W^D\}, \quad 0 \leq \gamma \leq 1,$$

which is a weighted average of a Benthamite sum of utilities social welfare function and a Rawlsian social welfare function.

The government chooses transfers to unemployed and employed workers,  $b$  and  $g$ , to maximize social welfare. The unemployment insurance,  $b$ , is a passive benefit, because the worker has complete freedom on how it is spent. However, a general training subsidy,  $g$ , is an active measure in a sense, because it has to be used on a specific activity, namely human capital investment. Both transfers are paid by a lump sum tax,  $t$ . The government balances its budget by setting

$$t = \eta(bu^A + h^A g) + (1 - \eta)(bu^D + h^D g), \quad (4)$$

where  $u^A$  and  $u^D$  indicate the unemployment rates of advantaged and disadvantaged workers and  $h^A$  and  $h^D$  are their respective human capital choices.

### 2.4 Equilibrium unemployment

We can now derive the equilibrium of the model. First we derive the unemployment rate. Consider the gross labour market flows for a group of workers

that have market tightness given by  $\phi$ . The fraction of workers employed in a period is given by

$$q_t = z_t + p(\phi)s_t,$$

where  $z_t$  is the fraction of all workers that are employed because they did not lose their jobs last period and  $p(\phi)s_t$  is the fraction of all workers that are employed because of a successful job search this period. At the end of each period each worker becomes unemployed with probability  $\delta$ . Thus

$$z_t = (1 - \delta)q_{t-1}.$$

The flows in and out of employment imply that the steady state fraction of workers engaged in job search in each period,  $s_h^i$ , is given by

$$s_h^i = \frac{\delta}{\delta + (1 - \delta)p(\phi_h^i)}. \quad (5)$$

Job searchers that do not find a job are unemployed. Thus the unemployment rate is given by

$$u_h^i = (1 - p(\phi_h^i))s_h^i = e^{-\phi_h^i} s_h^i. \quad (6)$$

## 2.5 Job search and human capital

The per period income of a worker investing in human capital can be written as follows

$$W_h^i = G_h^i - h(c - g) - t, \quad i \in \{A, D\}, h \in \{0, 1\}, \quad (7)$$

where  $G_h^i$  is the worker's labour market income as a function of their training decision.

For each type of worker,  $i \in \{A, D\}$ , the training decision is determined by the difference between the cost of training,  $c - g$ , and the benefit of training,  $G_1^i - G_0^i$ . Thus the optimal value of  $h$  for each group of workers is given by

$$h = \begin{cases} 1 & G_1^i - G_0^i \geq c - g. \\ 0 & \text{otherwise} \end{cases}. \quad (8)$$

Given that a competitive search equilibrium model matches jobs and labour constrained efficiently, the equilibrium is simple to derive if we assume that the discount factor approaches unity (see Appendix 2). In particular, for each type of worker, equilibrium market tightness,  $\phi_h^i \equiv v_h^i/s_h^i$ , is that which maximizes steady state output net of recruiting costs. Thus the workers' labour market income is given by

$$G_h^i = \max_{\phi_h^i} \{y_h^i(1 - u_h^i) + bu_h^i - kv_h^i\}, i = A, D, \quad (9)$$

where  $w_h^i = e^{-\phi_h^i} s_h^i$  and  $s_h^i = \delta / (\delta + (1 - \delta)(1 - e^{-\phi_h^i}))$ . Firms earn zero profits. Thus all income net of the cost of vacancies goes to workers. In competitive search equilibrium, this income is maximized and the solution is

$$k = \frac{y_h^i - b}{1 - (1 - \delta)(1 - \lambda(\phi_h^{i*}))} e^{-\phi_h^{i*}}, \quad (10)$$

where  $\phi_h^{i*}$  is the value of  $\phi_h^i$  that maximizes (9).

In the following we characterise the optimal labour market policy, in terms of optimal values of unemployment insurance  $b$  and a general training subsidy  $g$ . In the following section we examine how the optimal values of  $b$  and  $g$  varies with productivity levels of advantages and disadvantaged workers.

### 3 The case for laissez faire policy

In this section, we characterize the equilibrium of the model allowing the government a general training subsidy,  $g$ , in addition to its passive subsidy. The problem is solved sequentially. We use the competitive equilibrium allocation of jobs and skills derived above given a set of government transfers. We can then solve for the optimal government transfers while taking the decentralized (optimal) decision rules for jobs and human capital as given.

The government executes the transfers  $b, g$  and seeks to maximize the social welfare function, hence the government

$$\max_{b, g} Y = \gamma(\eta W^A + (1 - \eta)W^D) + (1 - \gamma) \min\{W^A, W^D\}, \quad 0 \leq \gamma \leq 1, \quad (11)$$

such that (i) the government budget is balanced, equation (4) is satisfied (ii) a participation constraint (PC) that all workers prefer participation to benefits

$$W_{h^A}^A, W_{h^D}^D \geq b \quad (12)$$

and (iii) the determination of  $W_{h^A}^A$  and  $W_{h^D}^D$  is given by the equilibrium outcome of the decentralized economy, which is described above.

We first establishes two results concerning the parameter,  $\gamma$ , where  $1 - \gamma$  indicates the governments' social preferences. The solution to the government's problem yields the following propositions about optimal labour market policy. Suppose that the government is only interested in wealth maximization, that is  $\gamma = 1$ . In this case, we have the following result.

**Proposition 1** *If social welfare is determined solely by aggregate wealth ( $\gamma = 1$ ), optimal government is laissez faire.*

**Proof.** Competitive search equilibrium ensures that any subsidy to one group of workers increases their output plus the subsidy an amount less than the cost of the subsidy. ■

If market tightness and human capital decisions are constrained efficient given the search frictions, a wealth maximizing government never gives subsidies that would distort these optimal decisions. Subsidies are only possible if the government evaluates a unit of income spent by a disadvantaged worker differently than a unit of income spent by an advantaged worker, that is, if  $\gamma < 1$ . Still, a training subsidy that leads to the adoption of training by both advantaged and disadvantaged workers is not optimal. This is shown by the following proposition.

**Proposition 2** *Assume  $\gamma < 1$ . A rational government never subsidizes the training of both advantaged and disadvantaged workers.*

**Proof.** If all agents adopt training, the cost of the subsidy is completely born by each group. Therefore, competitive search ensures that the optimal subsidy is zero for both groups of workers. ■

The direct implication of proposition (1) and (2) is that an optimal training subsidy must always exclude some workers, and it needs to be the advantaged workers that do not get subsidized training as the governments equity concern is the only possible motive for considering training. As we assume that the government cannot condition transfers on a particular workers type, the government will have to rely on self selection.

As  $g$  has no direct impact on labour market tightness we have the following results.

**Proposition 3** *Labour market tightness,  $\phi_h^{i*}$ , is increasing in the productivity level  $y_h^i$ , hence  $\phi_1^A > \phi_0^A > \phi_1^D > \phi_0^D$ .*

**Proof.**  $\phi_h^{i*}$  is positive if  $y_h^i - b > k$ . Likewise the right-hand side of equation 10 is monotonically decreasing in  $\phi_h^{i*}$  and equal to zero if  $\phi_h^{i*}$  is large. Therefore, there exists a unique equilibrium value of  $\phi_h^{i*}$  for each  $y_h^i$ . Hence for each  $b$  then  $\phi_h^i$  increases in  $y_h^i$ . By the maximisation problem (11),  $b$  serves to modify but not circumvent the difference inbetween advantaged and disadvantaged workers. ■

**Proposition 4** *Unemployment is decreasing in productivity such that unemployment is higher of disadvantaged trained workers than of advantaged untrained workers, which again is higher than unemployment of advantaged trained workers,  $u_0^D > u_1^D > u_0^A > u_1^A$ .*

**Proof.** This follows from proposition (3) and equation (6) as unemployment decreases in labour market tightness. ■



## 4 Complementarities between active and passive spending

Suppose that the government seeks to direct the training subsidy to the disadvantaged workers. This objective is met if the following two incentive compatibility constraints (ICCs) are obeyed:  $W_1^D \geq W_0^D$  and  $W_0^A \geq W_1^A$ . That is, the disadvantaged workers take up subsidized training while the advantaged workers do not. More explicit, the constraints are,

$$G_1^D - (c - g) \geq G_0^D \quad (\text{ICC(1)})$$

$$G_0^A \geq G_1^A - (c - g). \quad (\text{ICC(2)})$$

The second incentive compatibility constraint implies that the maximum active subsidy to disadvantaged workers is given by

$$g_{\max} = c - (G_1^A - G_0^A).$$

The behaviour of  $g_{\max}$  is closely related to the amount spent on passive subsidies as stated by the following proposition

**Proposition 5** *The maximum incentive compatible training subsidy for disadvantaged workers,  $g_{\max}$ , increases as the passive subsidy,  $b$ , increases.*

**Proof.** Comparative statics on equation (9) give  $\partial (G_1^A - G_0^A) / \partial b < 0$

■

The first ICC defines the minimum subsidy required to make disadvantaged workers train,

$$g_{\min} = c - (G_1^D - G_0^D).$$

Note that a government will subsidize a training programme only if

$$g_{\min} \leq g_{\max}.$$

This inequality is satisfied only if the marginal increase in labour productivity is greater for disadvantaged workers than advantaged workers.

**Proposition 6** *If the government assigns a weight to equity ( $\gamma < 1$ ) and disadvantaged workers are a sufficiently small part of the population ( $\eta$  is close to one), then the constraint optimal training subsidy is  $g_{\max}$ .*

**Proof.** If  $\eta$  is large, any training subsidy given by the government that does not lead to training by advantaged workers has virtually no effect on the level of taxation. In this case, the training subsidy can be treated purely as a reduction in training costs for the disadvantaged. This subsidy raises social welfare by an amount bounded away from zero if  $\gamma < 1$  with the

welfare change of advantaged workers going to zero as  $\eta$  approaching unity.

■

If disadvantaged workers are a large portion of the population, the optimal training subsidy is not necessarily  $g_{\max}$ . In this case, the result that the efficiency losses of training are small is not strictly valid. For example, if disadvantaged make up the entire population, the optimal general training subsidy is zero.

With a heterogenous population there is a case for a policy subsidizing training but it is halted by the fact that the government cannot discriminate between advantaged and disadvantaged. Although it is possible for the government to sort workers by incentive compatible self-selection schemes, this is still not providing a strong case for a training subsidy. Passive transfers are still the most efficient way of reducing income inequality in this case as we will illustrate below.

Here we will also show that the picture changes dramatically if the government can use an extra piece of information like, for instance, the individual workers unemployment risk. Then all of a sudden, training subsidies become an efficient tool in providing equity. As the disadvantaged workers face higher unemployment risk and thus are more likely to experience long-term unemployment, all the information the government needs for implementation is the duration of any unemployed workers current unemployment spell and then condition the training subsidies on the spell length.

There is, however, a complication to the use of unemployment experience as a screening criteria. Under such a policy it becomes an issue for the advantaged workers to try mimicking the disadvantaged workers in order to get subsidized training. When the training subsidy is offered unconditional this is of course not an issue.

The government needs to make sure that advantaged workers do not prefer subsidized training and long unemployment spells rather than no training subsidy and short unemployment spells. The government does not need to be concerned about the incentives of the disadvantaged workers as they simply cannot get re-employed fast enough to mimic the advantaged workers and neither would they gain anything from conducting such a behaviour.

## **5 Why active benefits are higher for the long-term unemployed**

Unemployment insurance is targeted to unemployed workers. Until now, we have assumed that training of unemployed workers, the idea behind active labour market programmes are not targeted to any particular group of unemployed workers, only to the unemployed workers in general. We then showed that it was optimal to set unemployment insurance and the training subsidy so as to prevent the already relatively advantaged workers from

acquiring training.

The type of active programmes that many countries have implemented are directed in particular, at the long-term unemployed workers. Hence, this corresponds in our model to a general targeting of the training subsidy to the group of workers with the highest unemployment rate, that is the disadvantaged workers. However, it may not be possible to distinguish the two worker types when offering the training subsidy to unemployed workers. Therefore, the government has to take into account that advantaged workers may find it optimal to mimic the disadvantaged workers when the government constructs the optimal policy.

When training subsidies are targeted on the unemployed workers with the highest unemployment rate, the government has to take into account that advantaged workers may not signal their true type to the government (disadvantaged workers cannot mimic the advantaged workers). Let  $W_h^A(u')$  be the average income to an advantaged worker with training  $h$  who has chosen unemployment  $u'$ . Now the government needs to make sure that the advantaged workers do not want to be burdened with the unemployment rate of trained disadvantaged workers  $u_1^D$  just to get the training subsidy, but rather prefer the unemployment rate of her own type  $u_0^A$ , that is,  $W_0^A(u_0^A) \geq W_1^A(u_1^D)$ . More explicitly, this ICC is  $y_0^A(1 - u_0^A) + bu_0^A - kv_0^A \geq y_1^A(1 - u_1^D) + bu_1^D - kv_1^D - (c - g)$ , where  $v_1^D$  is the equilibrium vacancy in the submarket for trained disadvantaged workers. Recall that  $y_0^A(1 - u_0^A) + bu_0^A - kv_0^A = G_0^A(u_0^A)$ . Thus, written in a form compatible with the ICC's of the previous section, we have,

$$G_0^A(u_0^A) \geq G_1^A(u_1^D) - (c - g). \quad (\text{ICC}(2'))$$

This constraint is more slack than the one needed in the previous section, which was  $W_0^A(u_0^A) \geq W_1^A(u_1^A)$ . This is so, because  $W_1^A(u_1^A) > W_1^A(u_1^D)$ , which follows from  $u_1^A < u_1^D$  corresponding to  $\phi_1^A > \phi_1^D$ , as  $\phi_h^i$  increases in productivity. Hence if advantaged workers should mimic disadvantaged workers, this corresponds to that they seem to have a lower productivity and thereby experience a higher unemployment rate as fewer vacancies are supplied.

Before we introduce the new ICC, (ICC(2')), into our model we will make an important simplification in order to facilitate the evaluating of a government training subsidies, which are targeted at workers who have a higher risk of unemployment. Suppose that  $b$  is constant and that  $g_j$  is given by 0 if individual  $j$  is not in a training programme and  $g$  if  $j$  is being activated, that is, in a training programme. Note that this assumption will make the active transfers dependent on the equilibrium unemployment rate. We approximate the non-linear relationship between (i) the unemployment rate of a particular type of workers and (ii) the average amount of active training subsidies paid out to such workers by the following simple step-wise

function

$$g(u) = \begin{cases} g & \text{if } u^i \geq u^* \\ 0 & \text{if otherwise} \end{cases}$$

This is only a crude representation of training subsidy that is conditioned on a sufficient unemployment duration. However, it should capture, to a close approximation, the essential non-linearity between benefit provision and the equilibrium unemployment rate of each group when benefits are determined by unemployment duration. The per period income of a worker investing in human capital is still given by equation (7) where now  $g$  is a function of unemployment. Hence, the model is unchanged except that the active transfer is paid only if the worker's type observes a sufficiently high unemployment rate. The equilibrium supply of jobs and human capital is approximated by the following static welfare optimization problem. The steady state welfare per worker per period of type  $i \in \{A, D\}$  is given by

$$G_h^i(u^*) = \max_{\substack{h \in \{0, 1\} \\ \phi_h^i \geq 0}} \begin{cases} y_h^i(1 - u_h^i) + bu_h^i - (c - g)h - kv_h^i & \text{if } u_h^i \geq u^* \\ y_h^i(1 - u_h^i) + bu_h^i - ch - kv_h^i & \text{otherwise} \end{cases}$$

Maximizing  $G^i(u^*)$  gives a first order condition which still is given by equation (10). The equilibrium training decision is given by

$$h = \begin{cases} 1 & \text{if } u_1^i \geq u^* \text{ and } G_1^i(u^*) - G_0^i \geq c - g, \\ 0 & \text{if } u_1^i \geq u^* \text{ and } G_1^i(u^*) - G_0^i < c - g, \\ 1 & \text{if } u_1^i < u^* \text{ and } G_1^i - G_0^i \geq c, \\ 0 & \text{if } u_1^i < u^* \text{ and } G_1^i - G_0^i < c. \end{cases}$$

The government can execute transfers  $b, g$ . The government seeks to maximize the social welfare function

$$Y' = \max_{b, g, u^*} \gamma(\eta W^A + (1 - \eta)W^D) + (1 - \gamma) \min\{W^A, W^D\}$$

such that the government balances its budget, that is fulfils, equation (4). The constraints on this maximization problem are the fact that  $b, g$  determine  $u^A, u^D, h^A, h^D$  by the equilibrium supply of jobs and human capital in the previous subsection. Welfare is always higher than in the basic model without targeted training, because activation gives the government an extra instrument to solve the incentive compatibility problem. In particular, the following policy menu is better.

1. For each value of the passive subsidy,  $a$ , compute the equilibrium unemployment rate,  $u^*$ , of trained disadvantaged workers.
2. Calculate the payoffs of (i) untrained advantaged workers and (ii) unsubsidized trained advantaged workers when the unemployment rate of advantaged workers is  $u^*$ .

3. Set the subsidy of disadvantaged workers equal to the difference of (i) and (ii) in 2.

The reason this scheme outperforms the scheme in the previous section is that the payoff of the unsubsidized constrained trained advantaged workers in 2 is lower than the payoff of unconstrained trained advantaged workers. In particular, for a given passive subsidy, the incentive compatibility constraint of active subsidies is weakened if they are targeted to the long term unemployed. The fact that advantaged workers must mimic the unemployment rate of disadvantaged workers (experience a large duration in unemployment), implies that a larger active subsidy can be paid to disadvantaged workers. Therefore, an incentive compatible training subsidy can be paid to disadvantaged workers even if the training yields significant productivity benefits for advantaged workers, who will otherwise go untrained if training is not subsidized.

In other words, we can show that, *ceteris paribus*, the impact on the training subsidy,  $g$ , from increasing the unemployment insurance,  $b$ , is larger in the targeted case than in the non-targeted case. All proofs are given in the appendix.

**Proposition 7** *The impact on the training subsidy,  $g$ , from increasing the unemployment insurance,  $b$ , is larger in the targeted case than in the non-targeted case, that is,  $\partial g/\partial a|_{\text{targeted case}} > \partial g/\partial a|_{\text{non-targeted case}}$ .*

## 6 Structural change

We now examine the optimal policy response to skill biased and general productivity shocks. First, we analyse skill biased productivity shocks affect optimal policy and unemployment rates when policy is not targeted. Then we consider the impact of these shocks when policy is targeted.

### 6.1 Skill biased productivity shocks with non-targeted policy

The following proposition holds unless unemployment of disadvantaged workers is very high.

**Proposition 8** *When disadvantaged trained workers face a positive productivity shock,  $y_1^D$  increases, unemployment of all workers falls, the optimal unemployment insurance and the optimal training subsidy fall. Unemployment distribution becomes less unequal. Unemployment insurance falls more than the training subsidy*

When disadvantaged workers face a positive productivity shock this has a direct positive effect on labour market tightness and thereby their transition

rate which directly reduces unemployment for this worker type. Furthermore, they are relatively better off compared to their advantaged working fellows. As the disadvantaged workers are the workers with the highest unemployment rates, them being in a better position implies that it is optimal to reduce unemployment insurance. The lower unemployment insurance implies that labour market tightness of both type of workers increase and thereby is unemployment reduced. The negative impact on disadvantage workers' unemployment rate is higher than the negative impact of advantaged workers' unemployment rate. Hence, distribution, in terms of unemployment, become less unequal. It is then optimal to reduce the training subsidy. Furthermore, the impact on unemployment insurance is larger than the impact on training as the latter impact is weighted by the difference inbetween the unemployment rates of the advantaged workers.

Next we consider the impact on the economy from a productivity shock for untrained advantaged workers

**Proposition 9** *When advantaged untrained workers face a positive productivity shock,  $y_0^A$  increases, unemployment for disadvantaged workers increases, unemployment of advantaged workers falls, the optimal unemployment insurance increases and the optimal training subsidy increases. Unemployment distribution becomes more unequal. Unemployment insurance most likely increases less than the training subsidy.*

When advantaged trained workers experience a productivity increase, then labour market tightness facing those workers and thereby their transition rate increases. This in turn reduces their unemployment rate. Disadvantaged workers are therefore relatively worse off which makes it optimal to increase unemployment insurance. This impact lowers the transition rate of disadvantaged workers and therefore increases unemployment for this worker type and inequality in terms of unemployment distribution worsens. As the transition rate of untrained advantaged workers increases, it is relatively more attractive to remain untrained. Hence, it is possible to increase the training subsidy and still deter advantaged workers from acquiring training. Therefore, the training subsidy most likely increases more than the unemployment insurance.

Finally, we consider the impact of a productivity increase for trained advantaged workers. In equilibrium unemployment insurance and the training subsidy are set such that advantaged workers do not train. However, the productivity of a trained advantaged worker raises the expected return and thereby the incentives to acquire skills for advantaged workers. Therefore, unemployment rates and optimal unemployment insurance and training subsidies are affected.

**Proposition 10** *When advantaged trained workers face a positive productivity shock,  $y_1^A$  increases, unemployment for all workers increases, the op-*

*timal unemployment insurance increases and the optimal training subsidy falls where the difference inbetween the two increases. Unemployment distribution becomes more unequal.*

When the productivity of advantaged trained workers increases, their transition rate would (if unemployment insurance and training subsidy are set so as to eliminate this market) increase and the government has to discourage advantaged workers from taking up training. They do so by increasing unemployment insurance and reducing the training subsidy. The higher unemployment insurance modifies the needed increase in  $g$ . However, a higher unemployment subsidy puts pressure on wages which reduces labour market tightness and thereby rises unemployment for both worker types. The impact on unemployment for disadvantaged workers is higher than the impact on unemployment of advantaged workers, whereby distribution in terms of unemployment rates becomes more unequal.

## 6.2 General shocks when policy is non-targeted

When all workers face a positive productivity shock the impact is as follows.

**Proposition 11** *When all workers are subject to a positive productivity shock, unemployment for all workers fall, the impact on the optimal unemployment insurance is ambiguous and the optimal training subsidy falls whereby the difference inbetween the two most likely increases.*

When the productivity of all workers increases, all workers' transition rates increase and thereby unemployment of all workers fall. The impact on unemployment insurance is ambiguous as disadvantaged trained workers are better off due to the higher  $y_1^D$  tending to reduce  $b$  and higher  $y^A$  tends to increase  $b$ . Considering the training subsidy there is again several impacts. A higher  $y^A$  reduces the optimal  $g$  as the direct impact from  $y_1^A$  which increases the value of acquiring trained as an advantaged workers dominates the positive impact through  $b$  as employment is higher than unemployment. The tendency to reduce the optimal unemployment insurance caused by a higher  $y_1^D$  tends to reduce the training subsidy which amplifies the negative impact on the optimal value of  $g$ .

## 6.3 Skill-biased Shocks when policy is targeted

When the productivity of disadvantaged trained workers increases, we observe the following.

**Proposition 12** *When disadvantaged trained workers face a positive productivity shock, unemployment of all workers falls, the optimal unemployment insurance and the optimal training subsidy decrease. Unemployment distribution becomes less unequal.*

When disadvantaged workers experience a positive productivity shock, their transition rate increases and their unemployment rate therefore falls. The higher productivity together with the higher transition rate reduces unemployment insurance. This happens as disadvantaged workers are then relatively better off. Lower unemployment insurance, in turn, raises the transition rate of advantaged untrained workers which lowers their unemployment rate. However, there is small modification of the reduction in unemployment insurance as the higher transition rate for advantaged untrained workers also tend to increase the unemployment insurance a bit as those workers, the workers being relatively best off, are even better off. The impact on the training subsidy is negative covering two divergent effects. First, the higher transition rate of disadvantaged trained workers tends to reduce the need for training subsidies. Second, the lower unemployment insurance 'punishes' the group with the highest unemployment rate, the disadvantages workers, most severely and therefore this tends to increase the optimal training subsidy. This latter effect dominates such that the optimal training subsidy is reduced. We can show that the negative impact on unemployment of the disadvantaged group of workers is higher than the negative impact on unemployment facing advantaged workers which reduces inequality in terms of unemployment rates.

**Proposition 13** *When advantaged untrained workers face a positive productivity shock, unemployment for disadvantaged and advantaged trained workers increase, unemployment of advantaged untrained workers falls, the optimal unemployment insurance and the optimal training subsidy increases. Unemployment distribution becomes more unequal.*

When advantaged untrained workers become more productive, more vacancies are supplied and thereby their transition rate increases. Consequently unemployment for untrained advantaged workers falls. The higher transition rate induces unemployment insurance to increase, to compensate the disadvantaged workers, as the advantaged workers, the workers being relatively best off, are even better off. The higher unemployment insurance imply that the transition rate of all trained workers decrease and thereby unemployment of these workers increases which then increases inequality measured in unemployment rates. There are three impacts on the training subsidy. The higher productivity directly tends to increase the training subsidy as being untrained becomes relatively more attractive. On the other hand, higher unemployment insurance makes it relatively more attractive for the advantaged to mimic the disadvantaged workers which tends to reduce the optimal training subsidy. Finally, the negative impact on disadvantaged workers' transition rate tends to increase the need for a higher subsidy. The positive impacts dominate whereby the optimal training subsidy increases.

**Proposition 14** *When advantaged trained workers face a positive produc-*



*tivity shock, unemployment for disadvantaged trained and advantaged untrained workers increase, the impact on unemployment of advantaged trained workers is ambiguous, the optimal unemployment insurance increases and the impact on optimal training subsidy is ambiguous. Unemployment becomes more unequal.*

Productivity increases for trained advantaged workers directly tend to increase their transition rate. However, unemployment insurance has to increase substantially to modify the relatively worse position of the disadvantaged trained and advantaged untrained workers. This will tend to reduce all transition rates. The overall impact on advantaged trained workers' transition rate is therefore ambiguous, but it is likely to increase. Concerning the training subsidy there are several impacts. There is a direct negative impact as the value of being trained increases. There is another negative impact through the increase in unemployment insurance as unemployment of disadvantaged trained workers is higher than unemployment of advantaged untrained workers. Both impacts are negative as they both give incentives to the advantaged workers to mimic the disadvantaged workers. There is also a positive impact on  $g$  as the reduction of disadvantaged trained workers transition rate reducing the attractiveness of training. The overall impact is ambiguous. Finally, unemployment of disadvantaged workers increases more than unemployment of advantaged workers, whereby the difference inbetween the two groups become even larger than before the shock.

#### 6.4 General shocks when policy is targeted

Finally, we consider the case where both disadvantaged trained workers and advantaged untrained workers experience a positive shock.

**Proposition 15** *When both disadvantaged trained workers and advantaged untrained workers experience a positive productivity shock, unemployment of all workers decrease, optimal unemployment insurance and training subsidy fall. Unemployment distribution becomes more equal.*

When the productivity of all workers increases, all workers' transition rates increase and thereby unemployment of all workers fall. The impact on unemployment insurance is unambiguously negative as the impact from a higher  $y_1^D$  dominates the impact from a higher  $y^A$ . Hence, as the policy is targeted, there is less need to increase  $b$ . Considering the training subsidy we observe the following. A higher  $y^A$  reduces the optimal  $g$  as the direct negative impact from  $y_1^A$  dominates the positive impact through  $b$  as employment is higher than unemployment.

#### 6.5 Discussion

All the impacts are summerised in the following table for convenience:

Non-targeted case, effects of productivity increases

increase in/effect on	$b$	$u_1^D$	$u_0^A$	$g$	$u_1^D/u_0^A$
$y_1^D$	-	-	-	-	-
$y_1^A$	+	+	+	-	+
$y_0^A$	+	+	-	+	+
$y_1^D, y_0^A, y_1^A$	?	-	-	-	-

Targeted case, effects of productivity increases

increase in/effect on	$b$	$u_1^D$	$u_0^A$	$g$	$u_1^D/u_0^A$
$y_1^D$	-	-	-	-	-
$y_1^A$	+	+	+	?	+
$y_0^A$	+	+	-	+	+
$y_1^D, y_0^A, y_1^A$	-	-	-	-	-

## 7 Conclusions

The massive and persistent emphasis put on activation and training of unemployed individuals in developed countries in general and in big-welfare-state countries in particular is a puzzle, because it has been difficult to identify positive effects - individual as well as macro-level effects - from the often huge spending on these programmes. This is surveyed by Martin (2000) Heckman, Lalonde, Smith (1999) and OECD (2003). So either politics are irrational or the profession has not been looking for effects in the right places. For instance, even if there are no effects at the mean for any of the programmes, there could be an effect at the macro level - e.g., less inequality - if it is the more disadvantaged workers who gain productivity from the programmes. This is conceivable as Martin (2000), Heckman, Lalonde, and Smith (1999) and OECD (2004) also conclude that some programmes have very significant effects for some groups of individuals. I OECD (2003) it is also suggested that activation policies have reduced poverty rates in some European countries.

Suppose income equality is a main objective for some countries along side with high average income. Could it then be that active programmes are favored by some countries because such programmes reduce inequality efficiently when used together with traditional passive programmes like UI benefits? This is the question that we have been discussing in this paper and the answer is in the affirmative. If income equality is a sufficiently strong objective to a government then it might well be rational to implement active training programmes for the long term unemployed together with passive benefit programmes like UI. This combination is far more effective than the combination of UI benefits and a general education subsidy. At the principal level, this could vindicate high spending on activation by countries with strong taste for equity. Our results also suggests that high passive and active spending goes hand in hand. Both these phenomenon can be observed in the data for the OECD countries.

These results are developed in a model with heterogeneous workers, human capital investment, and unemployment. The model is "pure" in the sense that 'laissez faire' is efficient: the privately chosen level of training is efficient and even though disadvantaged workers of low skills are the more unemployed ones, unemployment is efficient and reflects search and matching frictions. There are no externalities to justify training subsidies. We have also deliberately disregarded the traditional insurance aspect of passive policies by letting agents be risk neutral in our model. So it is not the usual missing insurance market that implies government spending on UI in optimum. The redistributive functioning of UI in this model with heterogeneous unemployment risk is enough to have passive transfers to unemployed entering the optimal policy packaged (of a government that maximizes a social welfare function that puts weight on both equity and income efficiency).

Furthermore, not only can we explain the joint use of passive and active subsidies, the model also shed light on the big variation in the labour market policies of OECD countries. Our results suggest that much of the variation in policy can be explained by different social objectives rather than by inefficient policy or differences in technology and human capital.

The analysis of this paper can be improved in two directions. First, the empirical assessment of the theory is only suggestive. An involved empirical study is needed to isolate the specific causes of policy variation across OECD countries. Second, the theory of the model could also be extended to incorporate a more detailed description of active labour market programmes. For example, different elements of active programmes, including different subsidies for training employed and unemployed workers, could be studied. We leave these improvements for further research.

## 8 Appendix

This appendix establishes results used in the body of the paper.

### 8.1 Solution Equivalence

This appendix shows that the decentralized economy is equivalent to the solution of a simple static maximization problem if the discount factor approaches unity. (1) The decentralized asset equations are given by

$$\begin{aligned}\Lambda &= \frac{y + \beta\delta(V + \Pi)}{1 - \beta(1 - \delta)} \\ \Pi &= 0 \\ V &= V(u) + (1 - e^{-\phi} - \phi e^{-\phi})(\Lambda - V(u)) \\ \Pi &= -k + e^{-\phi}(\Lambda - V(u)), \\ V(u) &= a + \beta V\end{aligned}$$

These equations for  $\Lambda$ ,  $V$ ,  $V(u)$ ,  $\Pi$  and  $\phi$  can be rewritten to get a single expression for  $\phi$ .

$$k = ye^{-\phi} + k(1 - \delta)\beta(e^{-\phi} + \phi e^{-\phi})$$

and that in the limit as  $\beta$  approaches 1 we get

$$k = ye^{-\phi} + k(1 - \delta)(e^{-\phi} + \phi e^{-\phi}) \quad (\text{A1})$$

(2) Now consider the simple static problem of maximizing steady state output less recruiting costs. In this case

$$W = \max_{\phi} y(1 - u) + au - kv$$

such that

$$\begin{aligned}\phi &= v/s \\ s &= \frac{\delta}{\delta + (1 - \delta)(1 - e^{-\phi})} \\ u &= s(1 - e^{-\phi})\end{aligned}$$

The solution to this problem is the same as A1.

## 8.2 Relative slopes, $dg/db$ and signs.

The slope,  $dg/db$  in the targeted case is

$$\begin{aligned} \frac{dg}{db}\Big|_{\text{targeted case}} &= (y_1^A - y_1^D) \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\partial \phi_1^D}{\partial y_1^D} + (u_0^A - u_1^D) > \\ & (y_1^A - y_1^D) \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\partial \phi_1^D}{\partial y_1^D} + (u_1^A - u_1^D) \end{aligned}$$

If  $y_1^A = y_1^D$  then  $(y_1^A - y_1^D) \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\partial \phi_1^D}{\partial y_1^D} + (u_1^A - u_1^D) = 0$ . Increasing  $y_1^A$  then gives

$$\frac{\partial (y_1^A - y_1^D) \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\partial \phi_1^D}{\partial y_1^D} + (u_1^A - u_1^D)}{\partial y_1^A} = -\frac{(u_1^A)^2}{1 - (1 - \delta) e^{-\phi_1^A}} \frac{1}{\delta k} + \frac{(u_1^D)^2}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{1}{\delta k} > 0 \quad (13)$$

Then

$$\frac{dg}{db}\Big|_{\text{targeted case}} > 0$$

The slope,  $dg/db$  in the non-targeted case is

$$\frac{dg}{db}\Big|_{\text{non-targeted case}} = u_0^A - u_1^A > 0$$

Relative slopes:

$$\begin{aligned} \frac{dg}{db}\Big|_{\text{targeted case}} &> \frac{dg}{db}\Big|_{\text{non-targeted case}} \text{ iff} \\ (y_1^A - y_1^D) \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\partial \phi_1^D}{\partial y_1^D} + (u_0^A - u_1^D) &> u_0^A - u_1^A \text{ iff} \\ (y_1^A - y_1^D) \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\partial \phi_1^D}{\partial y_1^D} + (u_1^A - u_1^D) &> 0 \end{aligned}$$

which is positive by (13).

## 8.3 Non targeted policy

The equilibrium equations are

$$\begin{aligned}
& \frac{y_1^D - b}{1 - (1 - \delta) e^{-\phi_1^D} (1 + \phi_1^D)} e^{-\phi_1^D} - k = 0 \\
& \frac{y_0^A - b}{1 - (1 - \delta) e^{-\phi_0^A} (1 + \phi_0^A)} e^{-\phi_0^A} - k = 0 \\
& \frac{y_1^A - b}{1 - (1 - \delta) e^{-\phi_1^A} (1 + \phi_1^A)} e^{-\phi_1^A} - k = 0 \\
& \frac{(1 - \gamma) \eta \left( \frac{\delta e^{-\phi_D}}{1 - (1 - \delta) e^{-\phi_D}} - \frac{\delta e^{-\phi_1^A}}{1 - (1 - \delta) e^{-\phi_1^A}} \right)}{\eta \frac{\delta (e^{-\phi_0^A})^2}{(1 - (1 - \delta) e^{-\phi_0^A})^3 k} + (1 - \eta) \frac{\delta (e^{-\phi_1^D})^2}{(1 - (1 - \delta) e^{-\phi_1^D})^3 k}} = b \\
& c + \left( y_0^A \frac{1 - e^{-\phi_0^A}}{1 - (1 - \delta) e^{-\phi_0^A}} + b \frac{\delta e^{-\phi_0^A}}{1 - (1 - \delta) e^{-\phi_0^A}} - k \phi_0^A \frac{\delta}{1 - (1 - \delta) e^{-\phi_0^A}} \right) \\
& - \left( y_1^A \frac{1 - e^{-\phi_1^A}}{1 - (1 - \delta_b) e^{-\phi_1^A}} + b \frac{\delta e^{-\phi_1^A}}{1 - (1 - \delta) e^{-\phi_1^A}} - k \phi_1^A \frac{\delta}{1 - (1 - \delta) e^{-\phi_1^A}} \right) = g
\end{aligned}$$

### 8.3.1 Skill-biased shocks

Differentiating the equilibrium equations with respect to  $\phi_1^D, \phi_0^A, \phi_1^A, b, g, y_1^D, y_0^A$  and  $y_1^A$  gives

$$\begin{aligned}
\frac{d\phi_{D1}}{dy_1^D} &= \frac{\frac{k\delta}{u_0^A} \left( \frac{k\delta}{u_1^A} + f \right) + \frac{k\delta}{u_1^A} \beta}{D_1} > 0, \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy_1^D} < 0 \\
\frac{d\phi_0^A}{dy_1^D} &= -\frac{\frac{k\delta}{u_1^A} m}{D_1} > 0 \text{ if } m < 0, \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy_1^D} < 0 \\
\frac{d\phi_1^A}{dy_1^D} &= \frac{-\frac{k\delta}{u_0^A} m}{D_1} > 0 \text{ if } m < 0, \frac{du_1^A}{d\phi_1^A} \frac{d\phi_1^A}{dy_1^D} < 0 \\
\frac{db}{dy_1^D} &= \frac{\frac{k\delta}{u_0^A} \frac{k\delta}{u_1^A} m}{D_1} < 0 \text{ if } m < 0 \\
\frac{dg}{dy_1^D} &= (u_0^A - u_1^A) \frac{db}{dy_1^D} < 0 \\
\left| \frac{db}{dy_1^D} \right| - \left| \frac{dg}{dy_1^D} \right| &= (1 - u_0^A + u_1^A) \left| \frac{db}{dy_1^D} \right| > 0
\end{aligned}$$

$$\begin{aligned}
\frac{d\phi_1^D}{dy_0^A} &= -\frac{\frac{k\delta}{u_1^A}\beta}{D_1} < 0, \quad \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy_0^A} > 0 \\
\frac{d\phi_0^A}{dy_0^A} &= \frac{\frac{k\delta}{u_1^D}f + \frac{k\delta}{u_1^A}\left(m + \frac{k\delta}{u_1^D}\right)}{D_1} > 0, \quad \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy_0^A} < 0 \\
\frac{d\phi_1^A}{dy_0^A} &= -\frac{\frac{k\delta}{u_1^D}\beta}{D_1} < 0, \quad \frac{du_1^A}{d\phi_1^A} \frac{d\phi_1^A}{dy_0^A} > 0 \\
\frac{db}{dy_0^A} &= \frac{\frac{k\delta}{u_1^D} \frac{k\delta}{u_1^A}\beta}{D_1} > 0, \\
\frac{dg}{dy_0^A} &= 1 - u_0^A + (u_0^A - u_1^A) \frac{db}{dy_0^A} > 0 \\
\frac{db}{dy_0^A} - \frac{dg}{dy_0^A} &= \frac{db}{dy_0^A} (1 - (u_0^A - u_1^A)) - (1 - u_0^A) < 0 \text{ most likely}
\end{aligned}$$

$$\begin{aligned}
\frac{d\phi_1^D}{dy_1^A} &= -\frac{\frac{k\delta}{u_0^A}f}{D_1} < 0, \quad \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy_1^A} > 0 \\
\frac{d\phi_0^A}{dy_1^A} &= -\frac{\frac{k\delta}{u_1^D}f}{D_1} < 0, \quad \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy_1^A} > 0 \\
\frac{d\phi_1^A}{dy_1^A} &= \frac{\frac{k\delta}{u_1^D}\beta + \frac{k\delta}{u_0^A}\left(m + \frac{k\delta}{u_1^D}\right)}{D_1} > 0, \quad \frac{du_1^A}{d\phi_1^A} \frac{d\phi_1^A}{dy_1^A} < 0, \\
\frac{db}{dy_1^A} &= \frac{\frac{k\delta}{u_1^D} \frac{k\delta}{u_0^A}f}{D_1} > 0 \text{ where } 1 > \frac{\frac{k\delta}{u_1^D} \frac{k\delta}{u_0^A}f}{D_1} > 0, \\
\frac{dg}{dy_1^A} &= -\frac{1 - e^{-\phi_{A1}}}{1 - (1 - \delta)e^{-\phi_{A1}}} + \left( \frac{\delta e^{-\phi_{A0}}}{1 - (1 - \delta)e^{-\phi_{A0}}} - \frac{\delta e^{-\phi_{A1}}}{1 - (1 - \delta)e^{-\phi_{A1}}} \right) \frac{db}{dy_1^A} \\
&= -(1 - u_{A0}) + (u_1^A - u_0^A) \left( 1 - \frac{db}{dy_1^A} \right) < 0 \\
\frac{db}{dy_0^A} - \frac{dg}{dy_0^A} &> 0
\end{aligned}$$

### 8.3.2 General Shock

$$\begin{aligned}
\frac{d\phi_1^D}{dy} &= \frac{\frac{k\delta}{u_0^A} \frac{k\delta}{u_1^A}}{D_1} > 0, \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy} < 0 \\
\frac{d\phi_0^A}{dy} &= \frac{\frac{k\delta}{u_1^A} \frac{k\delta}{u_1^D}}{D_1} > 0, \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy} < 0 \\
\frac{d\phi_1^A}{dy} &= \frac{\frac{k\delta}{u_0^A} \frac{k\delta}{u_1^D}}{D_1} > 0, \frac{du_1^A}{d\phi_1^A} \frac{d\phi_1^A}{dy} < 0 \\
\frac{d\phi_1^D}{dy} &> \frac{d\phi_1^A}{dy} > \frac{d\phi_0^A}{dy} \\
\frac{db}{dy} &= \frac{\frac{k\delta}{u_0^A} \frac{k\delta}{u_1^A} m + \frac{k\delta}{u_1^D} \frac{k\delta}{u_1^A} \beta + \frac{k\delta}{u_1^D} \frac{k\delta}{u_0^A} f}{D_1} \\
\frac{dg}{dy} &= (u_0^A - u_1^A) \left( \frac{db}{dy} - 1 \right) = -\frac{(u_0^A - u_1^A)}{D_1} \frac{k\delta}{u_0^A} \frac{k\delta}{u_1^A} \frac{k\delta}{u_1^D} < 0 \\
\frac{db}{dy} - \frac{dg}{dy} &= \frac{db}{dy} - (u_0^A - u_1^A) \left( \frac{db}{dy} - 1 \right)
\end{aligned}$$

where

$$\begin{aligned}
m &= \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{-(1 - \gamma) \eta + b(1 - \eta) (u^D)^2 \frac{2 + (1 - \delta) e^{-\phi_D}}{\delta (1 - (1 - \delta) e^{-\phi_D})^2 k}}{\eta \frac{(u_0^A)^2}{(1 - (1 - \delta) e^{-\phi_{A0}}) \delta k} + (1 - \eta) \frac{(u_1^D)^2}{(1 - (1 - \delta) e^{-\phi_D}) \delta k}} < 0 \text{ unless } u_1^D \text{ is very high,} \\
\beta &= \frac{b \eta (u_0^A)^2 \frac{2 + (1 - \delta) e^{-\phi_0^A}}{(1 - (1 - \delta) e^{-\phi_0^A})^2 k}}{\eta \frac{(u_0^A)^2}{(1 - (1 - \delta) e^{-\phi_{A0}}) \delta k} + (1 - \eta) \frac{(u_1^D)^2}{(1 - (1 - \delta) e^{-\phi_D}) \delta k}} > 0 \\
f &= \frac{(1 - \gamma) \eta \frac{u_1^A}{1 - (1 - \delta) e^{-\phi_1^A}}}{\eta \frac{(u_0^A)^2}{(1 - (1 - \delta) e^{-\phi_{A0}}) \delta k} + (1 - \eta) \frac{(u_1^D)^2}{(1 - (1 - \delta) e^{-\phi_D}) \delta k}} > 0
\end{aligned}$$

For stability we need that

$$\left| \frac{d\phi_1^D}{db} \Big|_{\phi_1^D(b)} \right| > \left| \frac{d\phi_1^D}{db} \Big|_{b(\phi_1^D)} \right| = m + \frac{k\delta}{u_1^D} > 0$$

Then

$$D_1 = \frac{k\delta}{u_0^A} \frac{k\delta}{u_1^A} \left( m + \frac{k\delta}{u_1^D} \right) + \frac{k\delta}{u_1^D} \left( \frac{k\delta}{u_1^A} \beta + \frac{k\delta}{u_0^A} f \right) > 0$$



### 8.3.3 Impact on unemployment distribution

When  $y_1^D$  increases, the negative impact on unemployment of disadvantaged workers is higher than the impact on advantaged workers' unemployment rate:

$$\left| \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy_1^D} \right| > \left| \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy_1^D} \right| \text{ iff}$$

$$\frac{\frac{k\delta}{u_0^A} \left( \frac{k\delta}{u_1^A} + f \right) + \frac{k\delta}{u_1^A} \beta}{D_1} \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} > \frac{-\frac{k\delta}{u_1^A} m}{D_1} \frac{u_0^A}{1 - (1 - \delta) e^{-\phi_0^A}}$$

which is true as given stability is  $\frac{k\delta}{u_1^D} + m > 0$  and  $\frac{k\delta}{u_0^A} > \frac{k\delta}{u_1^D}$ .

When  $y_0^A$  increases the unemployment distribution becomes more unequal as  $u_1^D$  increases and  $u_0^A$  falls.

When  $y_1^A$  increases, the unemployment distribution becomes more unequal:

$$\frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy_1^A} > \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy_1^A} \text{ iff } \frac{\frac{k\delta}{u_0^A} f}{D_1} \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} > \frac{\frac{k\delta}{u_1^D} f}{D_1} \frac{u_0^A}{1 - (1 - \delta) e^{-\phi_0^A}}$$

true as  $u_1^D > u_0^A$ .

When all productivity levels increase then unemployment distribution become more equal:

$$\left| \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy} \right| > \left| \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy} \right| - \text{ iff}$$

$$\frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\frac{k\delta}{u_0^A} \frac{k\delta}{u_1^A}}{D_1} > \frac{u_0^A}{1 - (1 - \delta) e^{-\phi_0^A}} \frac{\frac{k\delta}{u_1^A} \frac{k\delta}{u_1^D}}{D_1}$$

which holds.

## 8.4 Targeted policy

The equilibrium equations are

$$\begin{aligned}
& \frac{y_1^D - b}{1 - (1 - \delta) e^{-\phi_1^D} (1 + \phi_1^D)} e^{-\phi_1^D} - k = 0, \\
& \frac{y_0^A - b}{1 - (1 - \delta) e^{-\phi_0^A} (1 + \phi_0^A)} e^{-\phi_0^A} - k = 0, \\
& \frac{y_1^A - b}{1 - (1 - \delta) e^{-\phi_1^A} (1 + \phi_1^A)} e^{-\phi_1^A} - k = 0, \\
& \frac{\eta(1 - \gamma)(y_1^A - y_1^D) \frac{1}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{(u_1^D)^2}{\delta k}}{\eta \frac{1}{1 - (1 - \delta) e^{-\phi_0^A}} \frac{(u_0^A)^2}{\delta k} + (1 - \eta) \frac{1}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{(u_1^D)^2}{\delta k}} = b, \\
& c + \left( y_0^A \frac{1 - e^{-\phi_0^A}}{1 - (1 - \delta) e^{-\phi_0^A}} + b \frac{\delta e^{-\phi_0^A}}{1 - (1 - \delta) e^{-\phi_0^A}} - k \phi_0^A \frac{\delta}{1 - (1 - \delta) e^{-\phi_0^A}} \right) \\
& - \left( y_1^A \frac{1 - e^{-\phi_1^D}}{1 - (1 - \delta) e^{-\phi_1^D}} + b \frac{\delta e^{-\phi_1^D}}{1 - (1 - \delta) e^{-\phi_1^D}} - k \phi_1^D \frac{\delta}{1 - (1 - \delta) e^{-\phi_1^D}} \right) = g.
\end{aligned}$$

### 8.4.1 Skill biased shocks

Differentiating the equilibrium equations with respect to  $\phi_1^D, \phi_0^A, b, g, y_0^A$  and  $y_1^D$  gives:

$$\begin{aligned}
\frac{d\phi_{D1}}{dy_1^D} &= \frac{\frac{k\delta}{u_0^A} + \rho + \frac{k\delta}{u_0^A} (y_1^A - y_1^D)}{D_2} > 0, \quad \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy_1^D} < 0, \\
\frac{d\phi_0^A}{dy_1^D} &= -\frac{-\tau + \frac{\delta k}{u_1^D} (y_1^A - y_1^D)}{D_2} > 0, \quad \frac{du_0^A}{d\phi_1^D} \frac{d\phi_0^A}{dy_1^D} < 0, \\
\frac{d\phi_1^A}{dy_1^D} &= \frac{u_1^A k\delta - \tau + \frac{\delta k}{u_1^D} (y_1^A - y_1^D)}{k\delta u_0^A D_2} > 0, \quad \frac{du_1^A}{dy_1^D} \frac{d\phi_1^A}{dy_1^D} < 0, \\
\frac{db}{dy_1^D} &= -\frac{k\delta - \tau + \frac{\delta k}{u_1^D} (y_1^A - y_1^D)}{u_0^A D_2} < 0, \\
\frac{dg}{dy_1^D} &= -\frac{(y_1^A - y_1^D) u^D}{1 - (1 - \delta) e^{-\phi^D}} \left( \frac{\frac{\delta k}{u_0^A} \left( 1 + \tau \frac{u_1^D}{\delta k} \right) + \rho}{D_2} \right) + \left( \frac{(y_1^A - y_1^D) u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{u_1^D}{\delta k} + (u_0^A - u_1^D) \right) \\
\left| \frac{db}{dy_1^D} \right| - \left| \frac{dg}{dy_1^D} \right| &= \left| \frac{db}{dy_1^D} \right| \left( 1 - \left( \frac{(y_1^A - y_1^D) u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{u_1^D}{\delta k} + (u_0^A - u_1^D) \right) \right) - \frac{(y_1^A - y_1^D) u^D}{1 - (1 - \delta) e^{-\phi^D}} \left( \frac{\frac{\delta k}{u_0^A} (1 - \tau \frac{u_1^D}{\delta k})}{D_2} \right)
\end{aligned}$$

as  $\left( 1 + \tau \frac{u_1^D}{\delta k} \right) > 0$  by the stability condition.

$$\begin{aligned}
\frac{d\phi_1^D}{dy_0^A} &= -\frac{\rho}{D_2} < 0, \quad \frac{du_1^D}{d\phi_0^A} \frac{d\phi_1^D}{dy_0^A} > 0, \\
\frac{d\phi_0^A}{dy_0^A} &= \frac{\tau + \frac{\delta k}{u_1^D}}{D_2} > 0, \quad \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy_0^A} < 0, \\
\frac{d\phi_1^A}{dy_0^A} &= -\frac{u_1^A \rho \frac{\delta k}{u_1^D}}{k\delta D_2} < 0, \quad \frac{du_1^A}{dy_1^A} \frac{d\phi_1^A}{dy_0^A} > 0, \\
\frac{db}{dy_0^A} &= \frac{\rho \frac{\delta k}{u_1^D}}{D_2} > 0, \\
\frac{dg}{dy_0^A} &= 1 - u_1^D + (u_1^D - u_0^A) \left(1 - \frac{db}{dy_0^A}\right) - \frac{(y_1^A - y_1^D) u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{d\phi_1^D}{dy_0^A} > 0, \\
\frac{db}{dy_0^A} - \frac{dg}{dy_0^A} &= \frac{db}{dy_0^A} - (1 - u_1^D) - (u_1^D - u_0^A) \left(1 - \frac{db}{dy_0^A}\right) + \frac{(y_1^A - y_1^D) u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{d\phi_1^D}{dy_0^A} \\
\\
\frac{d\phi_1^D}{dy_1^A} &= -\frac{k\delta (y_1^A - y_1^D)}{u_0^A D_2} < 0, \quad \frac{du_1^D}{d\phi_1^A} \frac{d\phi_1^D}{dy_1^A} > 0, \\
\frac{d\phi_0^A}{dy_1^A} &= -\frac{k\delta (y_1^A - y_1^D)}{u_1^D D_2} < 0, \quad \frac{du_0^A}{d\phi_1^A} \frac{d\phi_0^A}{dy_1^A} > 0, \\
\frac{d\phi_1^A}{dy_1^A} &= \frac{u_1^A D_2 - \rho \frac{k\delta}{u_1^D} (y_1^A - y_1^D)}{k\delta D_2} > 0 \text{ unless } (y_1^A - y_1^D) \text{ is very big,} \\
\frac{db}{dy_1^A} &= \frac{\frac{k\delta}{u_0^A} \frac{k\delta}{u_1^D}}{D_2} (y_1^A - y_1^D) > 0, \\
\frac{dg}{dy_1^A} &= \left( u_0^A - u_1^D + \frac{(y_1^A - y_1^D) u^D}{1 - (1 - \delta) e^{-\phi^D}} \frac{\partial \phi^D}{\partial y^D} \right) \frac{db}{dy_1^A} - (1 - u_1^D), \\
\frac{db}{dy_1^A} - \frac{dg}{dy_1^A} &= \frac{db}{dy_1^A} - \left( u_0^A - u_1^D + \frac{(y_1^A - y_1^D) u^D}{1 - (1 - \delta) e^{-\phi^D}} \frac{\partial \phi^D}{\partial y^D} \right) \frac{db}{dy_1^A} + (1 - u_1^D) \text{ most likely pos.}
\end{aligned}$$

### 8.4.2 General Shock

$$\begin{aligned}
\frac{d\phi_1^D}{dy} &= \frac{k\delta}{u_0^A} \frac{1}{D_2} > 0, \quad \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy} < 0 \\
\frac{d\phi_0^A}{dy} &= \frac{k\delta}{u_1^D} \frac{1}{D_2} > 0, \quad \frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy} < 0 \\
\frac{d\phi_1^A}{dy} &= \frac{u_1^A}{u_0^A} \frac{k\delta}{u_1^D} \frac{1}{D_2} > 0, \quad \frac{du_1^A}{d\phi_1^A} \frac{d\phi_1^A}{dy} < 0 \\
\frac{db}{dy} &= \frac{\frac{k\delta}{u_0^A} \tau + \frac{k\delta}{u_1^D} \rho}{D_2} < 0, \\
\frac{dg}{dy} &= -\frac{k\delta}{u_0^A} \frac{k\delta}{u_1^D} \frac{1}{D_2} \left( (u^{A0} - u^D) + \frac{(y_1^A - y_1^D) u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\partial \phi^D}{\partial y_1^D} \right) < 0, \\
\frac{db}{dy} - \frac{dg}{dy} &= \frac{\frac{k\delta}{u_0^A} \tau + \frac{k\delta}{u_1^D} \rho}{D_2} + \frac{k\delta}{u_0^A} \frac{k\delta}{u_1^D} \frac{1}{D_2} \left( (u^{A0} - u^D) + \frac{(y_1^A - y_1^D) u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{\partial \phi^D}{\partial y_1^D} \right)
\end{aligned}$$

where

$$\begin{aligned}
\rho &= \frac{b\eta (u_0^A)^2 \frac{2+(1-\delta)e^{-\phi_0^A}}{(1-(1-\delta)e^{-\phi_0^A})^2} k}{\eta \frac{1}{1-(1-\delta)e^{-\phi_0^A}} \frac{(u_0^A)^2}{\delta k} + (1-\eta) \frac{1}{1-(1-\delta)e^{-\phi_1^D}} \frac{(u^D)^2}{\delta k}} > 0, \\
\tau &= -\frac{\frac{2+(1-\delta)e^{-\phi_D}}{1-(1-\delta)e^{-\phi_D}} \eta b \frac{1}{(u_1^D)^2} \frac{(u_0^A)^2}{(1-(1-\delta)e^{-\phi_{A0}})}}{\eta \frac{1}{1-(1-\delta)e^{-\phi_0^A}} \frac{(u_0^A)^2}{\delta k} + (1-\eta) \frac{1}{1-(1-\delta)e^{-\phi_1^D}} \frac{(u^D)^2}{\delta k}} < 0
\end{aligned}$$

and  $D_2 = \frac{k\delta}{u_0^A} \left( \tau + \frac{k\delta}{u_1^D} \right) + \rho \frac{k\delta}{u_1^D} > 0$ . The determinant is positive as for stability we need

$$\left| \frac{d\phi_1^D}{db} \Big|_{\phi_1^D(b)} \right| > \left| \frac{d\phi_1^D}{db} \Big|_{b(\phi_1^D)} \right| \iff \left( \tau + \frac{k\delta}{u_1^D} \right) > 0.$$

### 8.4.3 Impact on unemployment distribution

When  $y_1^D$  increases, then the impact on unemployment of disadvantaged workers is higher than that of advantaged workers

$$\begin{aligned}
\left| \frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy_1^D} \right| &> \left| \frac{du_0^A}{d\phi_1^D} \frac{d\phi_0^D}{dy_1^D} \right| \\
\frac{\frac{k\delta}{u_0^A} + \rho + \frac{k\delta}{u_0^A} (y_1^A - y_1^D)}{-D} \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} &> \frac{-\tau + \frac{\delta k}{u_1^D} (y_1^A - y_1^D)}{-D} \frac{u_0^A}{1 - (1 - \delta) e^{-\phi_0^A}}
\end{aligned}$$

true as  $\frac{k\delta}{u_0^A} + \tau > \frac{k\delta}{u_1^D} + \tau > 0$  by stability.

Unemployment distribution becomes more unequal when  $y_0^A$  increases as  $u_1^D$  increases and  $u_0^A$  falls.

When  $y_1^A$  increases then unemployment distribution becomes more unequal:

$$\frac{\frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy_1^A}}{\frac{k\delta}{u_0^A} \frac{(y_1^A - y^D)}{-D} \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}}} > \frac{\frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy_1^A}}{\frac{u_0^A}{1 - (1 - \delta) e^{-\phi_0^A}} \frac{k\delta}{u_1^D} \frac{(y_1^A - y^D)}{-D}}, \text{ iff}$$

which is true as  $u_0^A < u_1^D$

Impact of a general shock: unemployment distribution becomes more equal

$$\left| \frac{\frac{du_1^D}{d\phi_1^D} \frac{d\phi_1^D}{dy} \right| > \left| \frac{\frac{du_0^A}{d\phi_0^A} \frac{d\phi_0^A}{dy} \right| \text{ iff}$$

$$\frac{k\delta}{u_0^A} \frac{1}{D_2} \frac{1}{1 - (1 - \delta) e^{-\phi_1^D}} \frac{u_1^D}{1 - (1 - \delta) e^{-\phi_1^D}} > \frac{k\delta}{u_1^D} \frac{1}{D_2} \frac{1}{1 - (1 - \delta) e^{-\phi_0^A}} \frac{u_0^A}{1 - (1 - \delta) e^{-\phi_0^A}},$$

which is true.

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