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Working paper 5-2009

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by

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April 2009

Working Paper 09-01, Department of Economics,
College of Business Administration, University of Central Florida, 2009

ABSTRACT. Subjective probabilities play a role in many economic decisions. There is a large theoretical literature on the elicitation of subjective probabilities, and an equally large empirical literature. However, there is a gulf between the two. The theoretical literature proposes a range of procedures that can be used to recover subjective probabilities, but stresses the need to make strong auxiliary assumptions or "calibrating adjustments" to elicited reports in order to recover the latent probability. With some notable exceptions, the empirical literature seems intent on either making those strong assumptions or ignoring the need for calibration. We illustrate how one can *jointly estimate risk attitudes and subjective probabilities* using structural maximum likelihood methods. This allows the observer to make inferences about the latent subjective probability, calibrating for virtually any well-specified model of choice under uncertainty. We demonstrate our procedures with experiments in which we elicit subjective probabilities. We calibrate the estimates of subjective beliefs assuming that choices are made consistently with expected utility theory or rank-dependent utility theory. Inferred subjective probabilities are significantly different when calibrated according to either theory.

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Subjective probabilities about some event are operationally defined as those probabilities that lead an agent to make certain choices over outcomes that depend on that event. These choices could be as natural as placing a bet on a horse race, or as structured as responding to the payoffs provided by some scoring rule. In order to infer subjective probabilities from observed choices of this kind, however, one either has to make some strong assumptions about risk attitudes or jointly estimate risk attitudes and subjective probabilities. We show how the latter can be implemented by pairing several choices together, some of which identify risk attitudes and some of which identify the interplay between risk attitudes and subjective probabilities. Joint estimation of a structural model of choice across these two types of tasks allows one to then make inferences about subjective probabilities.

If inferred subjective probabilities are conditioned on knowing risk attitudes, then any statistical uncertainty in the estimation of risk attitudes would be expected to "propagate" into some additional uncertainty about subjective probabilities. Joint estimation allows these effects to occur, providing more reliable estimates of subjective probabilities, even if those estimates have large standard errors. In other words, it is possible that the choice task for eliciting subjective probabilities generates a point response that appears to be quite precise by itself, but which is actually not a very precise estimate of the latent subjective probability when one properly accounts for uncertainty over risk attitudes.

An important example is the response to a proper scoring rule, such as the quadratic scoring rule (QSR). A respondent might select 67% when faced with a QSR, and that would be the exact subjective probability if the subject were known to be *exactly* risk neutral. But if the subject was estimated to have risk attitudes in some interval, there would be a corresponding *range of subjective probabilities* to be inferred from that 67% *response* to the scoring rule task. If the estimate of the subject's risk attitudes spanned the possibility of risk neutrality, then the inferred subjective probability would include 67%, but would also include subjective probabilities on either side. If the estimate of the subject's risk attitudes revealed him to be clearly risk averse, with no statistically significant chance of being risk neutral, then the risk-adjusted subjective beliefs would actually be strictly higher than 67%. The intuition is clear: a response of 50% in the standard QSR removes all uncertainty over payoffs, and is a magnet for risk averse subjects. Sufficiently risk averse subjects would report close to 50% even if their latent subjective probability was 67%. So if we observe the subjects responding at 67%, and we know that they are risk averse to some degree, then we can infer that they must have actually held a latent subjective probability greater than 67% and responded to the pull of the magic 50% magnet.

Formalizing this intuition requires that one obtain estimates of risk attitudes from a task with *objective* probabilities as well as from a task whose outcomes depend on some *subjective* probability, and *then* untangles the effects of risk attitudes and subjective probability with a structural model of choice. We do so for the QSR, and also for a linear scoring rule (LSR).

In section 1 we briefly state the theory underlying our approach. The properties of the QSR and LSR, and the fact that responses to these are affected by risk attitudes, are well known. We assume throughout that the agent is acting in what is called a "probabilistically sophisticated" manner, although we do not restrict the characterization of risk attitudes to expected utility theory (EUT). We also consider the inference of subjective probability for a subject who is assumed to make decisions according to the rank-dependent utility (RDU) model. This extension is particularly appropriate in the case of eliciting subjective probabilities, because it involves allowing for probability weighting and non-additive decision weights on the utility of final outcomes. Given that one of these probabilities to be weighted is the subjective probability being estimated, one might expect estimates of the subjective probability to be sensitive to the correct specification of the model of risk attitudes employed.

In section 2 we describe the experimental task we posed to 140 subjects, split roughly equally across the QSR and LSR alternatives. Our subjects made choices over a number of standard lotteries, characterized by objective uncertainty over monetary outcomes between $0 and $100. They also gave responses to either a QSR or LSR choice task. The prizes on each of these scoring rule tasks also spanned $0 and $100, so that we were able to infer risk attitudes over the same prize domain as the scoring rule responses.

Section 3 formally sets out the econometric model used for estimating subjective probabilities, spelling out the manner in which we undertake joint estimation over all tasks in order to identify subjective probabilities. Section 4 then presents our estimates of the inferred subjective probabilities from these scoring rules, after adjusting for risk. We show how QSR and LSR results in subject responses which are far from the subjective probabilities one would *a priori* expect, but when inferring the subjective probabilities conditioned on the joint estimation or risk preferences and subjective probabilities the responses gives responses in line with intuition.

Section 5 reviews related literature, and some extensions to consider the case of subjective uncertainty, where the subject is not assumed to have "boiled down" his subjective belief into a crisp subjective probability. Section 6 draws some conclusions.

1. Scoring Rules

Scoring rules are procedures that convert a "report" by an individual into a lottery defined over the outcome of some event. For simplicity we assume throughout that the events in question only have two outcomes. A scoring rule asks the subject to make some report r, and then defines how an elicitor pays a subject depending on their report and the outcome of the event. This framework for eliciting subjective probabilities can be formally viewed from the perspective of a trading game between two agents: you give me a report, and I agree to pay you $X if one outcome occurs and $Y if the other outcome occurs. The scoring rule defines the terms of the exchange quantitatively, explaining how the elicitor converts the report from the subject into a lottery. We use the terminology "report" because we want to view this formally as a mechanism, and do not want to presume that the report is in fact the subjective probability π of the subject. In general, it is not.

For example, consider the log scoring rule proposed by Good [1952; p.112] and extended by Bernardo [1979]. In its simplest version, this scoring rule assigns a number, $\log(\theta)$, to the report θ for some event, constraining θ to lie in the unit interval. Hence we can refer to this as a reported probability. Since $0 \leq \theta \leq 1$, the simple log score is a penalty score, a negative number. Also, since the log function is increasing, higher reported probabilities θ for an event receive higher scores (lower penalties), with a maximum score (minimum penalty) of zero when the reported probability for the event that occurs is 1, i.e, $\theta=1$.[1]

The popular QSR is defined in terms of two positive parameters, α and β that determine the fixed reward the subject gets and the penalty for error. Assume that the possible outcomes are A or B, where B is the complement of A, that θ is the reported probability for A, and that Θ is the true binary-valued outcome for A.[2] Then the subject is paid $S(\theta \mid A \text{ occurs}) = \alpha - \beta(\Theta-\theta)^2 = \alpha - \beta(1-\theta)^2$ if event A occurs and $S(\theta \mid B \text{ occurs}) = \alpha - \beta(\Theta-\theta)^2 = \alpha - \beta(0-\theta)^2$ if B occurs. In effect, the score or payment penalizes the subject from the squared deviation that an omniscient seer would report in these two cases, which is 1 and 0, respectively. The fixed reward is a convenience to ensure that subjects are willing to play this trading game, and the penalty function simply accentuates the penalty from not being an omniscient seer. In our experiments $\alpha = \beta = \$100$, so subjects could earn up to

[1] The more general version of the log scoring rule (Bernardo and Smith [1994; p. 73]) is a state-dependent linear transformation of the simple log score: $k(s) + \tau \times \log [b(s)/\theta(s)]$, where b(s) is a probability report for state S=s from a discrete set of states and a probability mass function b=[b(1), b(2),....b(n)] such that $b(s)>0$, $\sum_s b(s)=1$; $\theta(s)$ can be thought of as a reference probability report such that $\theta(s)>0$, $\sum_s \theta(s)=1$; and k(s) and $\tau>0$ are constants.

[2] That is, $\Theta=1$ if A occurs, and $\Theta=0$ if it does not occur.

$100 or as little as $0. If they reported 1 they earned $100 if event A occurred or $0 if event B occurred; if they reported ¾ they earned $93.75 or $43.75; and if they reported ½ they earned $75 no matter what event occurred.

It is intuitively obvious, and also well known in the literature (e.g., Winkler and Murphy [1970] and Kadane and Winkler [1988]), that risk attitudes will affect the incentive to report one's subjective probability "truthfully" in the QSR.[3] A sufficiently risk averse agent is clearly going to be drawn to a report of ½, and varying degrees of risk aversion will cause varying distortions in reports from subjective probabilities. If we knew the form of the (well-behaved) utility function of the subjects, and their degree of risk aversion, we could infer back from any report what subjective probability they must have had. Indeed, this is exactly what we do in the sequel, recognizing that we only ever have estimates of their degree of risk aversion.

The LSR is also defined in terms of two positive parameters, γ and λ, that serve as fixed rewards and penalties, respectively. The subject is paid a fixed reward less some multiple of the absolute difference between their report and what actually happened, which is also what an omniscient seer would have reported. Thus the payment is $S(\theta | \text{A occurs}) = \gamma - \lambda(1-\theta)$ if event A occurs and $S(\theta | \text{B occurs}) = \gamma - \lambda(\theta-0)$ if B occurs. We again set $\gamma = \lambda = \$100$, generating payoffs of $100 or $0 for a report of 1; $75 and $25 for a report of ¾; and $50 no matter what the outcome for a report of ½. The LSR is not a favorite of decision theorists, since a risk neutral subject would jump to corner-solution reports of 1 or 0 whenever their true beliefs were either side of ½. But when the subject is (even modestly) risk averse, an interior solution is obtained, and we face the

[3] There exist mechanisms that will elicit subjective probabilities without requiring that one correct for risk attitudes, such as the procedure proposed by Karni [2009]. However, it is not apparent that the rationale to truthfully report can be easily explained to subjects, or that the incentives for truthful reporting are strong in the neighborhood of the true subjective probability. These behavioral properties of the procedure should be explored in controlled comparisons with more traditional scoring rules, such as the QSR.

same issues of inference as with the QSR. The LSR is a favorite of experimental economists because of the simplicity of explaining the rule: the score for a report and an event is linear in the reported probability report, so there is no need for elaborate tables showing cryptic payoff scores for discrete reports.[4]

2. Experimental Design

We recruited 140 subjects from the student population of the University of Central Florida in late October 2008 to participate in these experiments. Complete instructions are provided in appendix A.

Figure 1 illustrates the lottery choice that subjects were given. Each subject faced 45 such choices, where prizes spanned the domain $0 up to $100. One choice was selected to be paid out at random after all choices had been entered. Choices of indifference were resolved by rolling a die and picking one lottery, as had been explained to subject. This interface builds on the classic design of Hey and Orme [1994], and is discussed in greater detail in Harrison and Rutström [2008; Appendix B]. The lotteries were presented sequentially in 3 blocks of 15, where each block had prizes in one of three intervals between $0 and some higher level. One level was between $0 and $1, the other level was between $0 and $10, and the third level was between $0 and $100. We presented the lotteries sequentially so that the subject could see that all of the lotteries in one block were for a given scale. The sequence of blocks was randomized across subjects.

Figure 2 shows the interface for the QSR as it was presented to subjects on a computer screen and in printed instructions. The interface was explained with these instructions:

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[4] Hanson [1996; p. 1224] provides a useful reminder that discrete implementations of proper scoring rules can also engender piecewise linear opportunity sets. He points out that certain regions of the QSR implemented by McKelvey and Page [1990] were actually LSR, and that *risk-neutral* subjects would then rationally report a probability at the extremes of that linear region, and not at the discrete alternative closest to their true belief.

> You place your bets by sliding the bar at the bottom of the screen until you are happy with your choice of a probability report. The computer will start at some point on this bar at random: in the above screen it started at 71%, but you are free to change this as much as you like. In fact, you should slide this bar and see the effects on earnings, until you are happy to confirm your choice.
>
> In this hypothetical example the maximum payoff you can earn is $1,000. In the actual tasks the maximum payoff will be lower than that, and we will tell you what it is when we come to those tasks. But the layout of the screen will be the same.
>
> In this demonstration, the event you are betting on is whether a Ping Pong ball drawn from a bingo cage will be Orange or White. We have a bingo cage here, and we have 20 ping-pong balls. 15 of the balls are white, and 5 of the balls are orange. We will give the cage a few turns to scramble them up, and then select one ball at random.
>
> What we want you to do is place a bet on which color will be picked. At the top of your screen we tell you what the event is: in this case, it is **Picking a Ping-Pong Ball**, and you need to bet on whether you think it will be **Orange or White**.
>
> **Your possible earnings are displayed in the two bars in the main part of the screen, and also in numbers at the bottom of each bar.** For example, if you choose to report 71% you can see that you would earn $915.90 if the Ping Pong Ball was ORANGE, and $495.90 if the Ping Pong Ball was WHITE.

The subject was then taken through displays of their payoffs if they chose to report 0% or 100%.

We then concluded the main instructions in this manner:

> Summarizing, then, there are two important points for you to keep in mind when placing your bets:
>
> 1. **Your belief about the chances of each outcome is a personal judgment that depends on information you have about the different events.** In this practice example, the information you have consists of the total number of Orange balls and White balls.
>
> 2. **Your choices might also depend on your willingness to take risks or to gamble.** There is no right choice for everyone. For example, in a horse race you might want to bet on the longshot since it will bring you more money if it wins. On the other hand, you might want to bet on the favorite since it is more likely to win something.
>
> For each task, your choice will depend on two things: your judgment about how likely it is that each outcome will occur, and how much you like to gamble or take risks.

Each subject then participated in a training choice, in which they were told the number of orange balls in the a bingo cage that was on public display, and asked to make a report and confirm it. We deliberately adopted an extremely high scale of a maximum $1000 payoff to ensure that the subjects understood that this was to be a trainer.

Each subject then participated in 7 belief elicitation tasks, knowing that one would be selected for payment. The first 3 were repetitions of the training task with orange and white ping pong balls: subjects were told that there were 60 balls in all in a publicly visible, but initially covered, bingo cage, but were not told the number of orange or white balls. The urn was uncovered and spun for 10 rotations, and then the subject had to make a report that a ball drawn at random would be orange. We do not consider these events here. The fourth task was based on the outcomes of a test in psychology for empathy known as The Eyes Test (e.g., Baron-Cohen [2003]). All subjects had completed this test at the outset of the session, and the event they were asked about was whether the score that a randomly chosen man got on the Eyes Test was equal to or greater than the score that a randomly chosen woman would get. The final three tasks were based on the 2008 U.S. Presidential Election, which was to be held about one week after the session. One task was whether the outright winner of the Presidency would be a Democrat or a Republican, one task was whether the winning share of the popular vote would be at 5 percentage points or more greater than the losing share, and the final task was whether the winning share of the popular vote would be at 10 percentage points or more greater than the losing share.[5] Our own *a priori* expectations for these subjective probabilities were just below 50%, around 80%, around 65% and less than 10%, respectively.

The exact phrasing of these events was explained in written instructions, which were also

[5] These events compare to similar events employed in popular prediction markets, inspired by Forsythe, Nelson, Neumann and Wright [1992]. See http://www.biz.uiowa.edu/iem/ for the current version of this market, and the contracts traded in the 2008 Presidential Election.

read out loud. The event based on the Eyes Test was explained as follows:

The Eyes Test

> At the beginning of today's experimental session we asked you to answer 36 questions, called **<u>The Eyes Test</u>**. These questions were designed by psychologists to measure a person's ability to "read someone else's mind" by just looking at their eyes.
>
> Each and everyone of you were given the same 36 Eyes Test questions in today's experiment and a total score was recorded for each and every one of you in this experiment.
>
> Now we come to the outcome we want you to place bets on in this portion of the experiment. We will pick one man and one woman in the room. Do you think the man who is selected will have a higher score on the Eyes Test than the woman who is selected?
>
> After everyone in the experiment has made their bets for this event we will randomly select one man, and randomly select one woman from this experimental session. We will use the cards we collected, and sort then into one pile for men and one pile for women. Each pile will be shuffled, and one card drawn from each pile. We will then compare the score for the man that is drawn with the score for the woman that is drawn, and write these scores up on the board for you to see.
>
> We therefore pose the following outcome for you to bet on now:
>
> **That the man we select at random will have a higher score on the Eyes Test than the woman we select at random.**
>
> Do you have any questions?

After subjects had completed their bets on the bingo cage and Eyes Test tasks, the final three events were explained as follows:

2008 Presidential Elections

> We want you to place bets on some questions about the U.S. Presidential Elections being held in a few weeks:
>
> 1. Will the next President of the United States be a Democrat or a Republican?
>
> 2. Will the popular vote for the winning candidate be 5 or more percentage points greater than the popular vote for the losing candidate?
>
> 3. Will the popular vote for the winning candidate be 10 or more percentage

points greater than the popular vote for the losing candidate?

It is important that you understand that the first question is about the outcome of the Electoral College vote, and not the popular vote. The popular vote is just the sum of all votes across the United States. We are only referring to the Presidential Election, and not to any other elections that might occur on the same day.

For the second and third question, we are asking if you think that the winner of the popular vote will beat the loser by 5 or 10 percentage points or more. For example, if the winner of the popular vote gets 51% of the vote and the loser gets 49%, then this is a 2 percentage point difference. If the winner gets 53% and the loser gets 47%, then this is a 6 percentage point difference.

The election will be on Tuesday, November 4, 2008. To use a widely respected public source for the outcome, we will use the New York Times of Friday, November 7, 2008 as the official source used to determine your payoffs. In the event that there is a drawn out determination of the next President, such as in the 2000 election, we will delay payments until Inauguration Day, which is on January 20, 2009.

You will be paid for your bets in checks that will be mailed out on Monday, November 10, assuming we know who the next President will be at that time.

Please go ahead now and place your bets for this event, unless you have any questions.

The subjects then completed their belief elicitation tasks for these Presidential election events, and went on to the lottery choice tasks described earlier.

The experiments were conducted in the week of Monday October 27 through Friday October 31, in the week prior to the 2008 election. Including other belief elicitation treatments, a total of 354 subjects participated, earning a total of $32,101 for an average of just over $90 per subject. Each session lasted around 2 hours. There was considerable variation in earnings, with one subject taking home $3 and another subject taking home $205.

3. Econometric Model

We develop the econometric model to be estimated in three stages. First we present the specification of risk attitudes assuming an EUT model of latent choice, where the focus is entirely on the concavity of the estimated utility function. Second, we present the specification of risk attitudes assuming a RDU model of latent choice, so that risk attitudes are determined by the interplay of concave utility functions and non-linear probability weighting.[6] Third, we consider the joint estimation of risk attitudes and subjective probability, using either the EUT or the RDU specification.

A. Risk Attitudes under Expected Utility Theory

We assume an Expo-Power (EP) utility function originally proposed by Saha [1993]. Following Holt and Laury [2002], the EP function is defined as

$$U(y) = [1\text{-exp}(-\alpha y^{1-r})]/\alpha, \tag{1}$$

where α and r are parameters to be estimated, and y is income from the experimental choice. The EP function can exhibit increasing or decreasing relative risk aversion (RRA), depending on the parameter α: RRA is defined by $r + \alpha(1-r)y^{1-r}$, so RRA varies with income if $\alpha \neq 0$ and the estimate of r defines RRA at a zero income. This function nests CRRA (as $\alpha \rightarrow 0$) and CARA (as $r \rightarrow 0$).

The utility function (1) can be estimated using maximum likelihood and a latent EUT structural model of choice. Let there be K possible outcomes in a lottery; in our lottery choice task $K \leq 4$. Under EUT the probabilities for each outcome k in the lottery choice task, p_k, are those that are induced by the experimenter, so expected utility is simply the probability weighted utility of each outcome in each lottery i:

$$EU_i = \sum_{k=1,K} [\ p_k \times u_k\ ]. \tag{2}$$

The EU for each lottery pair is calculated for a candidate estimate of r and α, and the index

[6] We could just develop an RDU model and test if the estimated probability weighting is the identity function, in which case the RDU model collapses to an EUT model. However, the exposition is, in our view, simpler if one develops the models separately.

$$\nabla EU = eu_R/(eu_L + eu_R) \quad (3)$$

calculated, where

$$eu_i = \exp(EU_i), \quad (4)$$

for i= {R, L}, and where EU_L is the "left" lottery and EU_R is the "right" lottery, as displayed to the subject and illustrated in Figure 1. This latent index, based on latent preferences, is already in the form of a probability.[7]

Thus the likelihood of the observed responses, conditional on the EUT and EP specifications being true, depends on the estimates of r and α given the above statistical specification and the observed choices. If we ignore responses that reflect indifference[8] the log-likelihood is then

$$\ln L(r, \alpha; y, \mathbf{X}) = \sum_i [(\ln \nabla EU \times \mathbf{I}(y_i = 1)) + (\ln (1-\nabla EU) \times \mathbf{I}(y_i = -1))] \quad (5)$$

where $\mathbf{I}(\cdot)$ is the indicator function, $y_i = 1(-1)$ denotes the choice of the Option R (L) lottery in risk aversion task i, and **X** is a vector of individual characteristics reflecting age, sex, race, and so on. When we pool responses over subjects the **X** vector will play an important role to allow for some heterogeneity of preferences.

To allow for subject heterogeneity with respect to risk attitudes, the parameters r and α are each modeled as linear functions of observed individual characteristics of the subject. For example, assume that we only had information on the age and sex of the subject, denoted Age (years of age) and Female (0 for males, and 1 for females). Then we would estimate the coefficients r_0, r_1 and r_2 in

[7] It is well known, but useful to note, that (3) is equivalent to $\Lambda(EU_R - EU_L)$ where $\Lambda(\cdot)$ is the logistic cumulative density function. Thus (3) embodies a statistical "link function" between the difference in the EU of the two lotteries and the probability of the observed choice.

[8] In our lottery experiments the subjects are told at the outset that any expression of indifference would mean that the experimenter would toss a fair coin to make the decision for them if that choice was selected to be played out. Hence one can modify the likelihood to take these responses into account either by recognizing this is a third option, the compound lottery of the two lotteries, or alternatively that such choices implies a 50:50 mixture of the likelihood of choosing either lottery, as illustrated by Harrison and Rutström [2008; p.71]. We do not consider indifference here because it was an extremely rare event.

$r = r_0 + r_1 \times Age + r_2 \times$ Female. Therefore, each subject would have a different estimated r, $\hat{r}$, for a given set of estimates of r_0, r_1 and r_2 to the extent that the subject had distinct individual characteristics. So if there were two subjects with the same sex and age, to use the above example, they would literally have the same $\hat{r}$, but if they differed in sex and/or age they would generally have distinct $\hat{r}$. In fact, we choose to use 4 individual characteristics to model heterogeneity in our estimated risk preferences. Apart from a dummy for peopled aged over 22 years and gender, these include binary indicators for subjects that self-declare as having a high GPA (over 3.75) and self-declare as graduate students.

An important extension of the core model is to allow for subjects to make some errors. The notion of error is one that has already been encountered in the form of the statistical assumption (4) that the probability of choosing a lottery is not 1 when the EU of that lottery exceeds the EU of the other lottery.[9] By varying the shape of the link function implicit in (4), one can informally imagine subjects that are more sensitive to a given difference in the index ∇EU and subjects that are not so sensitive. We use the contextual error specification proposed by Wilcox [2009]. It posits the latent index

$$eu_i = \exp[(EU_i/\nu)/\mu], \tag{4'}$$

instead of (4), where ν is a normalizing term for each lottery pair L and R, and $\mu>0$ is a structural "noise parameter" used to allow some errors from the perspective of the deterministic EUT model. The normalizing term ν is defined as the maximum utility over all prizes in this lottery pair minus the minimum utility over all prizes in this lottery pair, and ensures that the *normalized* EU difference

[9] This assumption is clear in the use of a link function between the latent index ∇EU and the probability of picking one or other lottery; in the case of the normal CDF, this link function is $\Phi(\nabla EU)$. If the subject exhibited no errors from the perspective of EUT, this function would be a step function: zero for all values of $\nabla EU<0$, anywhere between 0 and 1 for $\nabla EU=0$, and 1 for all values of $\nabla EU>0$. Harrison [2008; p.326] illustrates the implied CDF, referring to it as the CDF of a "Hardnose Theorist."

$[(EU_R - EU_L)/\nu]$ remains in the unit interval. As $\mu \to \infty$ this specification collapses ∇EU to 0 for any values of EU_R and EU_L, so the probability of either choice converges to ½. So a larger μ means that the difference in the EU of the two lotteries, conditional on the estimate of r, has less predictive effect on choices. Thus μ can be viewed as a parameter that flattens out, or "sharpens," the link functions implicit in (4). This is just one of several different types of error story that could be used, and Wilcox [2008] provides a masterful review of the implications of the strengths and weaknesses of the major alternatives.

Thus we extend the likelihood specification to include the noise parameter μ and maximize $\ln L(r, \alpha, \mu; y, \mathbf{X})$ by estimating r, α and μ, given observations on y and **X**.[10] Additional details of the estimation methods used, including corrections for "clustered" errors when we pool choices over subjects and tasks, is provided by Harrison and Rutström [2008; p.69ff].

B. Risk Attitudes under Rank Dependent Utility Theory

The RDU model extends the EUT model by allowing for decision weights on lottery outcomes. Instead of (1) we have

$$U(y) = [1-\exp(-\acute{\alpha} y^{1-\acute{r}})]/\acute{\alpha}, \tag{6}$$

where $\acute{r}$ and $\acute{\alpha}$ are the coefficient for the curvature of the utility function, comparable to r and α in (1). To calculate decision weights under RDU one replaces expected utility defined by (2) with RDU

$$RDU_i = \sum_{k=1, K} [\, w_k \times u_k \,] \tag{7}$$

where

$$w_i = \omega(p_i + \ldots + p_n) - \omega(p_{i+1} + \ldots + p_n) \tag{8}$$

for i=1,... , n-1, and

[10] The normalizing term ν is given by the value of r and the lottery parameters, which are part of **X**.

$$w_i = \omega(p_i) \tag{9}$$

for i=n, the subscript indicating outcomes ranked from worst to best, and where $\omega(p)$ is some probability weighting function. We adopt the simple "power" probability weighting function proposed by Quiggin [1982], with curvature parameter γ:

$$\omega(p) = p^{\gamma} \tag{10}$$

So $\gamma \neq 1$ is consistent with a deviation from the conventional EUT representation.

The same behavioral noise specification is used as in the EUT model, replacing EU_i with RDU_i, leading to the definition of a comparable latent index ∇RDU. The likelihood specification for the RDU model is therefore

$$\ln L(\acute{r}, \acute{\alpha}, \gamma, \mu; y, \mathbf{X}) = \sum_i [(\ln \Phi(\nabla RDU) \times \mathbf{I}(y_i=1)) + (\ln (1-\Phi(\nabla RDU)) \times \mathbf{I}(y_i= -1))] \tag{11}$$

and entails the estimate of $\acute{r}$, $\acute{\alpha}$, γ and μ using maximum likelihood. Individual heterogeneity is allowed for by estimating the parameters $\acute{r}$, $\acute{\alpha}$ and γ as linear functions of the demographic characteristics defined earlier.

Figure 3 shows the manner in which the parameter γ characterizes the probability weighting function and the decisions weights used to evaluate lottery choices. Since we assume γ=0.77<1 in this illustration, to anticipate our estimates, the probability weighting function $\omega(p)$ is concave. For simplicity here we assume lotteries with 2, 3 or 4 prizes that are equally likely when we generate the decision weights. So for the case of 2 prizes, each prize has p=½; with 3 prizes, each prize has p=⅓; and with 4 prizes, each prize has p=¼. For the 3-prize and 4-prize lottery we see the standard result, that the decision weights on the largest prizes are relatively greater than the true probability, and the decision weights on the smallest prizes are relatively smaller than the true probability. In the belief elicitation task there were only 2 prizes per lottery, so this value of the parameter γ puts greater decision weight on the higher prize.

Each panel in Figure 3 is important for our analysis. For the purposes of estimating γ from the observed lottery choices we only need the decision weights in the right panel of Figure 3. But for the purposes of recovering subjective beliefs subject to probability weighting, we only need the probability weighting function. In fact, we need it's inverse function, since it is the π in the $\omega(\pi)$ function that we are seeking to recover in that case. We do not directly observe $\omega(p)$ or $\omega(\pi)$, but we can estimate $\omega(\cdot)$ as part of the latent structure generating the observed choices in the two types of task, implicitly assuming that $\omega(p) = \omega(\pi)$. Once we have $\omega(\cdot)$ we can then recover π by directly applying the estimated probability weighting function, such as the one shown, for a typical γ, in the left panel of Figure 3.

C. Estimating the Subjective Probability

The responses to the belief elicitation task can be used to estimate the subjective probability that each subject holds if we are willing to assume something about how they make decisions under risk.

If they are assumed to be risk neutral, then we can directly infer the subjective probability from the report of the subject.[11] This result is immediate under the QSR, but raises a problem of interpretation under the LSR if the reports are not at the corner solutions of 0% and 100%. In that case the behavioral error story has a lot of explaining to do, if one wants to be formal. On the other hand, any minimal level of risk aversion will suffice, under the LSR, to generate interior responses, so we assume that the subjects indeed have some minimal level of risk aversion when we report "risk neutral subjective beliefs" for the LSR.

Moving to the models that allow for varying risk attitudes, we jointly estimate the subjective

[11] The expression "risk neutral" here should be understood to include the curvature of the utility function and the curvature of the probability weighting function. So it is not just a statement about the former, unless one assumes EUT.

probability and the parameters of the core model. Assume for the moment that we have an EUT specification. The subject that selects report θ from a given scoring rule receives the following EU

$$\begin{aligned} EU_\theta = \quad & \pi_A \times U(\text{payout if A occurs} \mid \text{report } \theta) + \\ & (1-\pi_A) \times U(\text{payout if B occurs} \mid \text{report } \theta) \end{aligned} \tag{12}$$

where π_A is the subjective probability that A will occur. The payouts that enter the utility function are defined by the scoring rule and of course the specific report θ, and span the interval [\$0, \$100]. For the QSR and a report of 75%, for example, we have

$$EU_{75\%} = \pi_A \times U(\$93.75) + (1-\pi_A) \times U(\$43.75) \tag{12$'$}$$

For the LSR, and the same report, we have:

$$EU_{75\%} = \pi_A \times U(\$75) + (1-\pi_A) \times U(\$25) \tag{12$''$}$$

and so on for other possible reports. We observe the report made by the subject, and we know that they had 101 possible reports defined over percentage points, so we can calculate the likelihood of that choice given values of r, π_A and μ. In this case the likelihood is the multinomial analogue of the binary logit specification used for lottery choices. We define

$$eu_\Theta = \exp[(EU_\Theta/\nu)/\mu] \tag{13}$$

for any report Θ, analogously to (4′), and then

$$\nabla EU = eu_\theta/(eu_{0\%} + eu_{1\%} + ... + eu_{100\%}) \tag{14}$$

for the specific report θ observed, analogously to (3).

We need r and α to evaluate the utility function in (12), we need π_A to calculate the EU_θ in (12) for each *possible* report Θ in {0%, 1%, 2%, ..., 100% }once we know the utility values, and we need μ to calculate the latent indices (13) and (14) that generate the subjective probability of observing the choice of specific report θ when we allow for some noise in that process. The *joint* maximum likelihood problem is to find the values of these parameters that best explain observed choices in the belief elicitation tasks as well as observed choices in the lottery tasks.

Exactly the same logic extends to the model in which we assume an RDU latent structure instead of an EUT latent structure. In effect, the lottery task allow us to identify r and α under EUT, and $\acute{r}$, $\acute{\alpha}$ and γ under RDU, since π_A plays no direct role in explaining the choices in that task. Individual heterogeneity is allowed for by estimating the π parameter as a linear functions of the demographic characteristics defined earlier, as well as allowing risk attitudes to vary with demographic characteristics.

4. Results

A. Raw Elicited Beliefs

Figure 4 displays the raw responses from each of the scoring rules for each event, in the form of kernel densities, and Table 1 shows summary statistics of the elicited responses. The summary statistics suggest that the QSR and LSR provided roughly the same responses, but the densities in Figure 3 do have some differences in shape. In part this simply alerts us to be aware of the non-Gaussian shape of these distributions.

The general location of the densities corresponds with our qualitative priors on the subjective beliefs that were to be expected for these events. For the Eyes Test it appears, from the observation that the modal response is around 0.4 (LSR) to 0.4 (QSR), that the sample did not expect the male score to exceed the female score, but that this was not an overwhelmingly strong belief. The sample appeared confident that Barack Obama would indeed win the election outright, but displayed a healthy sense of perspective on what the winning margin would be. For our purposes, the choices of a 5% and 10% threshold for the popular vote could not have worked out better, with a majority believing that a 5% margin would be attained but that a 10% margin would not. These results show, at a minimum, that responses were at least correlated with what we believe to be reasonably coherent subjective beliefs for these events.

The fact that the responses to the LSR are not at "corner" value of 0 or 1 shows that the subjects were not exactly risk neutral. But it does not show much more, because one would observe some interior response even for small amounts of risk aversion, as noted earlier.

With the exception of the bets on the outright winner of the Presidential election, the distribution of responses for the two scoring rules are roughly the same. This conclusion is supported by a Kolmogorov-Smirnov test of the null hypothesis that the two distributions are equal. In the case of the outright winner event, the *p*-value on this test is only 0.011, so we can reject that hypothesis in this instance. This finding provides some support for those that would prefer to use the LSR on the grounds that it is simpler to explain to subjects than the QSR. Of course, the real issue is whether they generate the same estimates of subjective probability when one allows for risk attitudes.

B. Characterizing Risk Attitudes

Looking just at the lottery choices under a maintained hypothesis of EUT for now, we find evidence of modest risk aversion at low stakes (since $r>0$, and r defines RRA at $y=0$), and evidence of increasing relative risk aversion as the prizes climb to $100 (since $\alpha>0$). Detailed results are provided in Appendix C, since they are only of indirect interest here. Given these parameter estimates we can calculate RRA at various prize levels: at $25, $50, $75 and $100 the RRA is estimated to be 0.49, 0.61, 0.71 and 0.81, respectively. Thus subjects exhibit much stronger risk aversion for the higher stakes in the belief task than they do for the lower stakes.

When we allow for an array of covariates to better characterize the heterogeneity of risk attitudes, we observe females to be significantly more risk averse, with rra .14 higher at the $0 level. Detailed estimates are provided in a Appendix C. The net effect of allowing for covariates is to slightly increase the range of RRA values for different prize levels: they span 0.29 at $0 up to 0.82 at

$100, with wider standard errors than when there were no covariates.

These results also suggest that one might see quite different effects of "risk-conditioning" the reports for scoring rules depending on the stakes involved. Our stakes are huge in relation to the literature, reviewed in detail in an Appendix: a maximum prize of $100, compared to common implementations in experiments of a maximum prize of less than $1. To be fair, those low-stake implementations are often in the context of the probability elicitation task being paired with another task, such as the choice of a strategy in a game, and the stake for the probability elicitation task is kept small to avoid the subject attempting to construct a portfolio of paired responses across the two tasks. Whatever the position one takes on the behavioral significance of the "payoff dominance" problems that arise from low payoffs, our results on risk attitudes for low stakes and high stakes imply that the extent of adjustment for risk attitudes is much greater for higher stake elicitations. It is a factor for both, since we estimate RRA to be positive for the lowest stakes, but it is not as serious a factor as when the stakes are higher. Thus one might expect to see much more variability of responses for lower-stake scoring rule behavior than for higher-stake scoring rule behavior, *ceteris paribus* the subjective probability.[12]

The results from estimating the RDU model are slightly different. Detailed estimates are again reported in Appendix C. The estimates of the utility function parameters $\acute{r}$ and $\acute{\alpha}$ are both smaller than their counterparts (r and α) under EUT, implying lower levels of risk aversion. We estimate the probability weighting parameter γ=0.72 without covariates, and can reject the hypothesis that this is equal to 1 (p-value<0.001). A likelihood ratio test of the hypothesis that the

[12] Using extremely low stakes for the QSR has the possible advantage that one might appeal to risk neutrality on *a priori* grounds, but the corollary is that one faces the risk of payoff dominance problems. When we say "extremely low," we mean it: many experimenters use a maximum payoff of 10 cents, or perhaps $1 on a generous day in the lab. We understand well the arguments advanced for low payoffs, and review them in an appendix, but they remain extremely low by any practical metric.

EUT model and the RDU model are the same when there are no covariates has a $\chi^2_1 = 35.47$ (p-value<0.01), so we reject that null. The same conclusion is true when we account for covariates and heterogeneity of responses. There are no strikingly different demographic characteristics, and the small changes across the board does not add up to a statistically significant difference. A likelihood ratio test of the hypothesis that the EUT model and the RDU model are the same when covariates are allowed has a $\chi^2_{14} = 47.74$ (p-value<0.01). The RDU model thus econometrically outperforms the EUT model when one assumes that the data is completely generated by one or the other.[13]

One noteworthy feature of these estimates is that one can reject the CRRA specification for both EUT and RDU models. Of course, one might accept CRRA if estimating risk attitudes over a much smaller income domain, such as between $0 and $10.

C. Estimating Subjective Probabilities

Table 2 lists the main results from estimating subjective probabilities for each of the four events considered here, and assuming either an EUT or RDU specification. Table 3 provides detailed estimates for one event from the model assuming EUT and using covariates for each parameter to capture observable individual heterogeneity. Figure 5 displays kernel density functions of the estimated subjective probability distributions from estimates such as those in Table 3, for two events, and assuming EUT. It also shows the empirical cumulative density functions from the estimated subjective probability distributions. Figure 6 does the same as Figure 5, but using the RDU model instead of the EUT model.

Given that we find evidence of risk aversion in our subjects over the domain of prizes on

[13] The alternative approach is to assume a finite mixture specification and evaluate the fraction of the choices consistent with EUT and the fraction consistent with RDU (e.g., Harrison and Rutström [2009]). For our purposes such a refined characterization of risk attitudes is not needed.

offer in their belief elicitation tasks, our estimated subjective probabilities are all translations of the raw responses away from the 50% response. The reason, again, is that risk averse subjects are drawn to respond toward 50% simply to reduce the uncertainty over payoffs, so evidence of risk aversion implies that their true, latent, subjective probabilities must be further away from 50% than their raw responses. Our maximum likelihood estimates simply impose some parametric structure on that qualitative logic, to be able to quantify the extent of the required translation and the precision of the resulting inference about the latent subjective probability. And for the RDU model, our approach allows the probability weighting function to directly affect the estimated subjective probabilities, quite apart from the effect it has on the decision weights for the QSR and LSR bets.

For the Eyes Test, we observe a small movement in subjective probabilities away from the raw responses under EUT, but this in large part follows from the fact that the raw responses themselves were already so close to 50%. The effect of probability weighting is significant, again in large measure due to the baseline values of the raw responses being close to 50%. The estimated value of the probability weighting parameter in this case is γ=0.72, which is the value used to generate Figure 3. By inspection of the left panel of Figure 3 we observe that probability values of just less than 50% would appear to be the most significantly affected by this functional form, explaining why the RDU model generates different estimates than the EUT model.

Figure 5 indicates that the distribution of estimated subjective probabilities is much more concentrated than the raw responses.[14] Our estimates indicate that one *might* arrive at roughly the same mean estimate of the subjective probability without adjusting for risk attitudes, but the precision might be very different after one correctly adjusts for risk.

For the three election events, we see more interesting effects of adjusting for risk aversion.

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[14] In part this may be due to the use of pooled predictions of the estimated subjective probability to generate this distribution instead of subject-specific responses.

In the "win by 5%" event, which is perhaps the most interesting one, we observe a marked translation from the raw response average of 59% to 73% under EUT, and from 50% to 73% under RDU. The 95% confidence interval on each of these estimate does include 59%. In the case of the outright winner event we estimate latent subjective probabilities close to 1, rather than the raw response average of 71%; similarly, in the case of the chance of the popular vote for the winner being more than 10% of the popular vote of the loser, we estimate subjective probabilities close to 0, rather than the raw response average of 28%. Interpreted literally as reflecting the beliefs of a single agent, these results are a puzzle: if the agent has (essentially) certain subjective beliefs about the outcome, then why would risk aversion affect them? As we see, however, these estimates mask some heterogeneity across a subset of the subjects in the underlying distribution of beliefs.

Table 3 lists the detailed EUT estimates for the model of the "win by 5%" event when we include covariates.[15] These estimates generated the results shown in Figure 5, which displays the distribution of estimated subjective probabilities for this sample, where the distribution here is across point estimates of the subjective probability for each subject. We observe a clear shift in the distribution from correcting for risk attitudes. In both Figures 5 and 6, the "win by 5% or more" risk-adjusted estimates are larger than the raw responses in the sense of first order stochastic dominance, and not merely on average. And both models, EUT and RDU, shift the estimates subjective probability in the same direction here. But on the Eyes Test, the EUT and RDU models of risk attitude leads to markedly different inferences. Raw responses indicate that about 15% of the sample have a subjective belief on this event being less than 0.3. However, conditional on EUT, the

[15] The estimates for the subjective probability π refer to a non-linear transform in which we actually estimate the parameter κ and then convert κ to π using $\pi = 1/(1+\exp(\kappa))$. Thus κ can vary between $\pm\infty$ and π is constrained to the open unit interval. To interpret these coefficients, κ=0 implies π=½, κ>0 implies π<½, and κ<0 implies π>½. The estimates subjective probabilities in Figures 5 and 6 have been converted back using this non-linear function and the "delta method" to correctly calculate standard errors (Oehlert [1992]).

chances of a subjective belief less than 0.3 are virtually zero after adjusting for risk, whereas, conditional on RDU, inferred chances of a subjective belief below 0.3 are in the range of 30% to 40%. This illustrates our motivating hypothesis that inferences about subjective belief could be sensitive to the specific model of risk attitudes specified, quite apart from them being sensitive to the correct for risk attitudes in general.

There is no major effect of allowing for observable individual heterogeneity on the point estimates for the beliefs about the Eyes test, but there is some effect for the election events. For the "win by 5%" event the average belief estimate changes from 73% to 66%, but there is no statistically significant difference between the two.

For the "win by 10%" and outright winner events, allowing for heterogeneity does shift the average belief so that it is not 0 or 1. The "win by 10%" average belief is estimated to be 0.060 when we allow for heterogeneous beliefs, with the Business majors having a significantly higher belief that this outcome will occur. These majors are 60% of the sample, and while this marginal effect is offset by other marginal effects[16], it accounts for the overall average being greater than zero. Similarly, for the outright winner event, the average belief is 0.870 when we allow for heterogeneous beliefs, with females having a 32.2 percentage point lower belief that this event will occur (*p*-value of 0.14 on this estimated *marginal* effect). Figure 7 illustrates the calibration involved, with the top two panels reflecting the use of an EUT model. Overall, women have average beliefs of 0.726 and men have average beliefs of 0.968; the density of beliefs for women are shown in the bottom left panel of Figure 7. Similarly, students that are not in their Senior year have much lower subjective probabilities than others, as shown in the bottom right panel of Figure 7. Thus there are significant sub-samples for these two events for which beliefs are not degenerate at 0 or 1, and for which risk aversion plays

[16] For example, females are 38% of the sample and have a significantly lower belief that this outcome will occur.

a role in affecting inferences about true, latent, subjective beliefs.

We observe no statistically significant effect from using the QSR or LSR, but there is no reason to expect one after we condition for risk attitudes. That is, the two scoring rules simply provide subjects with different lotteries with which to place bets about their subjective beliefs. So the same subjective belief should be estimated from each treatment, once one has conditioned on risk attitudes. Comparable estimation using the RDU model generate the results in Figure 6. Again, this provides support for those that would use the LSR over the QSR on the grounds that it is easier to explain to subjects.

To put these results into perspective, particularly those for the outright winner, it is important to note that in the week of the experiments the tide of public opinion clearly favored Barack Obama to win in a landslide. Eliciting a raw belief of only a 70% chance of Obama winning is therefore puzzling, and an *a priori* challenge for those that would use the raw results from a QSR or LSR procedure. For example, consider the Iowa Presidential Market, the current version of the prediction market developed by Forsythe, Nelson, Neumann and Wright [1992]. Figure 8 displays average daily prices on this market for the month of October 2008, and marks off the week in which our experiments were conducted. Average prices for the week of our experiments implied that the probability of Obama winning was around 85%. Moreover, as the average prices for this "winner take all" contract over the whole month show, this was the prevailing sense of the market for at least 2 weeks prior to our experiments. Indeed, the online Irish betting house, PaddyPower.com, was already paying out on over €1 million in pro-Obama bets as early as October 17!

Figure 8 also shows that the Iowa Presidential Market on the popular vote share suggested that a 10% difference was possible. On the other hand, one striking feature of these two contracts (the "winner take all" and the "popular vote share") is that the latter was poorly traded in comparison to the former, as measured by volume. Of course, that could reflect a market that has found an equilibrium price, but it also could reflect a market in which there is too much uncertainty about the outcome for traders to feel safe making a bet.

6. Conclusions

We demonstrate how one can make the theory of subjective probability elicitation practical. Our experimental interface allows subjects to see the tradeoff between alternative reports and payoff consequences, without the need for long lists of payoff tables to be printed out. Our experimental design shows how one can pair different types of choice tasks to allow estimation of risk attitudes, which can then be used to condition the inferences from responses to the scoring rule tasks. Finally, our structural econometric model shows how maximum likelihood methods can then be used to estimate subjective probabilities. We applied this approach to elicit subjective probabilities over four naturally occurring events from a sample of 140 subjects.

Our results show that one has to be sensitive to the risk attitudes of subjects before drawing inferences about subjective probabilities from responses to scoring rules. More accurately, we show that one cannot just directly treat the responses to the scoring rule as if it is a subjective probability, unless one is willing *a priori* to make striking assumptions about risk attitudes. Those assumptions are rejected in our data.

Quite apart from inferring the correct point estimate of subjective probability, uncertainty about risk attitudes affects the confidence that one should have in any point estimate. Even if subjects are "approximately risk neutral" on average, and the QSR is used, uncertainty about the precise level of risk attitudes should be properly reflected in uncertainty about inferences over subjective probabilities. Our analysis has demonstrated how to combine theory and econometrics to do just that. The choice task for eliciting subjective probabilities generates a point response that might appear to be quite precise by itself, but which is actually not a very precise estimate of the latent subjective probability when one properly accounts for uncertainty over risk attitudes.

Of course, although the estimation of subjective probabilities is an important objective in itself, the issue of how to best characterize subjective uncertainty, and attitudes towards it, involve deeper issues. Our estimation approach is clearly within the conventional Bayesian subjective probability framework of Savage [1971][1972]. That framework provides one point of departure for criticisms of conventional EUT that are motivated by an attempt to provide an explanation for the idea that subjective uncertainty should be characterized in a way that allows for the possibility that subjective beliefs are not "boiled down" to subjective probabilities. Exactly how one then models subjective beliefs is an open and important area of research (e.g., Smith [1969], Gilboa and Schmeidler [1989], Ghirardoto, Maccheroni and Marinacci [2004], Klibanoff, Marinacci and Mukerji [2005], Nau [2007] and Gilboa, Postlewaite and Schmeidler [2008]). It seems plausible that subjects exhibit some degree of "uncertainty aversion" in addition to traditional risk aversion when faced with making decisions about events that involve subjective probabilities rather than objective probabilities.

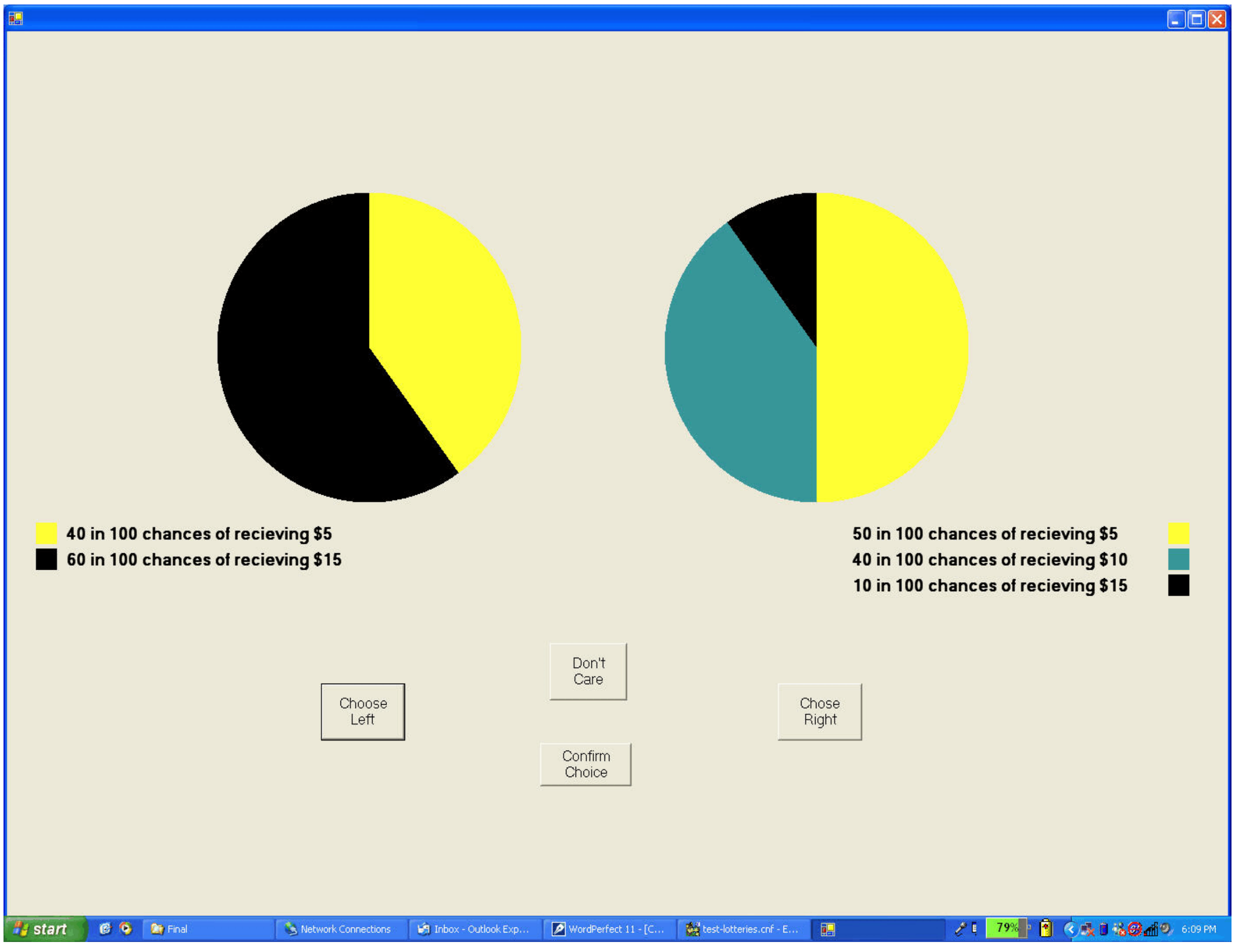


Figure 1: Illustrative Lottery Choice

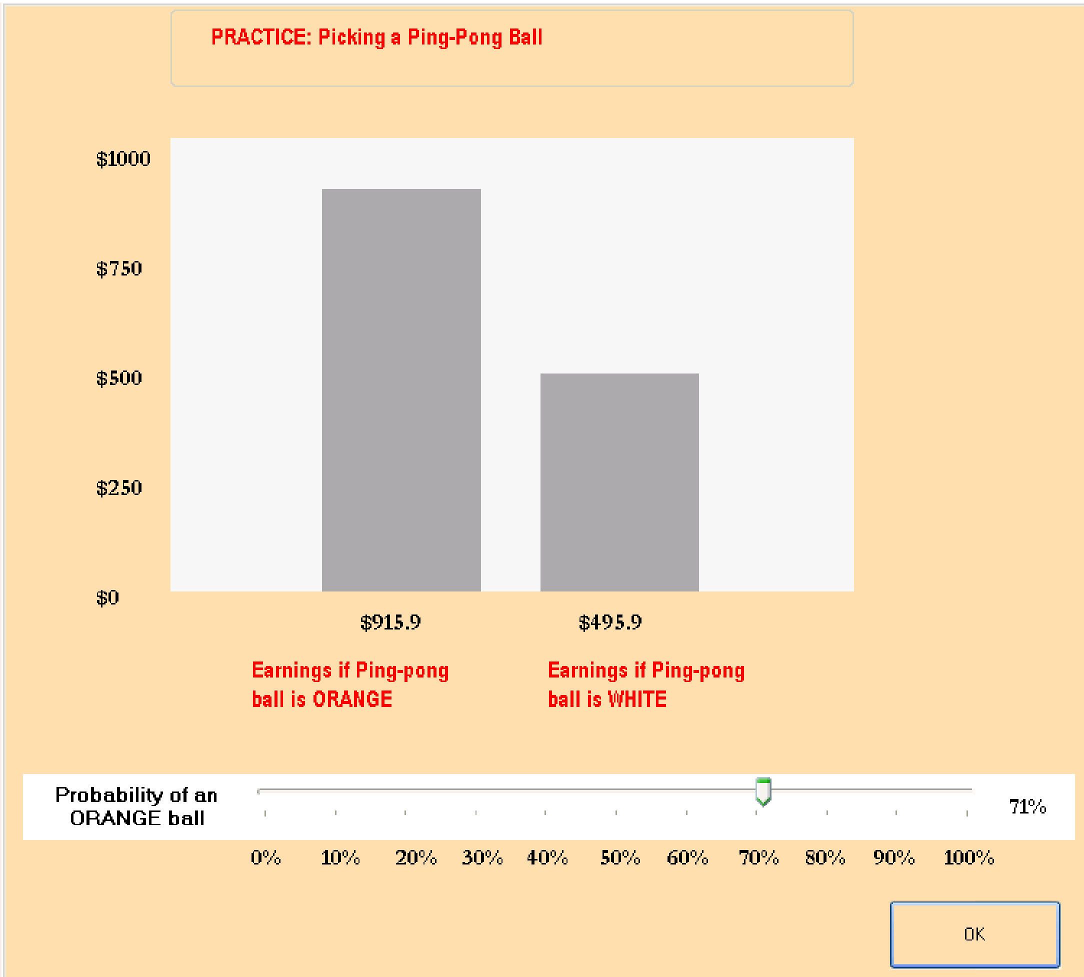


Figure 2: Illustrative Quadratic Scoring Rule Interface

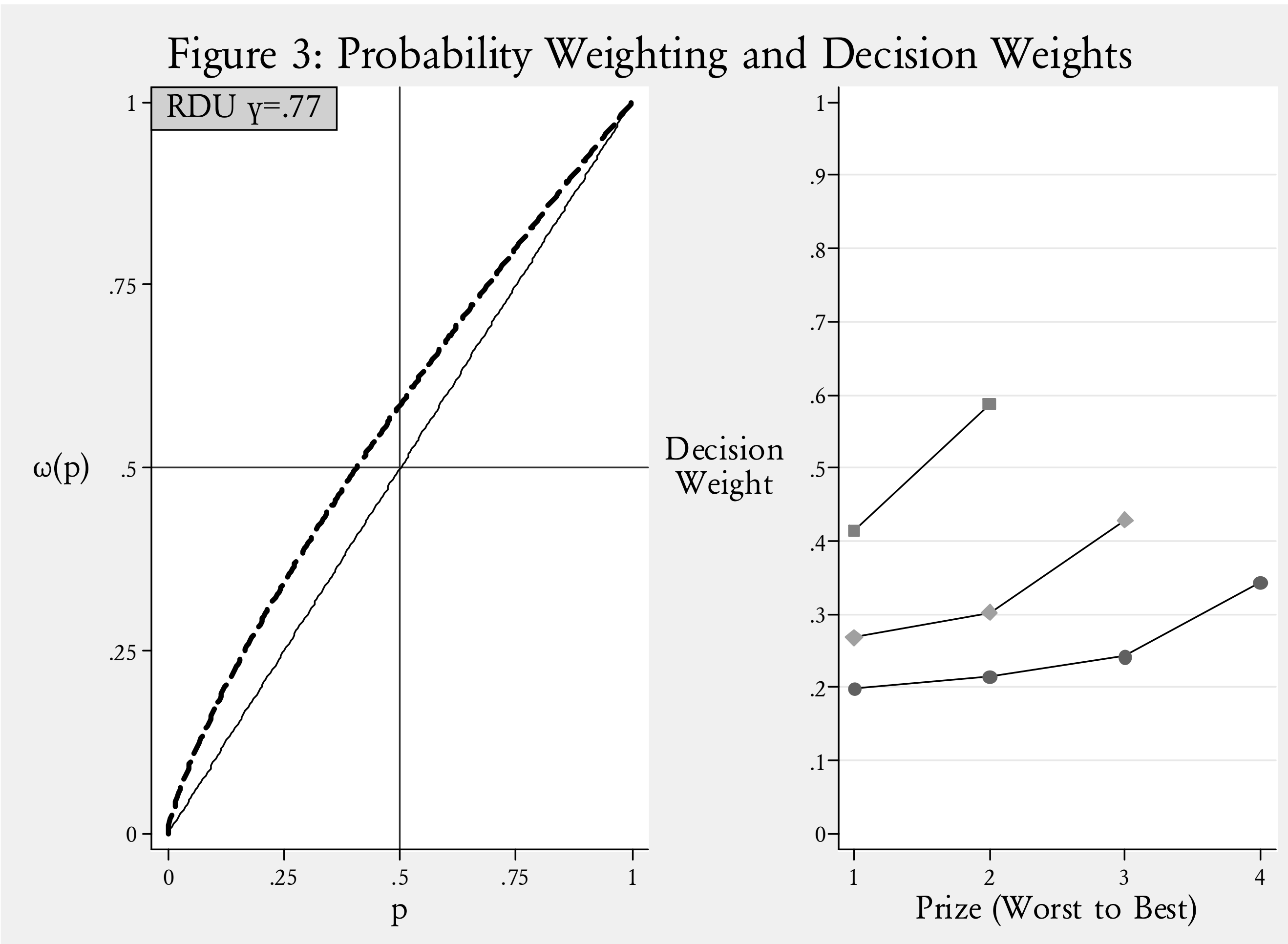


Figure 3: Probability Weighting and Decision Weights
RDU γ=.77
ω(p)
1
.75
.5
.25
0
0
.25
.5
.75
1
p
Decision
Weight
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
1
2
3
4
Prize (Worst to Best)

Table 1: Descriptive Statistics for Scoring Rule Responses

Event	Scoring Rule	Mean	Median	Standard Deviation
Eyes Test	Quadratic	0.43	0.40	0.19
	Linear	0.43	0.40	0.19
	Both	0.43	0.40	0.19
President	Quadratic	0.69	0.70	0.20
	Linear	0.74	0.75	0.22
	Both	0.71	0.70	0.21
Win by 5%	Quadratic	0.59	0.60	0.23
	Linear	0.58	0.60	0.26
	Both	0.59	0.60	0.24
Win by 10%	Quadratic	0.28	0.30	0.20
	Linear	0.29	0.26	0.21
	Both	0.28	0.30	0.21

Figure 4: Elicited Responses from QSR and LSR

Kernel density for QSR is solid line, and LSR is dashed line

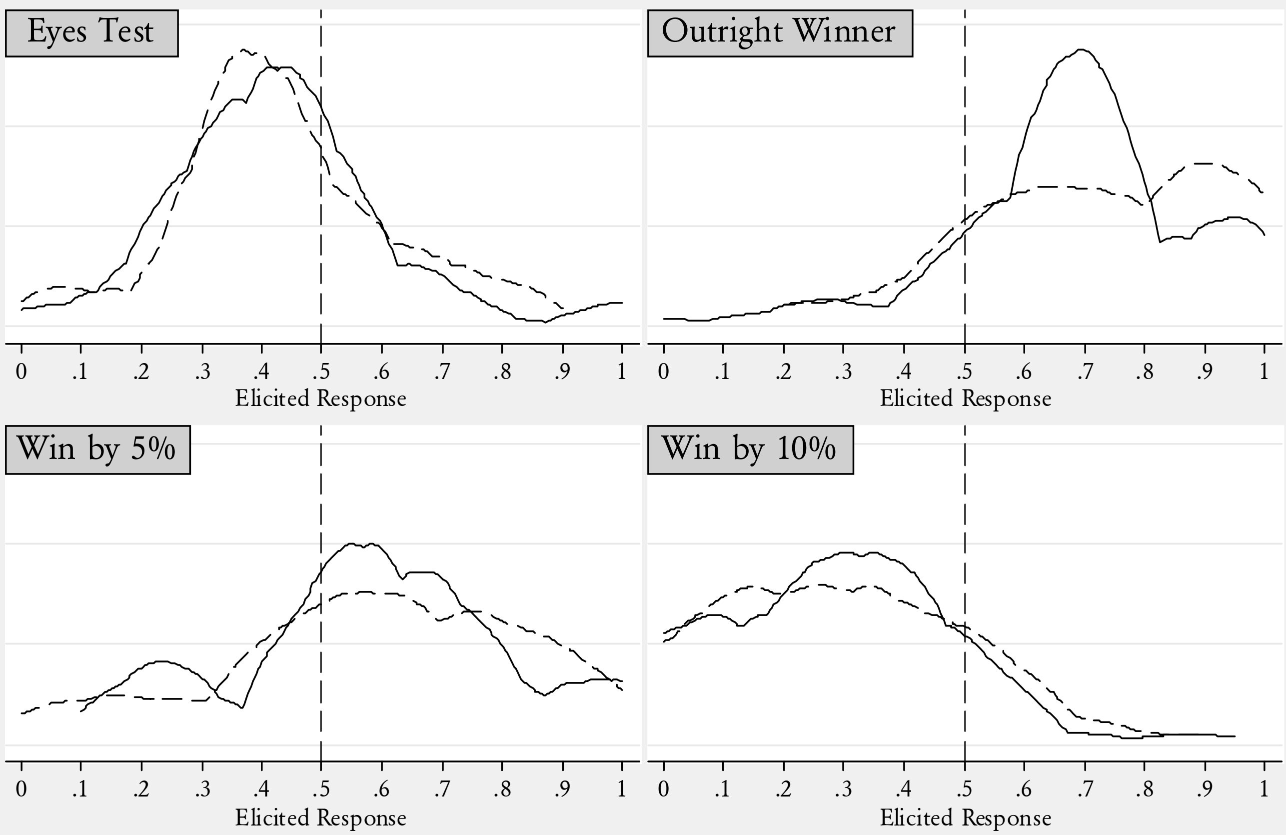


Table 2: Estimated Subjective Probabilities

Event	Specification (Log-Likelihood)	Point Estimate	Standard Error	95% Confidence Interval
Eyes Test	EUT (-4562.7)	0.41	0.023	0.37/0.46
	RDU (-4549.6)	0.30	0.037	0.23/0.38
	Raw Responses	0.43	0.19	0/0.95
President	EUT (-4547.0)	0.99	0.0004	0.99/1.00
	RDU (-4530.5)	0.99	0.0001	0.99/1.00
	Raw Responses	0.71	0.21	0.20/1
Win by 5%	EUT (-4579.3)	0.73	0.10	0.54/0.92
	RDU (-4562.8)	0.82	0.27	0.29/1.00
	Raw Responses	0.59	0.24	0.10/1
Win by 10%	EUT (-4546.5)	.0002	0.00018	0/0.0006
	RDU (-4528.8)	.0002	‡	‡
	Raw Responses	0.28	0.20	0/0.77

Notes: ‡ Due to numerical unreliability of the subjective probability at extreme "corner" values, the standard errors and confidence intervals could not be reliably estimated.

Table 3: Detailed EUT Estimates for Bets on Win Margin Being 5% or More

Variable	Description	Estimate	Standard Error	*p*-value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
r						
Constant		0.23	0.06	0.00	0.12	0.35
female	Female	0.14	0.07	0.04	0.01	0.28
Over22	Over 22 Years of age	0.09	0.10	0.36	-0.11	0.30
GPAhi	GPA great than 3.75	0.05	0.09	0.59	-0.13	0.23
Graduate	Graduate Student	-0.17	0.12	0.17	-0.40	0.07
α						
Constant		0.02	0.01	0.02	0.00	0.03
female	Female	0.02	0.01	0.18	-0.01	0.05
Over22	Over 22 Years of age	0.01	0.01	0.41	-0.02	0.04
GPAhi	GPA great than 3.75	0.00	0.02	1.00	-0.03	0.03
Graduate	Graduate Student	0.00	0.02	0.97	-0.03	0.03
μ_{RA}	Fechner Error on Risk Tasks	0.07	0.00	0.00	0.07	0.08
π						
Constant		-1.20	0.65	0.07	-2.47	0.08
female	Female	0.76	0.79	0.34	-0.80	2.31
Over22	Over 22 Years of age	-0.28	0.86	0.75	-1.97	1.42
GPAhi	GPA great than 3.75	0.32	0.46	0.49	-0.58	1.23
Graduate	Graduate Student	-0.48	1.55	0.76	-3.52	2.56
LSR	Linear Scoring Rule tasks	0.23	0.83	0.79	-1.40	1.85
μ_{BE}	Fechner Error on Belief Tasks	0.23	0.10	0.03	0.03	0.43

Figure 5: Estimated Subjective Probabilities and Raw Responses Assuming Expected Utility Model

Kernel density estimates, and empirical cumulative density.
Elicited responses in thin line, and risk-adjusted estimates in thick line.

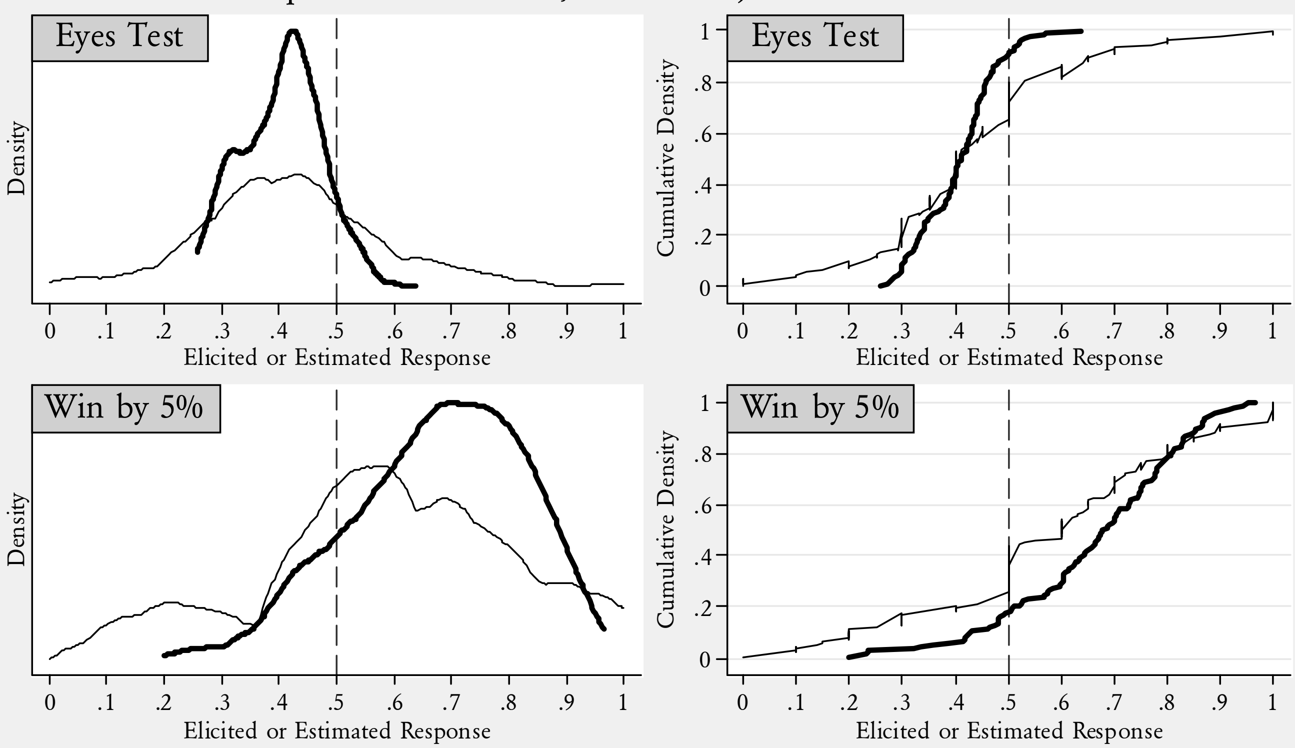


Figure 6: Estimated Subjective Probabilities and Raw Responses
Assuming Rank-Dependent Utility Model

Kernel density estimates, and empirical cumulative density.
Elicited responses in thin line, and risk-adjusted estimates in thick line.

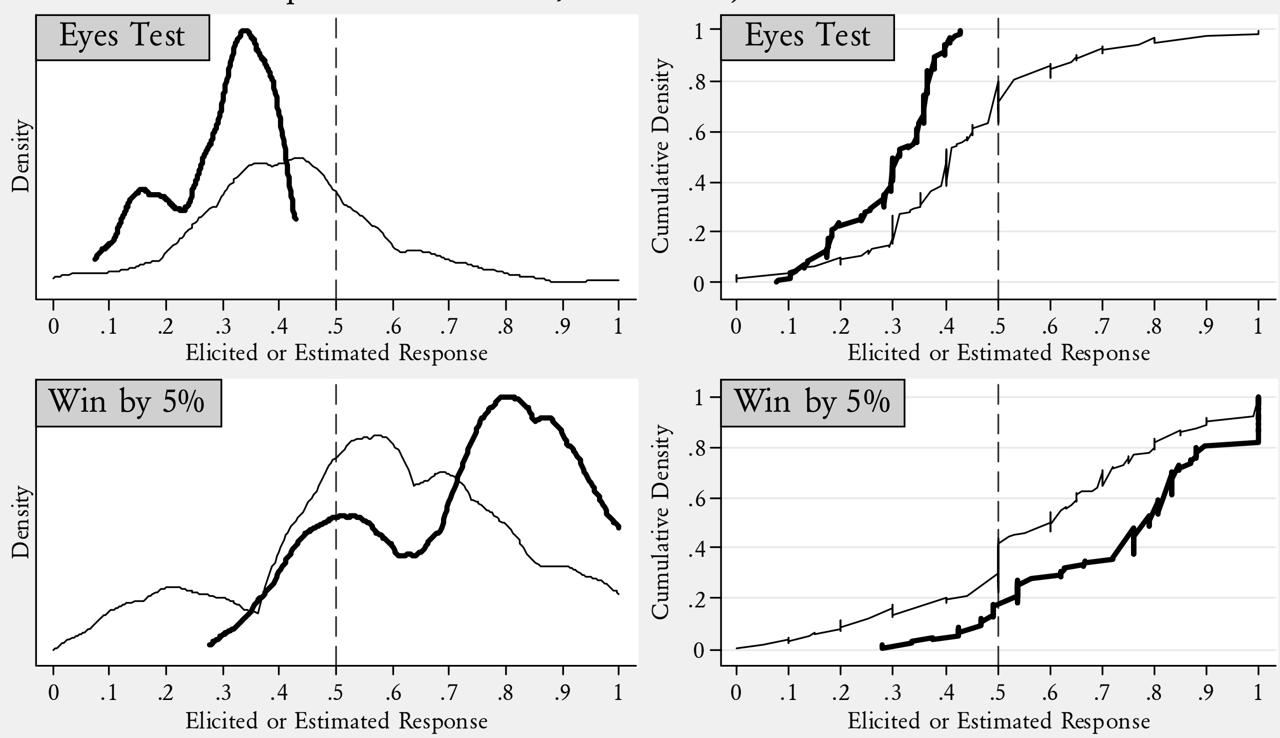
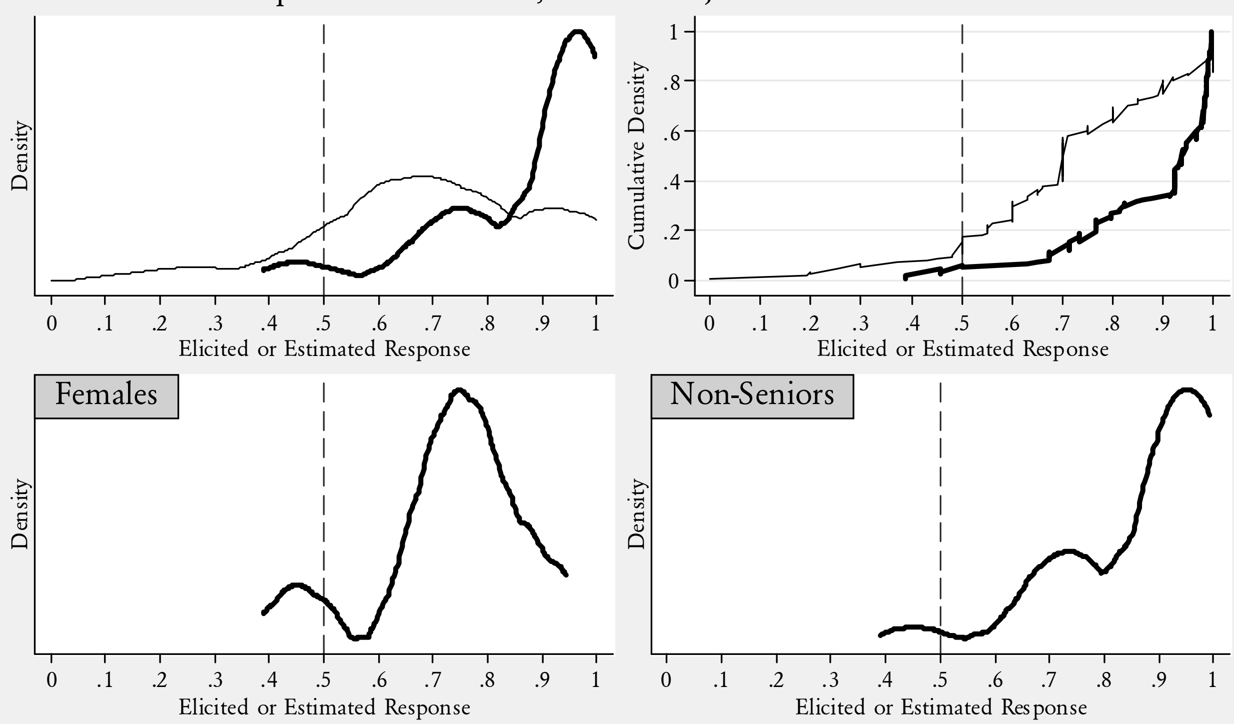


Figure 7: Calibration of Subjective Probabilities of an Outright Win

Assuming Expected Utility Model
Kernel density estimates, and empirical cumulative density.
Elicited responses in thin line, and risk-adjusted estimates in thick line.



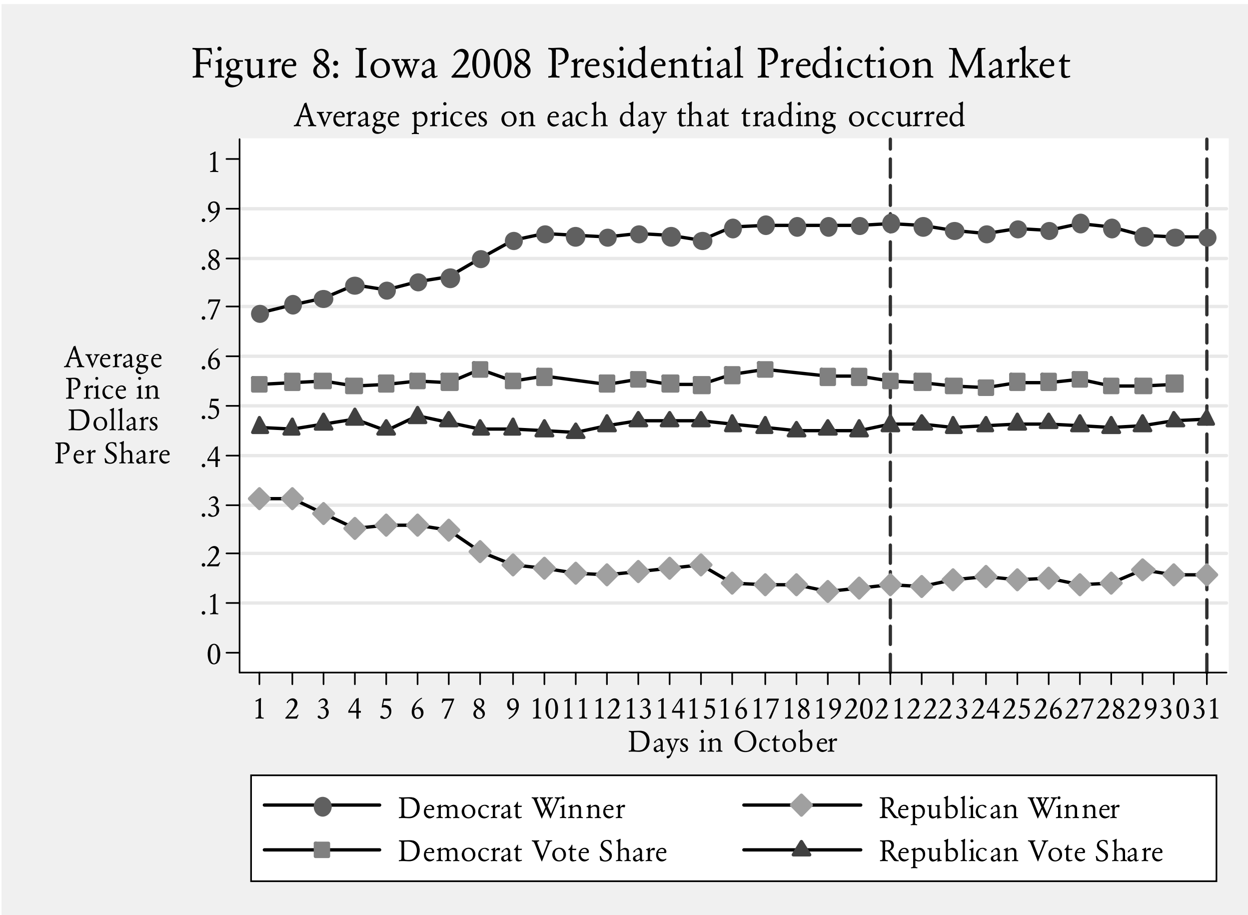


Figure 8: Iowa 2008 Presidential Prediction Market
Average prices on each day that trading occurred
Average Price in Dollars Per Share
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
Days in October
Democrat Winner
Republican Winner
Democrat Vote Share
Republican Vote Share

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Appendix A: Instructions

We provide complete instructions for the introduction to the session (marked I in the top, right corner of the first page of instructions, the quadratic scoring rule task trainer (marked q), the linear scoring rule trainer (marked L), the actual belief elicitation tasks for which subjects are paid (marked sr100), and then the lottery choice tasks (marked LOT). Copies of the exact instructions, which were printed in color, are available on request. Each subject received either the QSR or the LSR instructions.

YOUR INSTRUCTIONS

Welcome! Today we will be asking you several types of questions. Some of these will earn you cash, which we will pay you today. And some may earn you cash which we will pay you in a few weeks. You have already earned $5 for showing up and agreeing to participate.

There are basically four stages today:

1. We will ask you a series of questions about yourself, such as some basic information about your age. The computer will prompt you for these questions, and you should just work through them at your own pace when we log you in.

2. We will then pause, and provide some instructions on the next task, which involves you making some judgements about what someone in a picture is thinking. We will explain that task when we come to it.

3. We will then pause, and provide more instructions on the next task, which will involve you placing some bets on things that have yet to happen. In this stage we will take small breaks between the bets you place, so that we may explain the next specific thing that you are to bet on. These choices will directly affect your earnings. Nothing comes out of your own pocket.

4. We will then pause, and provide more instructions on some choices you are to make over different amounts of money that have different chances of occurring. These choices will also directly affect your earnings.

The instructions for the second, third and fourth stage will provide more information on the type of choices you are being asked to make.

The experimenters will then collate all of your earnings and pay you for the money you have earned, as well as provide a receipt for any earnings that will be paid in the future.

Your choices are private, and will only be associated with an ID that we will enter when we log you in to the computer. So your name, address and SSN will not be linked to any choices you make. We will pay you privately, one at a time, at the end to keep your earnings private.

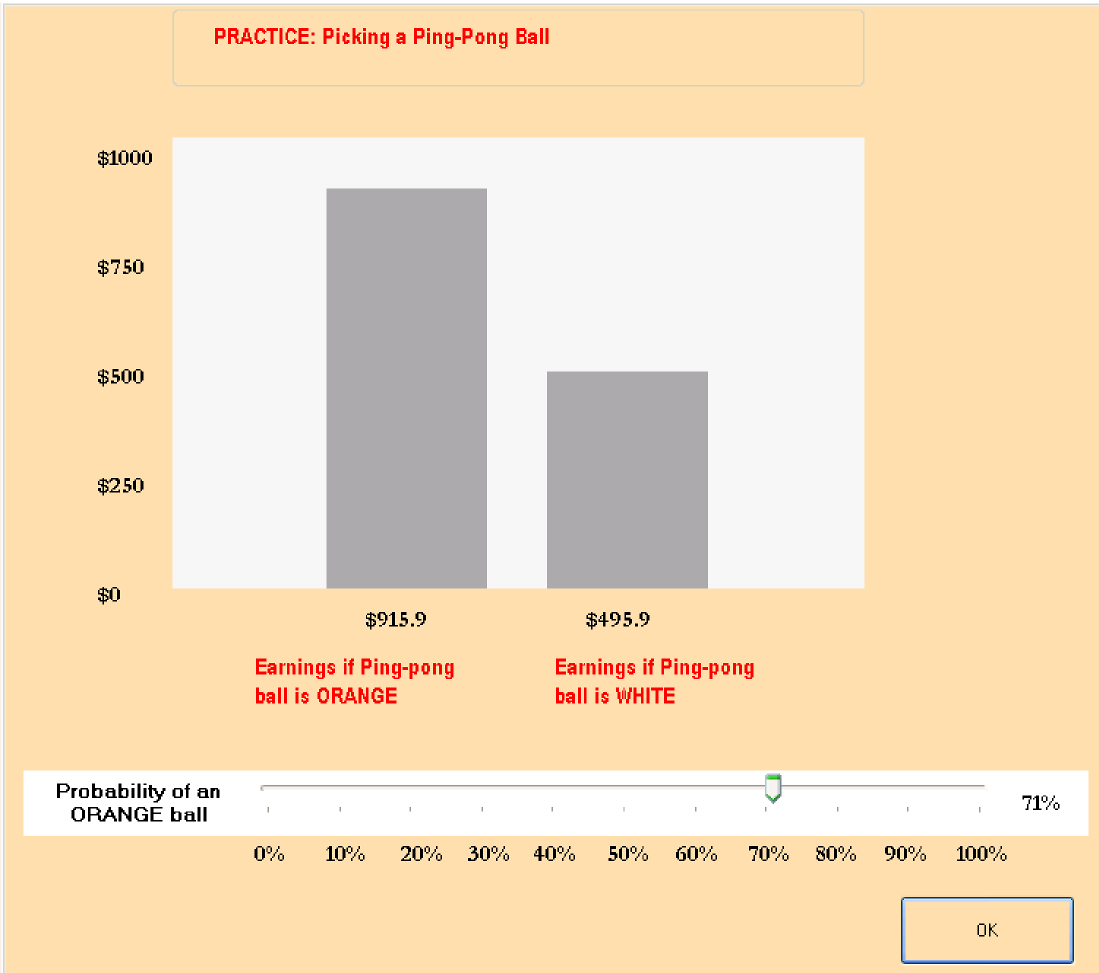
Are there any questions? If not, go ahead and answer the questions until the computer pauses and asks for a password. When everyone is finished this stage we will announce the password and we can go on to the second stage. There is no hurry, so take your time.

YOU MAY NOW PROCEED WITH THE FIRST STAGE.

INSTRUCTIONS (CONTINUED)

In this stage we will give you tasks where you will place bets on the outcome of events that will happen today or in the future. For example, who will be the next U.S. President? You can make more money the more accurately you can predict these outcomes.

You place these bets on a screen like the one below. In a moment we will let you practice with this screen on your computer. Remember, any betting you do today is with our money, not your money.



You place your bets by sliding the bar at the bottom of the screen until you are happy with your choice of a probability report. The computer will start at some point on this bar at random: in the above screen it started at 71%, but you are free to change this as much as you like. In fact, you should slide this bar and see the effects on earnings, until you are happy to confirm your choice.

ENTER THE PASSWORD THAT IS BEING ANNOUNCED NOW.

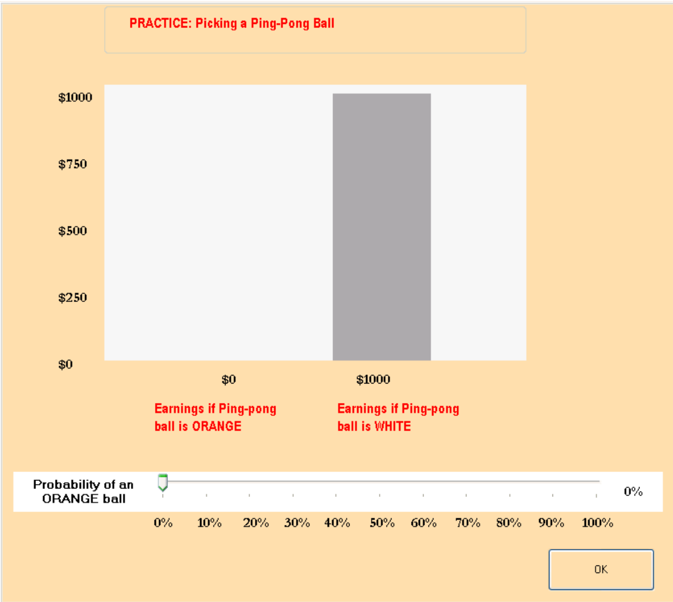
In this hypothetical example the maximum payoff you can earn is $1,000. In the actual tasks the maximum payoff will be lower than that, and we will tell you what it is when we come to those tasks. But the layout of the screen will be the same.

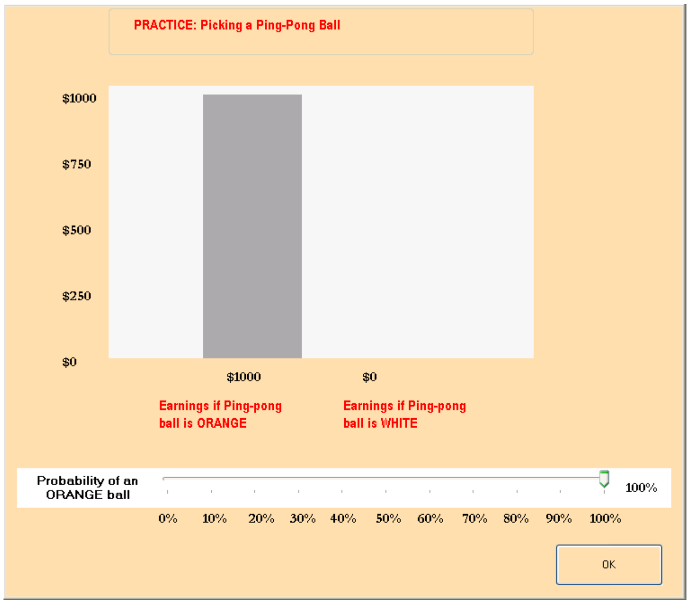
In this demonstration, the event you are betting on is whether a Ping Pong ball drawn from a bingo cage will be Orange or White. We have a bingo cage here, and we have 20 ping-pong balls. 15 of the balls are white, and 5 of the balls are orange. We will give the cage a few turns to scramble them up, and then select one ball at random.

What we want you to do is place a bet on which color will be picked. At the top of your screen we tell you what the event is: in this case, it is **Picking a Ping-Pong Ball**, and you need to bet on whether you think it will be **Orange or White**.

Your possible earnings are displayed in the two bars in the main part of the screen, and also in numbers at the bottom of each bar. For example, if you choose to report 71% you can see that you would earn $915.90 if the Ping Pong Ball was ORANGE, and $495.90 if the Ping Pong Ball was WHITE.

Lets see what happens if you make different reports. If you chose to report 0% or 100% here is what you would see, and earn:



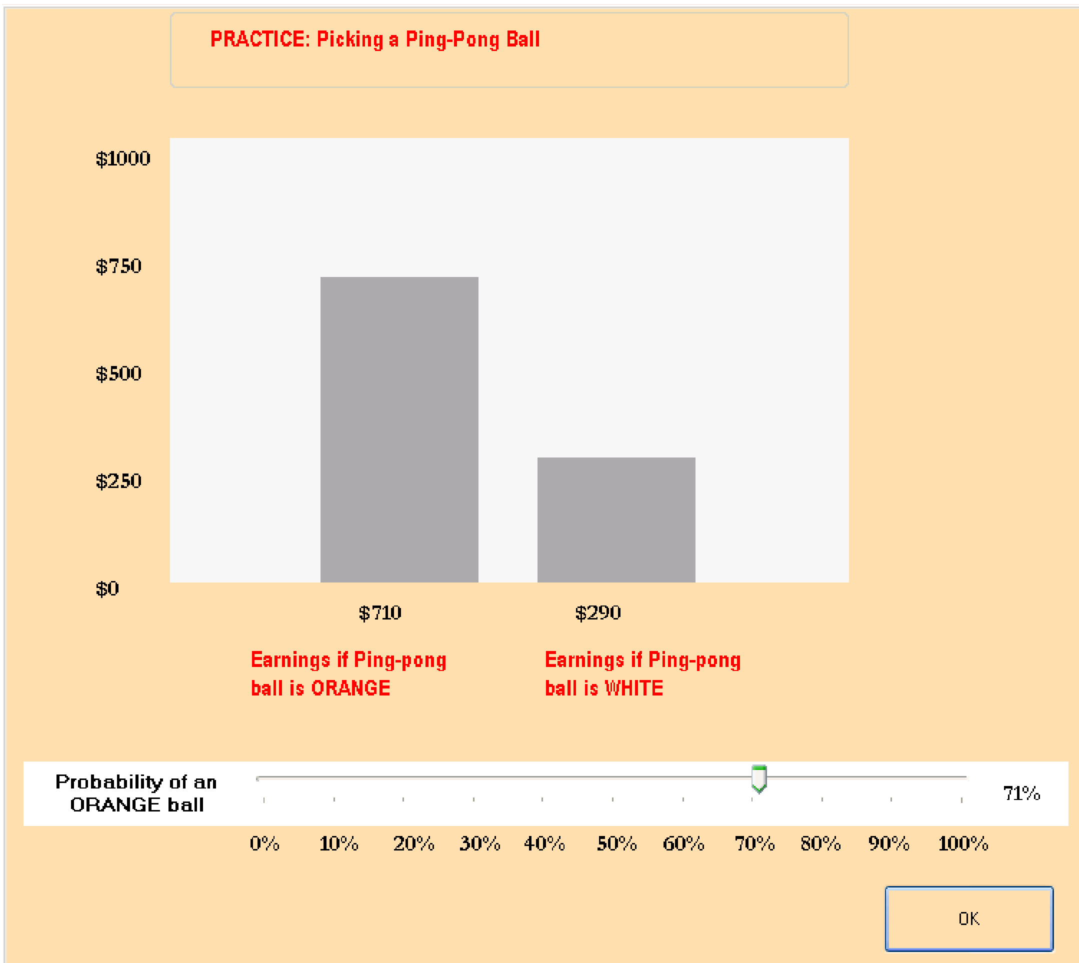


These screens are a little small, but you can see that these two reports lead to extreme payoffs. The “good news” is the possible $1,000 payoff, but the “bad news” is the possible $0 payoff. In between the reports of 0% and 100% you will have some positive payoff no matter what happens, but it will vary, as you can see from the report of 71%.

INSTRUCTIONS (CONTINUED) L

In this stage we will give you tasks where you will place bets on the outcome of events that will happen today or in the future. For example, who will be the next U.S. President? You can make more money the more accurately you can predict these outcomes.

You place these bets on a screen like the one below. In a moment we will let you practice with this screen on your computer. Remember, any betting you do today is with our money, not your money.



You place your bets by sliding the bar at the bottom of the screen until you are happy with your choice of a probability report. The computer will start at some point on this bar at random: in the above screen it started at 71%, but you are free to change this as much as you like. In fact, you should slide this bar and see the effects on earnings, until you are happy to confirm your choice.

ENTER THE PASSWORD THAT IS BEING ANNOUNCED NOW.

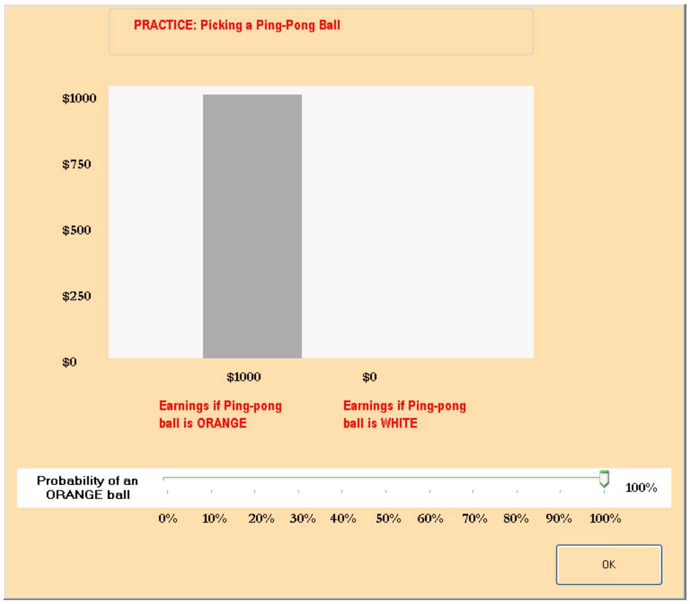
In this hypothetical example the maximum payoff you can earn is $1,000. In the actual tasks the maximum payoff will be lower than that, and we will tell you what it is when we come to those tasks. But the layout of the screen will be the same.

In this demonstration, the event you are betting on is whether a Ping Pong ball drawn from a bingo cage will be Orange or White. We have a bingo cage here, and we have 20 ping-pong balls. 15 of the balls are white, and 5 of the balls are orange. We will give the cage a few turns to scramble them up, and then select one ball at random.

What we want you to do is place a bet on which color will be picked. At the top of your screen we tell you what the event is: in this case, it is **Picking a Ping-Pong Ball**, and you need to bet on whether you think it will be **Orange or White**.

Your possible earnings are displayed in the two bars in the main part of the screen, and also in numbers at the bottom of each bar. For example, if you choose to report 71% you can see that you would earn $710 if the Ping Pong Ball was ORANGE, and $290 if the Ping Pong Ball was WHITE.

Lets see what happens if you make different reports. If you chose to report 0% or 100% here is what you would see, and earn:



These screens are a little small, but you can see that these two reports lead to extreme payoffs. The “good news” is the possible $1,000 payoff, but the “bad news” is the possible $0 payoff. In between the reports of 0% and 100% you will have some positive payoff no matter what happens, but it will vary, as you can see from the report of 71%.

Summarizing, then, there are two important points for you to keep in mind when placing your bets:

1. **Your belief about the chances of each outcome is a personal judgment that depends on information you have about the different events.** In this practice example, the information you have consists of the total number of Orange balls and White balls.

2. **Your choices might also depend on your willingness to take risks or to gamble.** There is no right choice for everyone. For example, in a horse race you might want to bet on the longshot since it will bring you more money if it wins. On the other hand, you might want to bet on the favorite since it is more likely to win something.

For each task, your choice will depend on two things: your judgment about how likely it is that each outcome will occur, and how much you like to gamble or take risks.

You will now make your report in this practice round. When you have chosen the report, confirm your bet by clicking on the OK tab.

After you click OK, a special box will come up which causes the program to pause. We will do this after every series of bets, and then explain what the next few bets are about. We will tell you what the password is when we are all ready to proceed. There is plenty of time, so there is no need to rush.

When everyone has placed their bets we will pick the ball and you will see what your earnings would have been if this had been for money. After that we will go on with the bets for which you can earn real money.

Does anyone have any questions?

ENTER THE PASSWORD THAT IS BEING ANNOUNCED NOW.

We are now ready to begin the choices for which you will be paid. There will be several sets of choices. In each case we will describe the bet to you, and then you can place your bets. Then we will explain the next couple of bets to you, and you place those bets.

Some of these bets will be about outcomes we know today, here and now, and some will be about outcomes we will only know in a few weeks. There will be 7 bets in all. We will pay you for one of these 7 bets. We will pick this bet at random after all bets are made, and tell you which one will be paid. You should view each bet as if it could be the one to determine your payoffs, since one of them actually will.

The maximum payoff for your bets today will be $100.

Ping Pong Balls Again

We will now repeat the task with Ping Pong balls a few times.

We have a number of ping pong balls in each of three bingo cages, which we have labeled Cage A, Cage B and Cage C. Some of the ping pong balls are Orange and some are White. We will roll each bingo cage and you can decide for yourself what fraction of Orange balls you think are in the cage. Of course, the balls will be rolling around, and you may not be able to tell exactly how many Orange balls are in the cage. You will be asked to bet on the color of one ping pong ball, selected at random after you all place your bets. For example, if there are 20 Orange balls and 80 White balls, the chance of an Orange ball being picked at random is 20 ÷ 100, or 20%.

We will do this task 3 times, with 3 different bingo cages. Just be sure that you check which cage you are placing a bet on. You can see this listed in the top left corner of your screen, where it refers to Cage A, Cage B or Cage C. We will show you each cage one at a time, and allow you to place your bets after we show it to you.

Do you have any questions?

The Eyes Test

At the beginning of today's experimental session we asked you to answer 36 questions, called **<u>The Eyes Test</u>**. These questions were designed by psychologists to measure a person's ability to "read someone else's mind" by just looking at their eyes.

Each and everyone of you were given the same 36 Eyes Test questions in today's experiment and a total score was recorded for each and every one of you in this experiment.

Now we come to the outcome we want you to place bets on in this portion of the experiment. We will pick one man and one woman in the room. Do you think the man who is selected will have a higher score on the Eyes Test than the woman who is selected?

After everyone in the experiment has made their bets for this event we will randomly select one man, and randomly select one woman from this experimental session. We will use the cards we collected, and sort then into one pile for men and one pile for women. Each pile will be shuffled, and one card drawn from each pile. We will then compare the score for the man that is drawn with the score for the woman that is drawn, and write these scores up on the board for you to see.

We therefore pose the following outcome for you to bet on now:

That the man we select at random will have a higher score

on the Eyes Test than the woman we select at random.

Do you have any questions?

2008 Presidential Elections

We want you to place bets on some questions about the U.S. Presidential Elections being held in a few weeks:

5. Will the next President of the United States be a Democrat or a Republican?

6. Will the popular vote for the winning candidate be 5 or more percentage points greater than the popular vote for the losing candidate?

7. Will the popular vote for the winning candidate be 10 or more percentage points greater than the popular vote for the losing candidate?

It is important that you understand that the first question is about the outcome of the Electoral College vote, and not the popular vote. The popular vote is just the sum of all votes across the United States. We are only referring to the Presidential Election, and not to any other elections that might occur on the same day.

For the second and third question, we are asking if you think that the winner of the popular vote will beat the loser by 5 or 10 percentage points or more. For example, if the winner of the popular vote gets 51% of the vote and the loser gets 49%, then this is a 2 percentage point difference. If the winner gets 53% and the loser gets 47%, then this is a 6 percentage point difference.

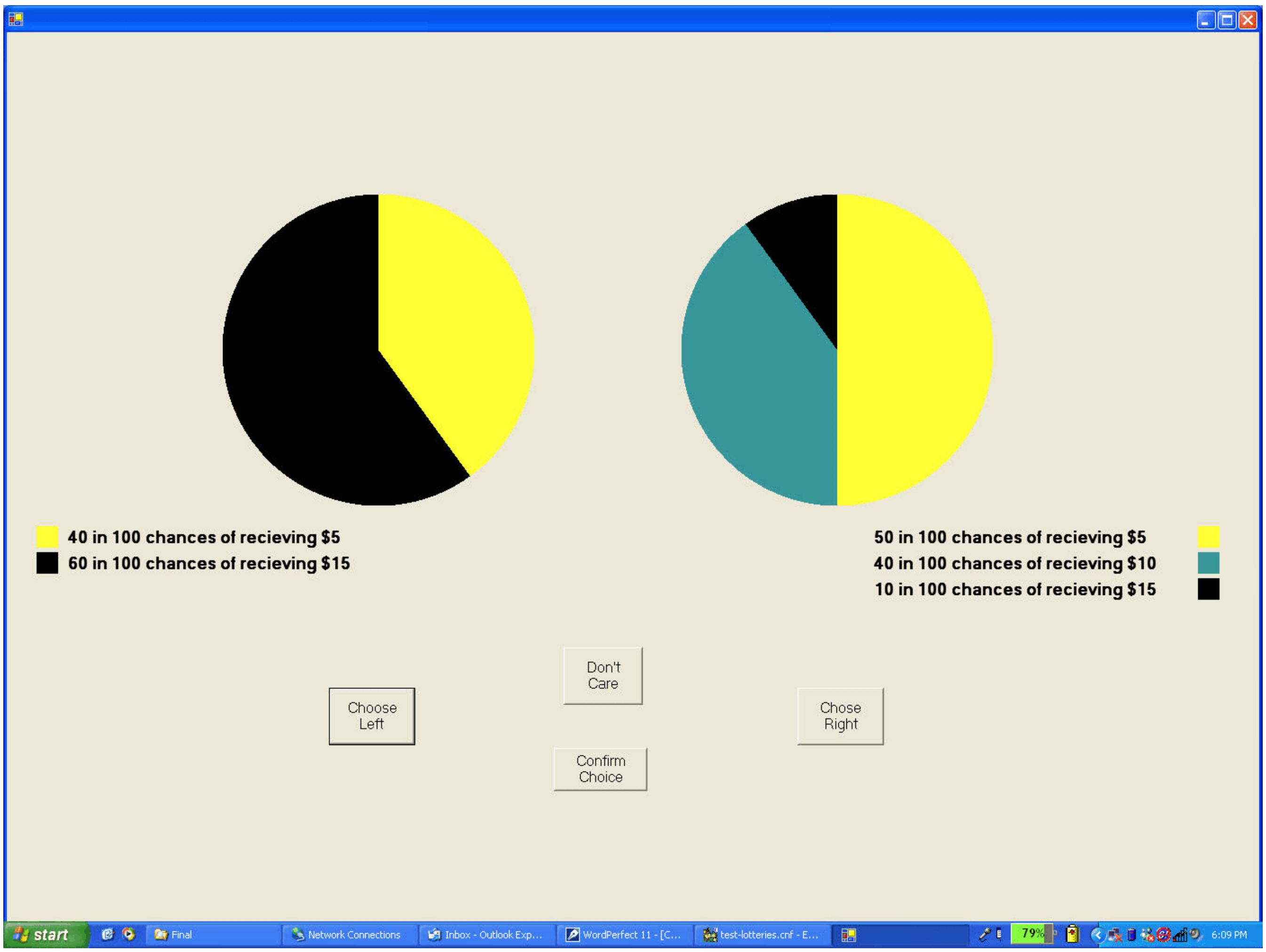
The election will be on Tuesday, November 4, 2008. To use a widely respected public source for the outcome, we will use the New York Times of Friday, November 7, 2008 as the official source used to determine your payoffs. In the event that there is a drawn out determination of the next President, such as in the 2000 election, we will delay payments until Inauguration Day, which is on January 20, 2009.

You will be paid for your bets in checks that will be mailed out on Monday, November 10, assuming we know who the next President will be at that time.

Please go ahead now and place your bets for this event, unless you have any questions.

This is the final stage of today's experiment. You will be asked to choose between lotteries with varying prizes and chances of winning. You will be presented with a series of lotteries where you will make choices between pairs of them. There are 45 pairs in the series. For each pair of lotteries, you should indicate which of the two lotteries you prefer to play. You will actually get the chance to play one of the lotteries you choose, and will be paid according to the outcome of that lottery, so you should think carefully about which lotteries you prefer.

Here is an example of what the computer display of such a pair of lotteries will look like. The display on your screen will be bigger and easier to read.



The outcome of the lotteries will be determined by the draw of a random number between 1 and 100. Each number between (and including) 1 and 100 is equally likely to occur. In fact, you will be able to draw the number yourself using two 10-sided dice.

In the above example the left lottery pays five dollars ($5) if the number on the dice that is rolled is between 1 and 40, and pays fifteen dollars ($15) if the number is between 41 and 100. The yellow color in the pie chart corresponds to 40% of the area and illustrates the chances that the number on the dice rolled will be between 1 and 40 and your prize will be $5. The black area in the pie chart corresponds to 60% of the area and illustrates the chances that the number on the dice rolled will be between 41 and 100 and your prize will be $15.

We have selected colors for the pie charts such that a darker color indicates a higher prize. White will be used when the prize is zero dollars ($0).

Now look at the pie in the chart on the right. It pays five dollars ($5) if the number on the dice rolled is between 1 and 50, ten dollars ($10) if the number is between 51 and 90, and fifteen dollars ($15) if the number is between 91 and 100. As with the lottery on the left, the pie slices represent the fraction of the possible numbers which yield each payoff. For example, the size of the $15 pie slice is 10% of the total pie.

Each pair of lotteries is shown on a separate screen on the computer. On each screen, you should indicate which of the lotteries you prefer to play by clicking on one of the three boxes beneath the lotteries. You should click the LEFT box if you prefer the lottery on the left, the RIGHT box if you prefer the lottery on the right, and the DON'T CARE box if you do not prefer one or the other.

You should approach each pair of lotteries as if it is the one out of the 45 that you will play out. If you chose DON'T CARE in the lottery pair that we play out, you will pick one using a 10-sided die, where the numbers 1-5 correspond to the left lottery and the numbers 6-10 to the right lottery.

After you have worked through all of the pairs of lotteries, raise your hand and an experimenter will come over. You will then roll two 10-sided die to determine which pair of lotteries that will be played out. You roll the die until a number between 1 and 45 comes up, and that is the lottery pair to be played. If you picked DON'T CARE for that pair, you will use the 10-sided die to decide which one you will play. Finally, you will roll the two 10-sided dice to determine the outcome of the lottery you chose.

For instance, suppose you picked the lottery on the left in the above example. If the random number you rolled was 37, you would win $5; if it was 93, you would get $15. If you picked the lottery on the right and drew the number 37, you would get $5; if it was 93, you would get $15.

Therefore, your payoff is determined by three things:

- by which lottery pair is chosen to be played out in the series of 45 such pairs using the two 10-sided die;

- by which lottery you selected, the left or the right, for that pair; and
- by the outcome of that lottery when you roll the two 10-sided die.

This is not a test of whether you can pick the best lottery in each pair, because none of the lotteries are necessarily better than the others. Which lotteries you prefer is a matter of personal taste. The people next to you will have different lotteries in front of them when you make your choices, and may have different tastes, so their responses should not matter to you. Please work silently, and make your choices by thinking carefully about each lottery.

All payoffs are in cash, and are in addition to the $5 show-up fee that you receive just for being here and any earnings from the previous stage.

We will now come around to your computer and get you started. When you are finished, please signal someone to come around to play out your lottery and record your earnings. As soon as you have finished the actual series, and after you have rolled the necessary dice, you will be asked to check with someone in the next room to make sure that your earnings sheet is complete, and then you will be paid. You are then free to go. Thanks again for your participation today!

Appendix B: Related Literature

A. Theory

The notion that subjective probabilities can be usefully viewed as prices at which one might trade has been a common one in statistics, and is associated with de Finetti [1937][1970] and Savage [1971]. It is also clear, of course, in the vast literature on gambling, particularly on the setting of odds by bookies and parimutuel markets (Epstein [1977; p. 298ff.]). The central insight is that a subjective probability is a marginal rate of substitution between contingent claims, where the contingency is the event that the probability refers to. There are then a myriad of ways in which one can operationalize this notion of a marginal rate of substitution.[17]

The formal link between scoring rules and optimizing decisions by agents is also familiar, particularly in Savage [1971], Kadane and Winkler [1987][1988] and Hanson [2003]. Jose, Nau, and Winkler [2008] stress the interpretation of several popular scoring rules from the perspective of an expected utility maximizing agent with preferences derived from familiar utility functions. Their approach may be viewed as complementary to ours: if one knows the utility function of the agent, they show which scoring rule is incentive compatible. We start with an arbitrary utility function and belief betting game, which can be viewed as derived from a particular scoring rule, and draw statistical inferences about subjective beliefs.

Karni [2009] proposes a procedure that has the considerable advantage of eliciting true reports of subjective probabilities without requiring corrections for risk attitudes. The procedure is akin to the utility elicitation procedure of Becker, DeGroot and Marschak [1964] and the probability elicitation procedure of Marschak [1964], but with an innovative twist. Let there be two prizes, $x>y$.

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[17] For example, one could elicit the p that makes the subject indifferent between a lottery paying M with probability p and m with probability p, for $M>m$, and a lottery paying M if the event occurs and m if it does not (Marschak [1964; p. 107ff.]). This method formally requires that one elicit indifference, which raises procedural issues.

The subject reports a probability ξ, and a random number ζ is selected from the unit interval. If $\zeta \geq \xi$ the subject gets the lottery that pays off x if the event occurs, and y otherwise; if $\zeta < \xi$ the subject gets the lottery that pays off x with probability ζ and y with probability 1-ζ. If the agent is not satiated, and is probabilistically sophisticated, it is a dominant strategy to report the true subjective probability as ξ. The main problem is that this class of procedures is known to have very poor incentive properties in practice (Harrison [1992]). For example, assume that x=$100 and y=$0, which is the range of prizes used in our scoring rule experiments, let the number ζ be selected from a uniform distribution on the unit interval, and let the true subjective probability be ¾. Taking 4,000 random draws from this distribution, the average payoff from reporting the truth is $78.825.[18] But the expected earnings from reporting 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1 is $49.48, $56.90, $63.70, $68.10, $72.53, $75.65, $77.63, $78.58, $78.83, $77.45 and $75.23, respectively. This implies expected losses of $3.18, $1.20, $0.25, $0 and $1.38 from reports of 0.5, 0.6, 0.7, 0.8 and 0.9, respectively. These are small incentives for responding truthfully. To be sure, they are positive (or they are at greater decimal accuracy than reported here), which is all that is needed for the theoretical result. But they pose potential behavioral problems for encouraging truthful and accurate responses.[19] The upshot is a tradeoff worth further investigation empirically: is the advantage of not having to elicit risk attitudes, and then correct scoring rule responses for them, worth the loss in the strength of incentives to report truthfully and accurately? Theory cannot answer this question.

B. Experiments

Experimental economists have used several of the popular scoring rules, but with one

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[18] A spreadsheet with these calculations is available on request.

[19] One might apply the procedure over "points" and then convert earnings of points into currency in a non-linear way so as to sharpen incentives, but at the risk of an additional layer to the complexity of the explaining the procedure to subjects.

notable exception discussed below, none have corrected for any deviation from risk neutrality.

The QSR was apparently first used by McKelvey and Page [1990], and later by Offerman, Sonnemans and Schram [1996], McDaniel and Rutström [2001], Nyarko and Schotter [2002], Schotter and Sopher [2003], Rutström and Wilcox [2006] and Costa-Gomes and Weizsäcker [2007].[20] In each case the subject is implicitly or explicitly assumed to be risk-neutral. Schotter and Sopher [2003; p. 504] recognize the role of risk aversion, but appear to argue that it is not a factor behaviorally:

> It can easily be demonstrated that this reward function provides an incentive for subjects to reveal their true beliefs about the actions of their opponents. Telling the truth is optimal; however, this is true only if the subjects are risk neutral. Risk aversion can lead subjects to make a "secure" prediction and place a .50 probability of each strategy. We see no evidence of this type of behavior.

Of course, evidence of subjects selected the probability report of ½ only shows that the subject has *extreme* risk aversion. The absence of that extreme evidence says nothing about the role that risk aversion might play in general.

Scoring rules that are linear in the absolute deviation of the estimate have been used by Dufwenberg and Gneezy [2000] and Haruvy, Lahav and Noussair [2007]. Croson [2000] and Hurley and Shogren [2005] used scoring rules that are linear in the absolute deviation as well as providing a bonus for an exactly correct prediction. It is well-known that linear scoring rules elicit the *median* of the subjective predictive distribution for a risk-neutral agent, and of course this will also be the mean and mode if the distribution is unimodal and symmetric.

Scoring rules that provide a positive reward for an "exact" prediction and zero otherwise have been used by Charness and Dufwenberg [2006] and Dufwenberg, Gächter and Hennig-Schmidt [2007]. In each case the inferential objective has been to test hypotheses drawn from

[20] Hanson [1996] contains some important corrections to some of the claims about QSR elicitation in McKelvey and Page [1990].

"psychological game theory," which rest entirely on making operational the beliefs of players in strategic games. In the former study the "exact" prediction of a probability was defined as an estimate within 5 percentage points of the true outcome; in the latter study the estimates were over 41 finite contribution levels in a public good, so the prediction had to be the correct integer to receive the reward. It is easy to show that this scoring rule elicits the *mode* for a risk-neutral agent. In the case of contributions to a public good this is not at all likely to be a unimodal and symmetric distribution, given the expectation of a significant spike at the zero contribution level implied by perfect free-riding (e.g., see the histograms displayed in Dufwenberg, Gächter and Hennig-Schmidt [2007; Appendix B]). The rationale for this scoring rule, rather than the QSR, is provided by Charness and Dufwenberg [2006; p.1586]:

> Overall, we chose our belief-elicitation protocol mainly because it is simple and rather easy to describe in instructions. [...] Our idea is to get a rough-but-meaningful ballpark estimate of participants' degrees of belief.

We certainly accept that the QSR can be difficult to explain to subjects if one relies on explicit payoff tables showing the mapping from reports to payoffs, but do not know what the phrase "rough-but-meaningful ballpark estimate" means: either we take the incentives to report beliefs seriously, or we do not.

Most experimental economists embed the elicitation of probabilities in another experimental task that the subject is undertaking. Indeed, one of the hypotheses being studied is whether the effort to elicit beliefs will encourage players in a game to think more strategically (Croson [2000], Rutström and Wilcox [2006], Costa-Gomes and Weizsäcker [2007]). Of course, this violates the "no stakes condition" required for the QSR to elicit beliefs reliably unless one assumes that the subject is risk neutral. Only one study employs a "spectator" treatment in which players are asked to provide beliefs but do not take part in the constituent game determining the event outcome: study #2 of

Offerman, Sonnemans and Schram [1996].

The most serious concern with the experimental implementations of scoring rules is that the rewards are very, very small. For example, Nyarko and Schotter [2002] and Rutström and Wilcox [2006] gave each subject an endowment of 10 cents, from which their penalties are to be deducted. So the effect of the scoring rule is literally defined in terms of fractions of pennies, and the additional rewards are not very substantial for optimal reports as compared to reports near the optimum. Whatever position one takes on the issue of "flat payoff functions" raised by Von Winterfeldt and Edwards [1986] and Harrison [1989], these rewards for accuracy are disappointing.

C. Econometrics

Our statistical approach uses the notion of joint estimation of preference parameters from several complementary experimental tasks, such as applied in other experimental settings by Harrison and Rutström [2008; p.100ff.] and Andersen, Harrison, Lau and Rutström [2008]. One insight from this joint estimation approach is that uncertainty about the core parameters of utility functions and/or probability weighting functions affects inferences that can be made about behavior in risk-related domains, as they should.

Only one study attempts to recover elicited beliefs from observed choices, calibrating for non-linear utility functions and/or probability weighting: Offerman, Sonnemans, van de Kuilen and Wakker [2007; §6].[21] They provide a statement of some alternative ways in which this recovery could be undertaken, essentially the method we use, and then propose a new method. Like us, they consider the recovery of true subjective beliefs when the agent may be risk averse in the narrow sense of EUT, as well as the broader sense implied by an allowance for probability weighting. Their

[21] The need for some correction is also recognized by Offerman, Sonnemans and Schram [1996; p.824, fn.8] and Rutström and Wilcox [2007; p.11, fn.8].

preferred approach has a reduced form simplicity, and is actually agnostic about which structural model of decision making under risk one uses. Our approach is explicitly structural, and generates inferences about subjective beliefs that are conditional on the assumed model of decision making under risk. We see these as complementary approaches, and both have strengths and weaknesses.

One method they consider is by estimating or eliciting the functional forms of a model of choice under risk (e.g., EUT, RDU or CPT), then observing beliefs over a natural event in some task, and econometrically recovering the implied subjective beliefs by using the estimated model of choice under risk to "back out" the subjective probability that must have been used in the belief elicitation task. They dismiss this approach, which is the one we follow. They claim, without further discussion, that estimating or eliciting the functional forms is "laborious" and that it involves "complex multi-parameter estimations." It is certainly true that the joint likelihood involves several parameters, but such estimation is standard fare with maximum likelihood modeling, so that is hardly a concern (unless one wants to avoid writing out customized likelihoods). It is not clear in what sense this is a "complex" undertaking. The labor involved depends on how one undertakes the estimation or elicitation. In our case the subjects need to do one task, which consists of 60 binary choices over lotteries, and then all of the labor involved is by the computer estimating maximum likelihood models that have been well-studied for years (e.g., Harrison and Rutström [2008; §2] for a survey).[22]

The empirical method they use instead has an attractive reduced form simplicity. For a given subject, it uses the QSR to elicit reported probabilities for naturally occurring events, and then uses

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[22] On the other hand, if one uses other elicitation procedures, such as the Trade-Off design of Wakker and Deneffe [1996], Fennema and van Assen [1999], Abdellaoui [2000] and Abdellaoui, Bleichrodt and Paraschiv [2007], then the procedures can indeed become laborious for the subject. There are other reasons not to use these methods, the most significant of which is their lack of incentive compatibility as conventionally applied (Harrison and Rutström [2008; §1.5]). But these methods are not needed, and the stated concerns with this approach to recovering subjective beliefs are not substantial.

the QSR in a calibration task to elicit a "risk correction function" that allows them to recover the subjective probability that generated the report for the *naturally occurring event*. The risk correction function simply elicits reports for "objective probabilities," such as the chance that a single roll of a 100-sided die will come up between 1 and 25. Assume the subject report 0.30 for this event. Then, if the subject ever reported a 0.30 in the initial task for the naturally occurring event, they would infer that he had a subjective probability of 0.25 underlying it, since that was the objective probability that generated this report using the (same) scoring rule. Thus the difference between the report of 0.30 in the calibration task and the true underlying probability is attributed solely to the effects of non-linear utility and/or probability weighting. By eliciting a risk correction function for a wide range of probabilities, and with a sufficiently fine grid, one can recover any report with some reasonable accuracy.

This approach is attractive, and avoids the need for the researcher to "take a stand" on which model of choice under uncertainty determines betting behavior. To see the key assumption underlying their approach, let ϕ be the *actuarial* probability that the calibration event will occur. For some artefactual events, such as tossing coins and rolling die, ϕ is well defined, but for other events it is not so well defined. Let $\pi(\phi)$ be the function that summarizes the subjective belief that the subject actually holds that the calibration event will occur, and let $R(\pi(\phi))$ be the function transforming $\pi(\phi)$ into a report using the QSR, or any appropriate scoring rule. Offerman et al. [2007] first assume that $\pi(\phi) = \phi$ *in the calibration task*, so that the only reason that $R(\pi(\phi)) \neq \phi$ is that the subject has non-linear utility and/or undertakes probability weighting.[23] Why might $\pi(\phi) \neq$

[23] So there is no allowance for subjects to make decision errors in the calibration task, or the elicitation task for the naturally occurring event for that matter. These errors could be subsumed into some sampling error on estimates of $R(\pi(\phi))$ as an empirical function of ϕ, but then one is relying on the errors being well-behaved statistically. In fact, Offerman et al. [2007; equation (8.3)] do allow for an additive error term which they assume to be truncated normal to ensure that reported probabilities lie between 0 and 1. Their pooled estimates indicate that there is a need for some correction for non-linear utility, but that it is not

ϕ, for such simple tasks? Apart from concerns with loaded die, or certain cultures imbuing randomizing devices or colors on chips with some animist intent, we would be concerned with psychological editing processes based on similarity relations. To take a simplistic example, someone might "round down" to the nearest increment of 0.05 or 0.10 and then decide how to report using this subjectively edited probability $\pi(\phi)$ as the basis for any adjustments due to non-linear utility or probability weighting. Is the actuarial probability ϕ the one we really want to compare $R(\pi(\phi))$ to in such a case, or is it $\pi(\phi)$?

This might seem to be nit-picking when it comes to the rolling of a 100-sided die in the calibration task, and perhaps viewed as part of a latent structural psychological story underlying the idea of probability weighting. But it is surely more significant for naturally occurring events. Here is where the second assumption comes in: that $\pi(\phi) = \phi$ *in the belief elicitation task* where ϕ is defined (or not) over naturally occurring events. Thus, what if we accept that $\pi(\phi) = \phi$ is a reasonable assumption for the calibration task with the artefactual event, but cannot be so sure for the task with the naturally occurring event? Our position is that we are recovering $\pi(\phi)$, "warts and all" in terms of how the subject conceives of the event and defines the probability ϕ. Offerman et al. [2007] would appear to be recovering the "touched up" image of $\pi(\phi)$, ϕ, after the warts have been removed.

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so clear that probability weighting is an issue (the log-likelihood for row #1 in their Table 9.1, which allows for both effects, is virtually identical to the log-likelihood for row #5, in which no probability weighting is assumed). Although they allow for errors to vary with an incentives treatment applied between-subjects, it would be useful to extend their statistical analysis of the pooled data to allow for correlated errors at the level of the individual subject, rather than implicitly assume homoskedasticity.

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Appendix C. Additional Statistical Results

Table C1: Estimates of the Utility Function Under EUT

Variable	Description	Estimate	Standard Error	*p*-value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
		A. No Covariates				
r		0.30	0.03	0.00	0.23	0.36
α		0.03	0.01	0.00	0.02	0.04
μ		0.07	0.00	0.00	0.07	0.08
		B. Adding Covariates				
r						
Constant		0.23	0.06	0.00	0.12	0.35
female	Female	0.14	0.07	0.04	0.01	0.28
Over22	Over 22 Years of age	0.09	0.10	0.37	-0.11	0.29
GPAhi	GPA great than 3.75	0.05	0.09	0.57	-0.13	0.23
Graduate	Graduate Student	-0.16	0.12	0.17	-0.40	0.07
α						
Constant		0.02	0.01	0.02	0.00	0.03
female	Female	0.02	0.01	0.18	-0.01	0.05
Over22	Over 22 Years of age	0.01	0.01	0.39	-0.02	0.04
GPAhi	GPA great than 3.75	0.00	0.02	0.96	-0.03	0.03
Graduate	Graduate Student	0.00	0.02	0.95	-0.03	0.03
μ		0.07	0.00	0.00	0.07	0.08

Table C2: Estimates of the Utility Function Under RDU

Variable	Description	Estimate	Standard Error	*p*-value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
		A. No Covariates				
r		0.49	0.05	0.00	0.39	0.58
α		0.08	0.02	0.00	0.04	0.12
γ		0.72	0.06	0.00	0.60	0.83
μ		0.08	0.00	0.00	0.07	0.09
		B. Adding Covariates				
r						
Constant		0.45	0.07	0.00	0.31	0.58
female	Female	0.08	0.10	0.93	-0.18	0.20
Over22	Over 22 Years of age	0.02	0.15	0.87	-0.28	0.33
GPAhi	GPA great than 3.75	0.07	0.10	0.48	-0.12	0.26
Graduate	Graduate Student	0.18	0.25	0.46	-0.30	0.67
α						
Constant		0.05	0.02	0.04	0.00	0.09
female	Female	-0.01	0.04	0.82	-0.08	0.06
Over22	Over 22 Years of age	0.03	0.08	0.72	-0.12	0.18
GPAhi	GPA great than 3.75	0.07	0.07	0.30	-0.06	0.21
Graduate	Graduate Student	0.29	0.37	0.42	-0.42	1.01
γ						
Constant		0.72	0.07	0.00	0.59	0.86
female	Female	0.15	0.10	0.15	-0.05	0.35
Over22	Over 22 Years of age	0.05	0.25	0.85	-0.45	0.54
GPAhi	GPA great than 3.75	-0.09	0.13	0.46	-0.34	0.16
Graduate	Graduate Student	-0.47	0.41	0.25	-1.28	0.33
μ		0.07	0.01	0.00	0.06	0.08