Real-time Pricing in Power Markets: Who Gains?

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Abstract

We examine welfare effects of real-time pricing in electricity markets. Before stochastic energy demand is known, competitive retailers contract with final consumers who exogenously do not have real-time meters. After demand is realized, two electricity generators compete in a uniform price auction to satisfy demand from retailers acting on behalf of subscribed customers and from consumers with real-time meters. Increasing the number of consumers on real-time pricing does not always increase welfare since risk-averse consumers dislike uncertain and high prices arising through market power. In the Bertrand case, welfare is the same with all or no consumers on smart meters.

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1 Introduction

Real-time pricing of electricity for residential households and small businesses was for a long time technologically and economically not viable. Traditionally, final consumers had a meter that simply measured the total amount of electricity consumed without keeping track of when consumers actually consumed what amounts of electricity. For this reason it was not possible to differentiate prices to reflect the scarcity of electricity at each point in time, which made consumers unable to react to price signals. This lack of consumer response translates into highly inelastic market demand in electricity wholesale markets, which facilitates the exercise of market power especially in peak times, (see, e.g., Stoft, 2002, p.78f). In addition the absence of price signals prevents any consumption smoothing over time and thus aggravates the system operator's problem to constantly balance supply and demand. Since electricity is hardly storable, not achieving a balance results in costly blackouts and consumer rationing.

Recent technological developments and the rising need for more efficient power grids have however increased the attention on exploiting efficiency potentials through smarter metering. A number of firms have invented new meter technologies to reap such efficiency gains, which led to a drastic increase in venture capital for smart meter technologies.[1] This new development of *smart grids* and *smart meters* aims at allowing electricity providers to transmit time varying price signals, that in turn enable even residential households and small businesses to adjust their consumption over the day accordingly.[2] However, the installation of smart meters and smart grids changes the design of all current transmission networks and is extremely costly. Thus, there is considerable uncertainty in the welfare effects and the profitability of real-time metering technology. We ask how the introduction of real-time metering will benefit consumers, producers and overall welfare.

[1] See The Economist (2009b).

[2] See The Economist (2009a).

That prices should fluctuate if capacities cannot easily be adapted to fluctuating and uncertain demand is an insight gained from the peak-load pricing literature that started already in the fifties (for a survey see Crew et al. (1995)). More recently, Borenstein and Holland (2005) developed a model where firms invest in electricity generating capacity in the first stage and then compete in a perfectly competitive electricity market for a certain number of periods with a time-varying demand. They show that the market outcome is not efficient if not all consumers are on real time pricing schedules. Furthermore, the market outcome is less than second-best efficient, even if one takes into account that some consumers are priced only according to the average wholesale costs of serving them with a time-invariant price instead of paying the time-variant wholesale price in each single period. However, increasing the number of customers on real-time pricing does not necessarily increase social welfare, although having *all* customers on real-time pricing is always Pareto superior to having some of them on time-invariant rates.[3]

We derive efficiency effects of real-time pricing when generating firms have market power in the electricity wholesale market and consumers are risk-averse. We explicitly distinguish between the wholesale and the retail market of electricity, and assume market power in the wholesale market with only two firms generating and selling electricity. The retail sector is perfectly competitive. Like in Borenstein and Holland (2005), we assume that consumers who are on real-time pricing schedules can express their demand on the wholesale market either directly or indirectly via their competitive retailer. The consumers who are not on real-time meters need to contract with retailers before their own and the aggregate level of demand is known. Therefore they will finally pay the same price no matter what the level of demand will be. Joskow and Tirole (2006) and Joskow and Tirole (2007) both

[3]Holland and Mansur (2006) simulate the short-run efficiency gains without capacity investments from increasing the share of customers on real-time pricing in a model close to Borenstein and Holland (2005) for the PJM market and can only identify moderate efficiency increases for this case.

mainly focus on the retail market without taking into account repercussions to the potentially non-competitive wholesale market.[4] Contrary to the former and in line with the latter we abstract from monopoly distribution and assume that all retailers compete on a level playing field. Like Borenstein and Holland (2005), we model uniform retail prices whereas Joskow and Tirole (2006) and Joskow and Tirole (2007) allow for two-part tariffs.

Neither Borenstein and Holland (2005) nor the empirical studies that try to estimate the welfare effects of existing real-time pricing initiatives for large industrial customers (see Taylor et al. (2005)) take into account the insurance effect of fixed prices. Most analyses implicitly assume that the volatile demand is certain and therefore sum up the consumer surplus for all different time periods to determine consumer welfare. We instead assume demand uncertainty and consider concave surplus functions for our costumers when deriving welfare statements. Taking this risk aversion into account explicitly allows us to check whether the positive efficiency effects of real-time electricity pricing are potentially counteracted by the increase in price risks that risk-averse consumers dislike.

Our model is based on Boom and Buehler (2007). We introduce real time pricing and differentiated consumers, that is, each consumer demands a different quantity of electricity although they are all exposed to the same demand shock. Motivated by the observation that in most electricity markets larger consumers, e.g. private businesses, installed smart meters before smaller customers such as private households did, we assume that consumers with the highest demand will be served with real-time metering and pricing first.[5] As the degree of real-time pricing increases, the consumers that enter

[4] Joskow and Tirole (2007) derive optimal retail prices, rationing rules and capacity investments with price-sensitive and price-insensitive consumers.

[5] Empirical studies of existing real time pricing programs with the exception of Allcott (2009) focus only on large industrial customers (see Patrick and Wolak (2001), Taylor et al. (2005), Boisvert et al. (2007), and Zarnikau and Hallett (2008)). Allcott (2009) is the only one who reports on a small scale real-time pricing experiment with residential households in Chicago in 2003.

real-time pricing in our model will have a lower demand than those already in the program. Hence, our model set up also allows us to conclude whether real-time pricing is more beneficial for large or small customers.

The next section presents the modeling framework. Section three derives the model outcome and presents wholesale and retail market equilibria. In section four, we present comparative statics in the level of real-time pricing and derive welfare statements. Section five concludes.

2 The model

In a mass of N consumers with $N = 1$, each consumer can be of a different type α which is drawn from a uniform distribution on $\left[\frac{1}{2}, \frac{3}{2}\right]$. The preferences of a consumer of type α are represented by the consumer surplus function

$$V(x, \alpha, \varepsilon, p) = \alpha(x - \varepsilon) - \frac{(x - \varepsilon)^2}{2} - px, \tag{1}$$

where p is the electricity price, x the electricity consumed and ε a shock that affects all consumers alike and is drawn from a uniform distribution on $[0, 1]$. Maximizing the surplus with respect to the consumed electricity x yields the consumer's individual demand[6]

$$x(p, \alpha, \varepsilon) = \max\{\alpha + \varepsilon - p, 0\}. \tag{2}$$

We assume that the consumers with small demand, meaning $\alpha \leq \tilde{\alpha}$, do not have a smart meter and need to contract with one of the retailers and pay the retail price $p = r$. Consumers with a relatively large demand, defined by $\alpha > \tilde{\alpha}$ are on real-time meters and purchase their electricity directly on the wholesale market at the wholesale price $p = p^*$.[7] The threshold separating

[6]The demand is modeled similarly to Boom and Buehler (2007) and Boom (2009). However, there all consumers have $\alpha = 1$ and thus identical demand.

[7]Note that it does not matter whether customers on real-time meters bid their demand

large and small consumers, i.e. consumers with and without real-time meters, has to lie within the support of α, that is, $\frac{1}{2} \leq \tilde{\alpha} \leq \frac{3}{2}$.

There are $n \geq 2$ retailers who compete à la Bertrand. Consumers without a smart meter subscribe to the retailer with the lowest retail price r while their actual level of demand is still uncertain. For the sake of simplicity we assume zero retail costs. Retailers' marginal costs then equal the wholesale price for which they buy electricity. The retailers announce their customers' demand for electricity to the wholesale auction after they have observed the actual level of demand, that is the realization of ε. Retailers with supply obligations go out of business as soon as their marginal costs, the wholesale market price p^*, exceeds the retail price r. Then their customers will not be served with electricity, but the system operator is able to ration retail consumers and a blackout does not occur.[8]

Electricity is only produced by two electricity generating firms A and B. Each generator $i = A, B$ is capacity constrained and owns capacity K_i. Both generators use an identical technology with constant marginal costs c which are normalized to zero. Generating firms can produce up to their capacity K_i but not beyond that quantity. They can sell their electricity only via the wholesale market, run by the system operator as a uniform price auction. Before each firm submits its supply bid to the wholesale market, the total demand, meaning the level of ε, is publicly known. In the auction each firm only announces a price p_i at and above which they are willing to produce up to their total capacity. Fabra and von der Fehr (2006) show in their analysis that despite different optimal bidding strategies the market outcome would not change if we allowed for a finite but larger number of steps in the bidding

directly into the wholesale auction or whether they have a contract with a perfectly competitive retailer without retail costs. Borenstein and Holland (2005), Joskow and Tirole (2006) and (2007) also assume that consumers on real-time pricing pay the wholesale price.

[8]The latter assumption means that the system operator has perfect control over the grid and can selectively take customers off-line. This assumption is in line with Joskow and Tirole (2007) and will finally lead to efficient rationing. In the perfect smart grid scenario, efficient rationing is possible. However, today it is not implementable.

function of the generators.

The system operator runs a uniform price auction.[9] To clear the auction, the system operator first aggregates all submitted capacity at each price bid, and then finds the market clearing price, that equates supply and the level of demand stemming from the consumers on smart-meters and the ones that contract with a retailer and pay the retail price r. Three situations can occur:

1. The capacity of the low-bidding generator is sufficient to satisfy all demand at this low price. The wholesale price $p^* = p_i$ with $p_i \leq p_j$ and $i, j = A, B$ and only the low bidding firm is called to generate the amount of electricity necessary to satisfy demand $D(p^*, r, \tilde{\alpha}, \varepsilon)$.

2. The capacity of the low-bidding firm is insufficient to satisfy demand at this low price, but the total capacity of both firms is sufficient to satisfy the demand at the higher of the two prices. The wholesale price is $p^* = p_j$ with $p_i \leq p_j$ and $i, j = A, B$. The low-bidding firm can deliver its total capacity K_i whereas the high-bidding firm is rationed to the amount of electricity that is necessary to satisfy residual demand $(D(p^*, r, \tilde{\alpha}, \varepsilon) - K_i)$.

3. The capacity of the low-bidding firm is insufficient to satisfy demand at this low price and total capacity is also insufficient to satisfy the demand at the higher of the two prices. The wholesale price p^* is the price at which total demand satisfies total capacity $(D(p^*, r, \tilde{\alpha}, \varepsilon) = K_A + K_B)$. Both firms generate electricity at their capacity constraint.

All generators are paid the equilibrium price p^* for all the electricity they deliver no matter what their price bid was. Before this wholesale auction

[9] Multi-unit uniform price auctions are used in most major electricity markets in Europe and the US. The other alternative is a discriminatory auction format, to which the UK market switched in 2001 when introducing the New Electricity Trading Arrangements (NETA). For a theoretical comparison of both auction formats see Fabra and von der Fehr (2006).

is held, retailers contract with the final consumers. Figure 1 illustrates the timing of the model.

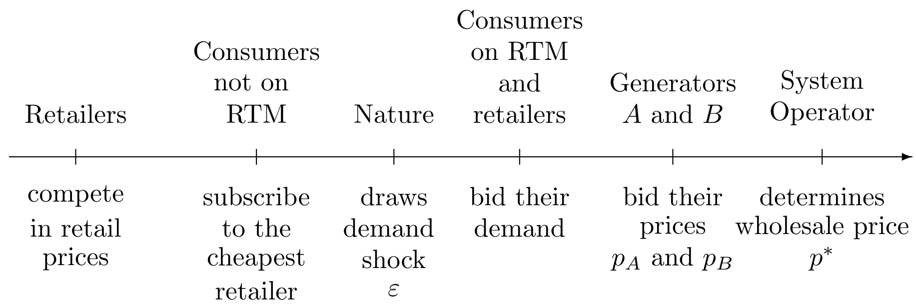

Figure 1: *Timing of the model*

In the first stage of the game before the level of demand is known retailers set their retail prices for customers without real time meters. These customers contract with the retailer who offers the lowest price.[10] Then, nature draws the demand shock ε and demand is known to the generators, the retailers, the consumers with real-time metering and the system operator. Consumers with real-time metering bid their demand, and non-bankrupt retailers the demand of their contracted customers. The two generators bid the prices at which they are willing to produce up to their total capacity, and finally the system operator determines the wholesale electricity price p^* as described above. We search for the subgame perfect equilibrium of this game.

3 Analysis of the model

Since we are looking for a subgame-perfect equilibrium of this game we start the analysis with the last stage of the game, the wholesale market. After

[10]The contract is a service contract and implies that the costumers are provided with as much electricity as they want as long as the retailer does not go out of business. Rationing rules as discussed in Joskow and Tirole (2007) are not part of the contract and are also not very common for residential households.

deriving the market outcome of the wholesale market for given retail prices, capacities and levels of smart metering we determine the retail price for those customers who do not have a smart meter for given capacities and levels of smart metering.

3.1 The wholesale market

By the time the wholesale market clears, the demand shock $\varepsilon \in [0, 1]$ is known to all market participants. The threshold $\tilde{\alpha} \in [\frac{1}{2}, \frac{3}{2}]$ defines the mass of consumers who are on pre-determined fixed retail prices and the mass of consumers who have a smart meter and can directly participate in the wholesale market. This threshold is exogenous and known to all market participants.

The group with $\alpha \leq \tilde{\alpha}$ buys electricity via their retailers and pays the pre-determined retail price r. Given r, their retailers demand a fixed volume of electricity which is derived from aggregating their individual demand, given in (2). The retailers' demand from consumers without a smart meter is represented by

$$D^R(r, \tilde{\alpha}, \varepsilon, p^*) = \begin{cases} \int_{\frac{1}{2}}^{\tilde{\alpha}} \alpha + \varepsilon - r d\alpha & \text{if } p^* \leq r \leq \frac{1}{2} + \varepsilon, \\ \int_{r-\epsilon}^{\tilde{\alpha}} \alpha + \varepsilon - r d\alpha & \text{if } \max\left\{p^*, \frac{1}{2} + \varepsilon\right\} \leq r \leq \\ & \quad \tilde{\alpha} + \varepsilon, \\ 0 & \text{if either } r < p^* \text{ or } r > \tilde{\alpha} + \varepsilon. \end{cases} \tag{3}$$

Retail demand is completely inelastic in the wholesale price p^*. As soon as the wholesale price exceeds the retail price, $r > p^*$, retailers stop demanding and serving their retail customers, because otherwise retailers suffer losses. The level of fixed retail demand depends on the pre-determined retail price r. For the retail price, we have to distinguish three cases: In the first case the retail price is small enough such that all customers with $\alpha < \tilde{\alpha}$ have a positive demand $r \leq \frac{1}{2} + \varepsilon$. With $\frac{1}{2} + \varepsilon < r \leq \tilde{\alpha} + \varepsilon$ some consumers without smart metering do not buy any electricity anymore because it is too costly and with $r > \tilde{\alpha} + \varepsilon$ no customer on traditional meters demands electricity.

Consumers with smart meters directly take part in the wholesale market. Aggregating their individual demand from (2) yields

$$D^W(p^*, \tilde{\alpha}, \varepsilon) = \begin{cases} \int_{\tilde{\alpha}}^{\frac{3}{2}} \alpha + \varepsilon - p^* d\alpha & \text{if } 0 \leq p^* \leq \tilde{\alpha} + \varepsilon, \\ \int_{p^*-\varepsilon}^{\frac{3}{2}} \alpha + \varepsilon - p^* d\alpha & \text{if } \tilde{\alpha} + \varepsilon < p^* \leq \frac{3}{2} + \varepsilon, \\ 0 & \text{if } \frac{3}{2} + \varepsilon < p^*. \end{cases} \tag{4}$$

Demand from consumers with smart meters is elastic in the wholesale price p^*. Again, we have to distinguish the three cases where all smart meter customers have a positive demand ($0 \leq p^* \leq \tilde{\alpha} + \varepsilon$), where some of them stop buying ($\tilde{\alpha} + \varepsilon < p^* \leq \frac{3}{2} + \varepsilon$), and where the price exceeds the reservation price and all of them stop buying electricity ($p^* > \frac{3}{2} + \varepsilon$).

Aggregate total demand then is the sum of the demand from the consumers with a predetermined retail price and from those on smart metering and is given by

$$D(p^*, r, \tilde{\alpha}, \varepsilon) = D^R(r, \tilde{\alpha}, \varepsilon, p^*) + D^W(p^*, \tilde{\alpha}, \varepsilon). \tag{5}$$

Total demand in the wholesale market is sketched in figure 2.

Total demand is discontinuous at $p^* = r$ if $r < \tilde{\alpha} + \varepsilon$, has the same constant slope for $0 \leq p^* < r$ and for $r < p^* < \tilde{\alpha} + \varepsilon$ and is convexly decreasing for $\tilde{\alpha} + \varepsilon \leq p^* \leq \frac{3}{2} + \varepsilon$.

The two generators A and B know the total realized demand when they bid their price into the market. Their optimal bidding strategies depend on their own and their rival's capacity K_A and K_B, on the retail price r, on the level of smart metering determined by $\tilde{\alpha}$ and on the level of the demand shock ε.

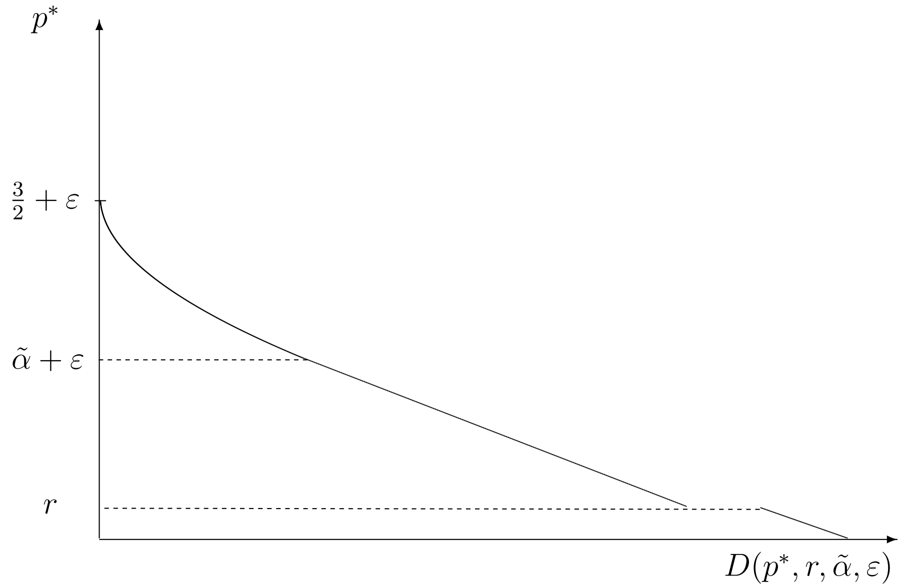

Figure 2: *Total Demand in the Wholesale Market with* $r < \tilde{\alpha}+\varepsilon$

Proposition 1 *With regard to the market equilibria on the wholesale market we can distinguish five cases.*

(i) If $K_i \geq D(0, r, \tilde{\alpha}, \varepsilon)$ *and* $K_j \geq D(0, r, \tilde{\alpha}, \varepsilon)$ *with* $i, j = A, B$ *the firms bid in the unique equilibrium* $p_i = 0$ *and* $p_j = 0$ *resulting in the uniform auction price of* $p^* = 0$.

(ii) If $0 \leq K_i < D(0, r, \tilde{\alpha}, \varepsilon)$ *and* $K_j > D(0, r, \tilde{\alpha}, \varepsilon)$ *with* $i, j = A, B$ *there are multiple equilibria. In all these equilibria the firms bid* $p_j = p_j^*$ *with*

$$p_j^* = \arg\max_p \{p[D(p, r, \tilde{\alpha}, \varepsilon) - K_i]\}$$

and $0 \leq p_i < \bar{p}_i < p_j^*$ *where* $\bar{p}_i$ *is implicitly defined by (17). The unique auction price is* $p^* = p_j^*$.

(iii) If $0 \leq K_i \leq K_j < D(0, r, \tilde{\alpha}, \varepsilon)$ *and* $D(p_j^*, r, \tilde{\alpha}, \varepsilon) - K_j \leq K_i < \underline{K_i}$ *we*

have the same equilibria as in (ii). $\underline{K_i}$ *is either defined in equation (34), (35) or (36).*

(iv) *If* $K_i \leq K_j < D(0, r, \tilde{\alpha}, \varepsilon)$ *and* $\underline{K_i} < K_i \leq K_j$ *with* $i, j = A, B$ *the uniform price auction has two types of equilibria, one type is identical with the one in (ii), in the other one the firms bid* $p_i = p_i^*$ *with*

$$p_i^* = \arg\max_p \{p[D(p, r, \tilde{\alpha}, \varepsilon) - K_j]\} \leq p_j^*$$

and $0 \leq p_j < \bar{p}_j < p_i^*$ *where* $\bar{p}_j$ *is implicitly defined by the equivalent to (17). The auction price in the latter type of equilibrium is* $p^* = p_i^*$*.*

(v) *If* $K_i < K_j$ *and* $K_i + K_j < D(p_j^*, r, \tilde{\alpha}, \varepsilon)$ *with* $i, j = A, B$ *there are multiple equilibria in which the two firms bid* $p_i \leq \hat{p}$ *and* $p_j \leq \hat{p}$ *with*

$$\hat{p} = \{p | K_i + K_j = D(p, r, \tilde{\alpha}, \varepsilon)\} > p_j^* \geq p_i^*.$$

The auction price is nevertheless unique and given by $p^* = \hat{p}$*.*

Proof: See Appendix A. □

Note that multiple equilibria occur as soon as firms are capacity constrained (cases (ii)-(v) of proposition 1). The multiplicity only leads to different equilibrium wholesale prices and different profits for the two generators if their capacities are of a relatively similar size and satisfy case (iv) of proposition 1. In this case there exist two types of equilibria where either the high capacity firm or the low capacity firm bids the high price in equilibrium. The high price maximizes the monopoly profit on the residual demand. If the firms' capacities differ more (cases (ii) and (iii)) it is always the firm with the larger capacity that bids high and serves the residual demand whereas the small firm bids low and sells its total capacity. In case (v) the market does not clear at the monopoly price on the residual demand. The demand cannot be served by the two firms at this price. The low capacity firm never has an incentive to bid a higher price than the large capacity firm and the system operator needs to increase the large capacity firm's bid to balance the market.

Whenever multiple equilibria occur, we select the equilibrium in which the larger firm is bidding the high price. For completely inelastic demand Boom (2008) argues that the equilibria with the large firm bidding the high price and the small firm undercutting it, risk-dominate the equilibria where the roles are reversed. It is beyond the scope of this paper to verify whether this selection can also be supported with elastic demand. Empirical findings by Wolfram (1998), however, show that for the UK electricity market it indeed is the larger firm that is the pivotal bidder and submits the market clearing price. With identical capacities we assume that each of the two firms is equally likely to choose the high price in equilibrium.

Figure 3 illustrates all equilibria of proposition 1. For a given demand shock ε, a given level of real time pricing $\tilde{\alpha}$ and a given retail price r the equilibrium auction price is a function of the capacity levels of the two firms. The equilibrium prices depend on each firm's capacities. As derived in Appendix A, the borders for which the large and high pricing firm finds it optimal to price above, at or below the retail price are denoted as K_1 and K_2, respectively. Whenever both firms can serve the entire market on their own and have capacities larger than D_0 the equilibrium price equals zero which is the Bertrand outcome. Figure 3 is drawn for relatively low retail prices because there exist capacity combinations $0 < \min\{K_A, K_B\} < K_1$ for which the equilibrium prices are above the retail price level r. For this case to be true K_1 as defined in (20) in Appendix A.2 needs to be positive which is equivalent to $0 < r < r_1$, where $r_1 = \{r | K_1 = 0\}$ is depicted in figure 4. If $K_1 < 0$, wholesale prices above the retail price cannot be an equilibrium, unless the system operator has to set the wholesale price. In the south west corner of figure 3 we always find an area where the system operator needs to set the

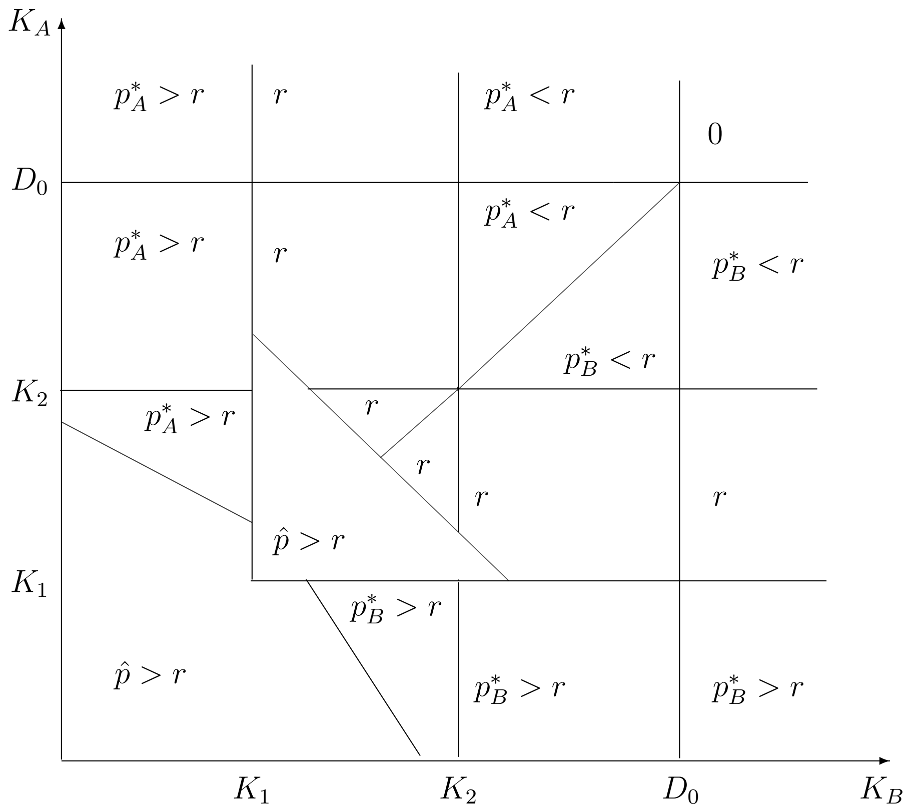

Figure 3: *The Auction Prices in the Wholesale Market for Low Retail Prices*

price above the highest price bid to clear the market.[11] The discontinuity of the system operator price regions at $\min\{K_A, K_B\} = K_1$ is due to the jump of the potential wholesale price from $p^* > r$ to $p^* = r$, which is induced by the kink in the demand curve due to the sudden inclusion of the customers without real time pricing (see the demand in figure 2). The overall pricing pattern described in figure 3 is intuitive. The larger the capacities the smaller is the wholesale price.

Considering the specific equilibrium prices, given in (19), (24) and (28) Appendix A.2, it becomes clear that the wholesale price depends only on the capacity of the smaller firm. This is because the smaller firm's capacity

[11]Borders for these areas are given by S_i in (31), (32) or (33) in Appendix A.3. As the retail price increases these borders shift inward for those areas with $\min\{K_A, K_B\} > K_1$. Higher retail prices reduce demand and therefore the system operator needs to interfere less often to ensure market clearing. When the market outcome is determined by capacities that satisfy $\min\{K_A, K_B\} < K_1$, increases in the retail price are irrelevant, because retailers have left the market.

determines the residual demand for the larger and pivotal firm that decides on the wholesale price in all cases where at least one firm is capacity constrained. For higher demand shocks capacities are relatively scarcer and hence the borders defining the equilibrium prices shift outwards. On the contrary for increasing retail prices, the borders shift inwards. Figure 4 shows which wholesale price regimes are relevant given the retail price and the level of smart metering. For $r < r_1$ all three wholesale price regimes depicted in figure 3 exist. For intermediate retail prices $r_1 < r < r_2$ figure 3 would simplify and wholesale prices above the retail price would no longer be possible. For $r_2 < r < \tilde{\alpha} + \varepsilon$ the critical capacity level K_2 or K_2', as defined in equation (21) or (25), respectively, are no longer positive.[12] In that case figure 3 simplifies even further and $0 \leq p^* < r$ must hold in equilibrium. Note that for $r > \tilde{\alpha} + \varepsilon$ the retail price does not matter any more for the level of the wholesale price because at these retail prices no retail customer has a positive demand. Figure 3 would have only one horizontal and vertical line, which would no longer be defined by $\min\{K_A, K_b\} = D_0'$ with D_0' defined in (26), but by $\min\{K_A, K_b\} = D_0''$ with D_0'' defined in (29) in Appendix A.2. Depending on whether $0 < \min\{K_A, K_b\} < D_0''$ holds or $\min\{K_A, K_b\} > D_0''$ we would either have $p^* > 0$ or $p^* = 0$.[13]

3.2 The retail market

Retailers compete in prices and do not have any other retail costs than the price they need to pay for electricity on the wholesale market. Therefore all retailers compete the price down to a level where they do not generate positive profits any more. Retailers have zero profits if they find themselves

[12] The critical retail price r_2 is defined by either $r_2 = \{r|K_2 = 0\}$ or $r_2 = \{r|K_2' = 0\}$ depending on whether it exceeds the level $r = \varepsilon + \frac{1}{2}$ or not, so on whether all retail customers have still a positive demand or not. The critical retail price r_2 increases and is continuous in $\tilde{\alpha}$ as is sketched in figure 4.

[13] With completely inelastic demand, as in Boom and Buehler (2007), $p^* > 0$ or $p^* = 0$ always are the only possible outcomes.

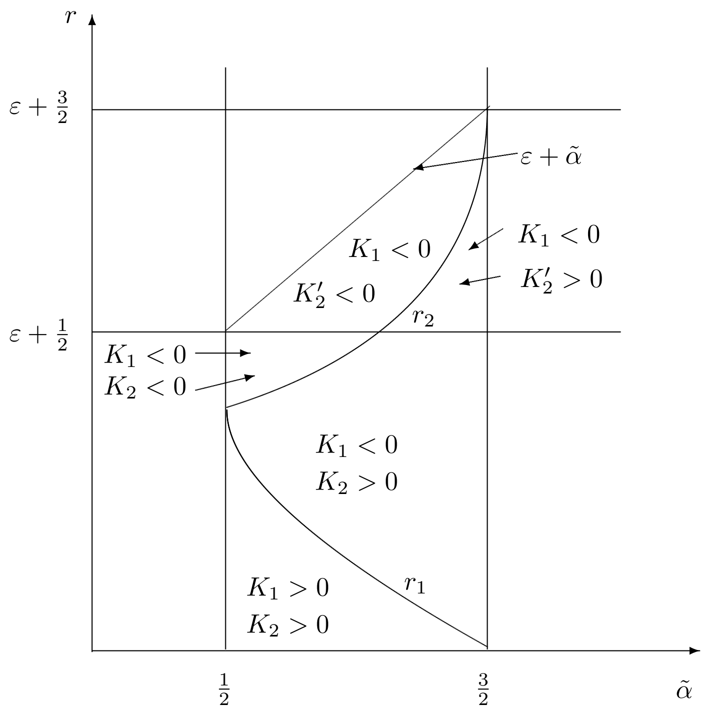

Figure 4: *Critical Retail Price Levels*

for every potential demand induced by ε in a situation where the wholesale price satisfies $p^* \geq r$. Looking at the lattice pattern in figure 3 it becomes obvious that this condition is satisfied if $K_2 > \min\{K_A, K_B\}$ or, if it becomes relevant, $K_2' > \min\{K_A, K_B\}$ for all $\varepsilon \in [0, 1]$. Retailers compete in the retail price until the generating firms' capacities ensure a wholesale price that equals the retail price at a demand shock of $\varepsilon = 0$. This condition guarantees zero profits for retailers for all $\varepsilon \in [0, 1]$ and all $p^* \geq r$. From this idea we can derive the following proposition which describes the retail price in equilibrium.

Proposition 2 *Assume $K_i \leq K_j$, then there is a unique subgame perfect equilibrium in which all retailers set $r = \bar{r} = 0$, if $K_i > 1$. If $K_i \leq 1$ then there are multiple subgame perfect Nash equilibria. In all these equilibria the* *retailers charge their customers retail prices which satisfy* $0 \leq r \leq \bar{r}$. *The level of* $\bar{r}$ *depends on the capacity levels* K_i *and* K_j *and on the level of smart metering reflected in* $\tilde{\alpha}$. *The definition of* $\bar{r}$ *is given by*

$$\bar{r} = \begin{cases} \frac{3}{2} - \sqrt{2(K_i + K_j)} & \text{if } 0 \leq K_i < \\ & \min\left\{ \frac{9}{8} - \frac{2K_j(9-(9-2\tilde{\alpha})\tilde{\alpha}-K_j)}{(3-2\tilde{\alpha})^2}, \frac{1-2K_j}{2} \right\}, \\ 3 - \tilde{\alpha} - \frac{1}{2}\sqrt{27 - 4\tilde{\alpha}(6-\tilde{\alpha}) + 8K_i} & \text{if } \max\left\{ \frac{9}{8} - \frac{2K_j(9-(9-2\tilde{\alpha})\tilde{\alpha}-K_j)}{(3-2\tilde{\alpha})^2}, 0 \right\} \\ & \leq K_i < \min\left\{ \frac{1}{2}(\tilde{\alpha} - \frac{1}{2}), K_j \right\}, \\ 1 - K_i - K_j & \text{if } \frac{1-2K_j}{2} \leq K_i \leq \min\{1 - \frac{(5-2\tilde{\alpha})K_j}{3-2\tilde{\alpha}}, \\ & K_j\}, \\ \frac{2(1-K_i)}{5-2\tilde{\alpha}} & \text{if } \max\left\{ \frac{1}{2}\left(\tilde{\alpha} - \frac{1}{2}\right), 1 - \frac{(5-2\tilde{\alpha})K_j}{3-2\tilde{\alpha}} \right\} \\ & \leq K_i < \min\{1, K_j\}, \\ 0 & \text{if } 1 \leq K_i \leq K_j. \end{cases}$$

Proof: See Appendix B. □

Note that we potentially have multiple equilibria. We follow the convention in economics that we assume that firms stop undercutting each others prices as soon as they generate zero profits. For retailers this condition translates into all retailers setting $r = \bar{r}$ if $0 \leq \min\{K_A, K_B\} < 1$ and $r = \bar{r} = 0$ otherwise. The relationship between the different capacity levels and the retail price is characterized in figure 5.

From proposition 2 it becomes clear that the retail price $\bar{r}$ only changes marginally in the level of smart metering if we are in the cases represented by the second and fourth line of its definition. These are the cases where the generating firms' capacities are sufficient such that the system operator does not need to interfere with the generators' price bidding on the electricity wholesale market for the smallest demand shock $\varepsilon = 0$. For these cases the retail price decreases if the level of smart metering increases because $\frac{\partial \bar{r}}{\partial \tilde{\alpha}} > 0$ and a lower $\tilde{\alpha}$ means more customers with smart meters. A larger number of

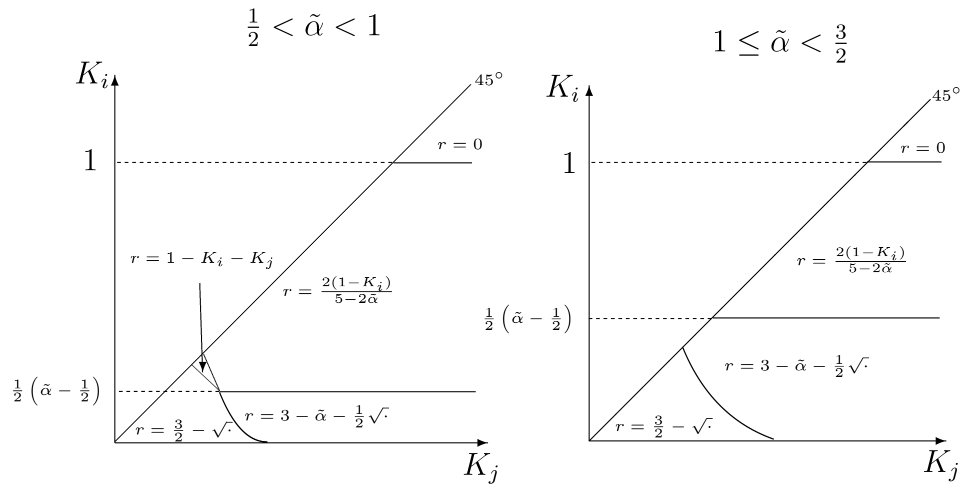

Figure 5: *Retail Price with Different Capacities for* $K_i \leq K_j$

smart meters decreases the retail demand and hence the retail price is lowered. In addition, because of the lower retail demand the retail market is now for more combinations of K_A and K_B fully covered instead of only partially covered. Thus, for the capacities for which $\frac{\partial \bar{r}}{\partial \tilde{\alpha}} > 0$ holds consumers without smart meters will always benefit from more (other) consumers having a smart meter and taking part directly in the wholesale market. The same effect has been found by Borenstein and Holland (2005). Since in their analysis all consumers were identical and more smart metering did not imply reducing the willingness to pay of the customers without a smart meter, this result is not simply driven by the lower willingness to pay of the customers without real time prices. Retail prices are determined by fierce price competition by the retailers who cannot just expropriate the consumers' rent.

If the capacities of the electricity generating firms are so low that the system operator needs to interfere with the price bidding of the generators for all possible levels of the demand shock $\varepsilon \in [0, 1]$ then the retail price does marginally not respond to a higher degree of metering. The main reason for this is that how consumers are split and how price responsive the wholesale demand is on the margin, does not influence the wholesale prices. Instead, wholesale prices are always determined by equalizing total demand with total capacity. Then, given that the retail price is determined by the lowest possible wholesale price being below or equal to the retail price, also the retail price solely depends on the firms' capacities. The higher the firms' total capacity is, the lower is the lowest possible wholesale and the resale price.

4 Comparative statics in the level of smart metering

Retailers always have zero profits and therefore do not have an impact on welfare as the level of smart metering changes. Only their competitive retail price and the wholesale market price effect welfare. Thus, we consider how a change in the level of smart metering changes retail and wholesale prices for all possible states of demand realizations. We use these prices to derive expected profits, consumer surplus and welfare ex ante of the demand realization. For the sake of tractability we only look at the cases in which the retail prices are indeed determined by $\tilde{\alpha}$ and the SO does not have to intervene in the market. This in turn assumes that the firms are always investing sufficiently in their capacity endowments and the market always clears at the residual monopoly price of the high bidding firm.

We use the consumer surplus function and the three equilibrium retail prices in proposition 2 that depend on $\tilde{\alpha}$ (cases (i), (ii) and (iv)) to calculate expected welfare. From the consumer surplus function in equation (1) we know that those consumers who are served will achieve a surplus of

$$V(\alpha + \varepsilon - p, \alpha, \varepsilon, p) = \alpha(\alpha - p) - \frac{(\alpha - p)^2}{2} - p(\alpha + \varepsilon - p), \tag{6}$$

where due to our assumption that the SO never has to intervene p is either the wholesale price $p^* = p_j^* > r$ that varies according to the state of demand ε or the predetermined retail price r that does not change with the demand realization. Those consumers who are either not served or who decide themselves that they do not want to consume realize a surplus of

$$V(0, \alpha, \varepsilon, p) = -\alpha\varepsilon - \frac{\varepsilon^2}{2}. \tag{7}$$

Customers on traditional meters always pay their contracted retail price and hence we have $p = r$ in equation (6) for all $\frac{1}{2} < \alpha < \tilde{\alpha}$. The wholesale price that consumers with smart meters pay can be either $p^* = p_j^*$ or $p^* = r$. We can have both prices for consumers with $\tilde{\alpha} < \alpha < \frac{3}{2}$. The level of the wholesale price depends on whether K_i is smaller or greater than K_1. If $K_i \geq K_1$ the residual monopoly price equals the retail price, while for $K_i < K_1$ the residual monopoly price lies on the linear downward sloping part of the demand curve above the retail price. To account for the different wholesale market prices in the welfare calculations we define a critical demand shock, ε^*. Whenever the demand shock is larger than

$$\varepsilon^* = \{\varepsilon \mid K_i = K_1\} \tag{8}$$

the low bidding firm's capacity is relatively scarce and the wholesale price becomes $p^* = p_j^*$. For lower demand shocks than ε^* the wholesale market price remains equal to the retail price. The critical shock ε^* depends on the retail price. In the following we distinguish between equilibrium retail prices of zero (case (i) in proposition 2), intermediate equilibrium retail prices (case (ii)) and high equilibrium retail prices (case (iv)).

4.1 Equilibrium retail prices of zero

When capacities satisfy $1 \leq K_i \leq K_j$ the retail price is zero. In this scenario the SO never has to intervene, because $K_i + K_j \geq 2 \geq D_0$ holds and the two firms can cover all demand at each price for all demand realizations. Figure 3 simplifies because $K_2 = D_0$, and the wholesale price can be either $p^* = p_j^*$

or $p^* = r = 0$. For $r = 0$ the critical demand shock in equation (8) becomes

$$\varepsilon_z^* = \frac{8K_i + 4\tilde{\alpha}^2 - 9}{12 - 8\tilde{\alpha}}. \tag{9}$$

In Appendix C.1 we derive ε_z^*. If the demand shock is larger than ε_z^* the wholesale price becomes $p^* = p_j^*$, while for lower demand shocks the wholesale market price is zero. Wholesale prices of $p^* = p_j^*$ never occur as long as $\varepsilon_z^* > 1$, which holds as long as $K_i > \max\{1, \frac{1}{8}(3 - 2\tilde{\alpha})(7 + 2\tilde{\alpha})\}$. If K_i exceeds this threshold the highest demand shock cannot be so large to make it optimal for the high bidding firm to price above the retail price. Then all consumers always pay a price of zero no matter whether they have a smart meter and participate in the wholesale market or whether they have a retail contract with a predetermined price. Therefore welfare is identical with aggregate consumer surplus which is defined by

$$CS = W = \int_0^1 \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{\alpha^2}{2} d\alpha d\varepsilon = \frac{13}{24}. \tag{10}$$

Generators do not earn any profits. If however $1 < K_i \leq \frac{1}{8}(3 - 2\tilde{\alpha})(7 + 2\tilde{\alpha})$ and $\varepsilon_z^* \leq 1$ then wholesale customers have to pay a positive price for some states of demand. In these states retail customers are not served because $p^* > r$. This happens if $\varepsilon^* = \varepsilon_z^* \leq \varepsilon \leq 1$. The consumer surplus is now

$$\begin{aligned} CS &= \int_0^{\varepsilon^*} \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{\alpha^2}{2} d\alpha d\varepsilon + \int_{\varepsilon^*}^1 \int_{\frac{1}{2}}^{\tilde{\alpha}} -\alpha\varepsilon - \frac{\varepsilon^2}{2} d\alpha d\varepsilon \\ &+ \int_{\varepsilon^*}^1 \int_{\tilde{\alpha}}^{\frac{3}{2}} \alpha(\alpha - p^*) - \frac{(\alpha - p^*)^2}{2} - p^*(\alpha + \varepsilon - p^*) d\alpha d\varepsilon. \end{aligned} \tag{11}$$

Substituting the relevant price which is always given by p_j^* from (19) we can show that $\frac{\partial CS}{\partial \tilde{\alpha}} > 0$ holds. Aggregate consumer surplus decreases if the level of smart metering increases (meaning that $\tilde{\alpha}$ decreases). While wholesale costumers face price risks in potentially having to pay positive duopoly prices, retail customers are not served for some demand realizations. By increasing the number of wholesale customers the first effect aggravates, whereas the second is softened. Since consumer surplus is reduced the first effect dominates the second. The producer surplus is given by

$$PS = \pi_i + \pi_j = \int_{\varepsilon_1}^{1} \int_{\tilde{\alpha}}^{\frac{3}{2}} p^*(\alpha + \varepsilon - p^*) d\alpha d\varepsilon. \tag{12}$$

The producer surplus increases with the level of smart metering since $\frac{\partial PS}{\partial \tilde{\alpha}} < 0$ holds. This is not surprising because more smart metering means more demand situations in which wholesale customers pay a positive price and, on top of it, there are more wholesale customers who have to pay the higher price. Because of the opposing nature of consumer and producer surplus, welfare is U-shaped in the level of smart metering. For small $\tilde{\alpha}$ we have $\frac{\partial W}{\partial \tilde{\alpha}} < 0$ while for larger we have $\frac{\partial W}{\partial \tilde{\alpha}} > 0$. Obviously the effect on the profits dominates welfare for small $\tilde{\alpha}$, whereas for large $\tilde{\alpha}$ the effect on consumer surplus dominates. Figure 6 depicts the welfare results for retail prices of zero and a given capacity of the low bidding firm.

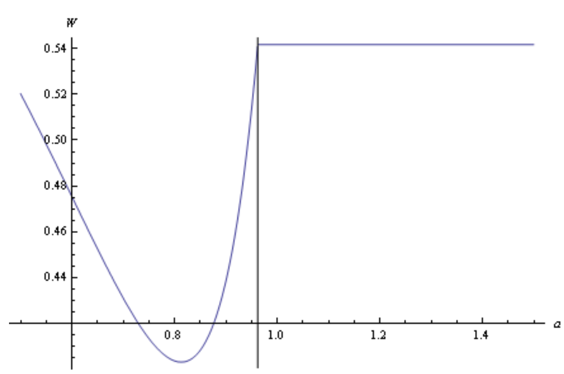

Figure 6: *Welfare depending on $\tilde{\alpha}$ for retail prices of zero.*

An increase in the level of smart metering from no smart metering at all ($\tilde{\alpha} = \frac{3}{2}$) does first not have an effect on welfare, consumer surplus or profits.

Welfare is constant as long as the degree of smart metering is low enough and $\tilde{\alpha}$ is above a certain threshold ($\tilde{\alpha} > \frac{\sqrt{25-8K_i}}{2} - 1$) that ensures that $\varepsilon_z^* > 1$ and $p* = r = 0$ always hold. Above this threshold the retail market is so large that the residual monopoly profit is always maximized at the retail price. For degrees of smart metering below this threshold the high bidding firm maximizes its profits by clearing the market above the retail price for at least some states of demand realization. Then consumer surplus and welfare decrease. The loss of consumer surplus due to uncertain prices above the marginal cost level cannot be compensated by the larger producer surplus and by the fact that fewer retail customers are sometimes not served. When smart metering is further extended, the two latter effects start dominating the first and finally, if all customers are on smart meters the wholesale price is approaching zero again. In the Bertrand case, when $K_j > K_i > 2 = D_0$, welfare is the same with all consumers on smart meters or none at all.

4.2 Intermediate equilibrium retail prices

When the two firms' capacities become scarcer the retail price is $\frac{2(1-K_i)}{5-2\tilde{\alpha}}$, as described in proposition 2. This retail price is always lower than $\frac{1}{2} + \varepsilon$, and hence all retail customers demand electricity. The SO might have to intervene, because $K_i + K_j < D(r)$ is possible for some states of the demand realization. To focus on the case where firms clear the market at their bid and the wholesale price can be either $p^* = p_j^*$ or $p^* = \frac{2(1-K_i)}{5-2\tilde{\alpha}}$ we introduce the following condition. As long as

$$K_i \geq D(r \mid \varepsilon = 1) - K_j = \frac{8 + 2\tilde{\alpha}(K_j - 2) - 5K_j}{3 - 2\tilde{\alpha}} \tag{13}$$

holds, the SO never has to set the price at $\hat{p} > r$. Equation (13) ensures that the two firms can cover the market at the retail price even for the highest demand shock. We derive this condition in Appendix C.2 and show that under this condition firms are also able to cover all possible states of demand at the optimal price above the retail price, $p^* = p_j^*$. Similarly to the case of zero retail prices we can now argue that whenever the demand shock is larger than ε_i^* the wholesale price becomes $p^* = p_j^*$, while for lower demand shocks the wholesale market price is $p^* = r = \frac{2(1-K_i)}{5-2\tilde{\alpha}}$. For the derivation of the critical shock with intermediate retail prices, ε_i^*, see Appendix C.1. Consumer surplus and profits can then be calculated equivalently to equations (11) and (12) respectively. Figure 7 illustrates the welfare results for a given capacity of the low bidding firm.

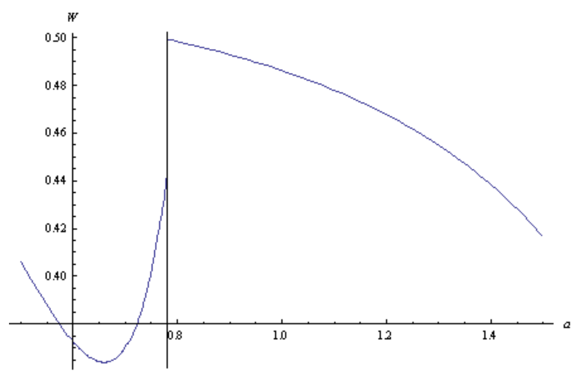

Figure 7: *Welfare depending on $\tilde{\alpha}$ for intermediate retail prices.*

Again, for a large enough $\tilde{\alpha}$ such that only $p^* = r$ applies, all consumers are always served and pay the same price no matter whether they are on smart metering or not. In this case all effects of a variation of $\tilde{\alpha}$ are driven by the change in the price. Due to $\frac{\partial \bar{r}}{\partial \tilde{\alpha}} > 0$, contrary to the case for zero retail prices, consumers like an increase in smart metering because they consume more and pay less ($\frac{\partial CS}{\partial \tilde{\alpha}} < 0$). We find that $\frac{\partial PS}{\partial \tilde{\alpha}} > 0$ and that producer surplus reduces as the level of smart metering increases because despite their increased electricity consumption consumers pay less. More smart metering increases welfare ($\frac{\partial W}{\partial \tilde{\alpha}} < 0$) because it reduces market power without any consumer being forced to leave the market. When the degree of smart metering is above the threshold the pivotal firm, depending on the demand shock, clears the market at or above the retail price. In this case more smart metering means a decrease in the consumer surplus ($\frac{\partial CS}{\partial \tilde{\alpha}} > 0$) and an increase in the producer surplus ($\frac{\partial PS}{\partial \tilde{\alpha}} < 0$). The level of welfare is U-shaped again. The same arguments that explain the U-shaped effect for zero retail prices also apply for intermediate retail prices.

4.3 High equilibrium retail prices

The retail price becomes $3-\tilde{\alpha}-\frac{1}{2}\sqrt{27-4\tilde{\alpha}(6-\tilde{\alpha})+8K_i}$ for relatively scarce capacities. Again, if $K_i \geq D(r \mid \varepsilon = 1) - K_j$ holds, the wholesale market always clears at the optimal bid of the pivotal firm. We only have wholesale prices equal to the retail price for low shocks that satisfy $\varepsilon < \varepsilon_h^*$. For demand shocks $\varepsilon_h^* < \varepsilon < 1$ the wholesale price is above the retail price. In Appendix C.1 and C.2 we derive ε_h^* and the functional form of the market clearing condition $K_i \geq D(r \mid \varepsilon = 1) - K_j$, for which the SO does not have to intervene and set scarcity prices. Opposing to the case of intermediate retail prices, for high retail prices we can have $r > \frac{1}{2} + \varepsilon$ and some retail consumers do not demand electricity. Given the retail price, whether all or only some retail customers demand electricity depends on the demand shock. Hence for $\varepsilon < \varepsilon_h^*$ we now derive consumer surplus as

$$\begin{aligned} CS \; = \; & \int_0^{\varepsilon^f} \int_{\frac{1}{2}}^{r-\varepsilon} -\alpha\varepsilon - \frac{\varepsilon^2}{2} d\alpha d\varepsilon \\ & + \int_0^{\varepsilon^f} \int_{r-\varepsilon}^{\tilde{\alpha}} \alpha(\alpha - r) - \frac{(\alpha - r)^2}{2} - r(\alpha + \varepsilon - r) d\alpha d\varepsilon \\ & + \int_{\varepsilon^f}^{1} \int_{\frac{1}{2}}^{\tilde{\alpha}} \alpha(\alpha - r) - \frac{(\alpha - r)^2}{2} - r(\alpha + \varepsilon - r) d\alpha d\varepsilon \\ & + \int_0^{1} \int_{\tilde{\alpha}}^{\frac{3}{2}} \alpha(\alpha - r) - \frac{(\alpha - r)^2}{2} - r(\alpha + \varepsilon - r) d\alpha d\varepsilon, \end{aligned} \tag{14}$$

where ε^f decides on wheter $r > \frac{1}{2} + \varepsilon$ or $r < \frac{1}{2} + \varepsilon$, so on whether the retail market is partially or fully covered. Since $\varepsilon < \varepsilon_h^*$ and $p^* = r$ for all demand shocks, only the effects of $\frac{\partial \bar{r}}{\partial \tilde{\alpha}}$ determine welfare. We again find $\frac{\partial CS}{\partial \tilde{\alpha}} < 0$ and $\frac{\partial W}{\partial \tilde{\alpha}} < 0$. More smart metering lowers the market price and increases consumer surplus and welfare. Producer surplus, that is derived within the same integrals as in equation (14), decreases as smart metering increases, however only if the negative effect on profits of the lowered retail price is offset by the positive effect that for lower retail prices the retail market is fully covered for more demand realizations. For high K_i, that lead to high retail prices and relatively greater losses if the retail market is not fully covered, the latter effect starts dominating and producer surplus becomes inverted U-shaped in the level of smart metering.

Whenever $\varepsilon_h^* < \varepsilon < 1$ and the wholesale price changes with the demand shock we derive consumer surplus as

$$\begin{aligned} CS \quad = \quad & \int_0^{\varepsilon^f} \int_{\frac{1}{2}}^{r-\varepsilon} -\alpha\varepsilon - \frac{\varepsilon^2}{2} d\alpha d\varepsilon \\ & + \int_0^{\varepsilon^f} \int_{r-\varepsilon}^{\tilde{\alpha}} \alpha(\alpha - r) - \frac{(\alpha - r)^2}{2} - r(\alpha + \varepsilon - r) d\alpha d\varepsilon \\ & + \int_{\varepsilon^f}^{\varepsilon_h^*} \int_{\frac{1}{2}}^{\tilde{\alpha}} \alpha(\alpha - r) - \frac{(\alpha - r)^2}{2} - r(\alpha + \varepsilon - r) d\alpha d\varepsilon \\ & + \int_{\varepsilon_h^*}^{1} \int_{\frac{1}{2}}^{\tilde{\alpha}} -\alpha\varepsilon - \frac{\varepsilon^2}{2} d\alpha d\varepsilon \\ & + \int_0^1 \int_{\tilde{\alpha}}^{\frac{3}{2}} \alpha(\alpha - p^*) - \frac{(\alpha - p^*)^2}{2} - p^*(\alpha + \varepsilon - p^*) d\alpha d\varepsilon. \end{aligned} \tag{15}$$

Then like for intermediate retail prices $\frac{\partial CS}{\partial \tilde{\alpha}} > 0$ holds and consumers dislike smart metering. Producer surplus is increasing in the amount of smart metering, unless the retail price is very high (for low K_i). If K_i is very low, the retail price is very high and producer surplus becomes slightly U-shaped, because a sufficient number of customers have to be on smart meters to level out the losses of retailers that pay high prices whenever $p = r$ but leave the market whenever the demand shock is high and $p > r$ holds. In line with the case of intermediate retail prices we find that welfare is U-shaped whenever the market outcome changes with the demand realization, while welfare is increasing if the wholesale market price equals the retail price for all demand shocks. Overall, the comparative statics of smart metering on welfare for high retail prices follow the patterns for intermediate retail prices.

5 Conclusion

This paper derives welfare effects of real-time pricing in electricity markets. When electricity generating firms have market power in the wholesale market and consumers are risk-averse, we show that real-time pricing does not have to be efficiency enhancing. Overall welfare implications depend on the level of firms' capacities and on the magnitude of stochastic demand shocks. With large capacities that always lead to Bertrand prices, we find no difference in welfare when all or no consumers are on smart meters. When firms' capacities are smaller such that market power arises, firms can price relatively high in times of high demand shocks. When this is the case, we show that for the main cases in which the system operator does not need to intervene and set prices, real-time pricing decreases consumer surplus, because risk-averse consumers dislike high and uncertain prices. At the same time real-time metering increases producer surplus, because more smart metering means more demand situations in which more wholesale customers pay a price above marginal costs. These two opposing effects lead to a U-shaped welfare in smart metering whenever the demand shock can change equilibrium prices. If however firms capacities are relatively large and the demand shock does not change the wholesale price, smart metering can increase consumer surplus and welfare. Our findings suggest that, before investing in smart meters and smart grids, dominant firm behavior and the welfare gain of fixed retail prices that insure risk-averse consumers against price fluctuations should be taken into consideration.

Appendix

A Proof or proposition 1

A.1 Case (i): $K_i > D(0, r, \tilde{\alpha}, \varepsilon)$, $K_j > D(0, r, \tilde{\alpha}, \varepsilon)$

This is the usual Bertrand case because non of the firms is effectively capacity constrained. If firm i bids $p_i = 0$ and firm j bids $p_j = 0$ with $i, j = A, B$, none of the two firms has an incentive to deviate because they could not improve on their profit of 0. If the firms bid $p_i = p_j = p > 0$ firm i's and firm j's profit would be identical and given by $\pi_i = \pi_j = \frac{1}{2} p D(p, r, \tilde{\alpha}, \varepsilon)$. Then each firm has an incentive to slightly undercut its rival because then it could realize instead $\pi_{i,j} = (p-\epsilon) D(p-\epsilon, r, \tilde{\alpha}, \varepsilon)$ with $\epsilon \to 0$. If the firms bid $p_i > p_j \geq 0$ then firm i's profit is zero and firm j's profit is $\pi_j = p_j D(p_j, r, \tilde{\alpha}, \varepsilon)$. Here firm i has again an incentive to slightly undercut firm j in order to realize $\pi_i = (p_j - \epsilon) D(p_j - \epsilon, r, \tilde{\alpha}, \varepsilon)$ with $\epsilon \to 0$ instead. Thus $p_i = p_j = 0$ is the only Nash equilibrium.

A.2 Case (ii): $K_i < D(0, r, \tilde{\alpha}, \varepsilon)$, $K_j > D(0, r, \tilde{\alpha}, \varepsilon)$

Here only firm i is capacity constrained. Suppose both firms bid $p_i = p_j = 0$ and have therefore zero profits, then only firm j has an incentive to deviate to a higher price $p_j > 0$. If it deviates it would serve the residual demand and would realize $\pi_j = p_j (D(p_j, r, \tilde{\alpha}, \varepsilon) - K_i) > 0$ if p_j were not too high. The optimal deviation would be to choose

$$p_j^* = \arg\max_p \{ p [D(p, r, \tilde{\alpha}, \varepsilon) - K_i] \} . \tag{16}$$

The same price p_j^* would also be a best response of firm j if firm i chooses p_i with $0 \leq p_i < p_j^*$ such that

$$(p_i - \epsilon) \min\{ D(p_i - \epsilon, r, \tilde{\alpha}, \varepsilon), K_j \} \leq p_j^* [D(p_j^*, r, \tilde{\alpha}, \varepsilon) - K_i] \tag{17}$$

with $\epsilon \to 0$. The capacity constrained firm i does never want to deviate to $p_i > p_j$ because it could not generate any positive demand for itself this way. The low-bidding firm j would serve the whole market and firm i would not increase its profits.

In order to determine p_j^* we need to take into account the different cases of the demand in equation (5) resulting from the situation on the retail market. We need to distinguish three cases:

Fully Covered Retail Market ($0 \leq r \leq \frac{1}{2} + \varepsilon$): All consumers without real-time pricing have a positive demand. The demand function is

$$D(p^*, r, \tilde{\alpha}, \varepsilon) = \begin{cases} 1 + \varepsilon - (\frac{3}{2} - \tilde{\alpha})p^* - (\tilde{\alpha} - \frac{1}{2})r & \text{if } 0 \leq p^* \leq r, \\ (\frac{3}{2} - \tilde{\alpha})(\frac{1}{2}(\frac{3}{2} + \tilde{\alpha}) + \varepsilon - p^*) & \text{if } r < p^* \leq \tilde{\alpha} + \varepsilon, \\ \frac{1}{2}(\varepsilon - p^* + \frac{3}{2})^2 & \text{if } \tilde{\alpha} + \varepsilon \leq p^* \leq \varepsilon + \frac{3}{2}, \\ 0 & \text{if } p^* > \varepsilon + \frac{3}{2}. \end{cases} \tag{18}$$

Solving for p_j^* yields the following solution

$$p_j^* = \begin{cases} \frac{3+2\tilde{\alpha}+4\varepsilon}{8} - \frac{K_i}{3-2\tilde{\alpha}} & \text{if } 0 \leq K_i < K_1, \\ r & \text{if } \max\{0, K_1\} \leq K_i < K_2, \\ \frac{2+2\varepsilon-2K_i-r(2\tilde{\alpha}-1)}{6-4\tilde{\alpha}} & \text{if } \max\{0, K_2\} < K_i < D_0, \end{cases} \tag{19}$$

where K_1, K_2 and D_0 are defined as

$$K_1 = \left(\frac{3}{2} - \tilde{\alpha}\right)\left(\frac{3+2\tilde{\alpha}}{4} + \varepsilon - 2r\right) - \frac{\sqrt{r(4(2-\tilde{\alpha})\tilde{\alpha} - 3)(1 + 2\tilde{\alpha} + 4\varepsilon - 4r)}}{2}, \tag{20}$$

$$K_2 = 1 + \varepsilon - \left(\frac{5}{2} - \tilde{\alpha}\right) r \text{ and} \tag{21}$$

$$D_0 = D(0, r, \tilde{\alpha}, \varepsilon) = 1 + \varepsilon - \left(\tilde{\alpha} - \frac{1}{2}\right) r. \tag{22}$$

Partially Covered Retail Market ($\frac{1}{2} + \varepsilon < r \leq \tilde{\alpha} + \varepsilon$): Some of the consumers without real-time pricing are priced out of the market. The demand function is

$$D(p^*, r, \tilde{\alpha}, \varepsilon) = \begin{cases} (\frac{3}{2} - \tilde{\alpha})(\frac{1}{2}(\frac{3}{2} + \tilde{\alpha}) + \varepsilon - p^*) & \\ \quad +\frac{1}{2}(\tilde{\alpha} + \varepsilon - r)^2 & \text{if } 0 \leq p^* \leq r, \\ (\frac{3}{2} - \tilde{\alpha})(\frac{1}{2}(\frac{3}{2} + \tilde{\alpha}) + \varepsilon - p^*) & \text{if } r < p^* \leq \tilde{\alpha} + \varepsilon, \\ \frac{1}{2}(\varepsilon - p^* + \frac{3}{2})^2 & \text{if } \tilde{\alpha} + \varepsilon \leq p^* \leq \varepsilon + \frac{3}{2}, \\ 0 & \text{if } p^* > \varepsilon + \frac{3}{2}. \end{cases} \tag{23}$$

Solving for p_j^* here yields the following

$$p_j^* = \begin{cases} r & \text{if } 0 \leq K_i < K_2', \\ \frac{1}{2}\left(\frac{1}{2}\left(\frac{3}{2} + \tilde{\alpha}\right) + \varepsilon + \frac{(\tilde{\alpha}+\varepsilon-r)^2 - 2K_i}{3-2\tilde{\alpha}}\right) & \text{if } \max\{0, K_2'\} < K_i \\ & \quad < D_0', \end{cases} \tag{24}$$

where K_2' and D_0' are defined as

$$K_2' = \frac{1}{2}\left(\left(\frac{3}{2} + \varepsilon\right)^2 - 2r(3 - \tilde{\alpha} + \varepsilon) + r^2\right) \text{ and} \tag{25}$$

$$D_0' = D(0, r, \tilde{\alpha}, \varepsilon) = \frac{1}{2}\left(\frac{3}{2} - \tilde{\alpha}\right)\left(\frac{3}{2} + \tilde{\alpha} + 2\varepsilon\right) + (\tilde{\alpha} + e - r)^2. \tag{26}$$

Uncovered Retail Market ($r > \tilde{\alpha} + \varepsilon$): All consumers without real-time prices are priced out of the market. The Demand function is

$$D(p^*, r, \tilde{\alpha}, \varepsilon) = \begin{cases} (\frac{3}{2} - \tilde{\alpha})(\frac{1}{2}(\frac{3}{2} + \tilde{\alpha}) + \varepsilon - p^*) & \text{if } 0 < p^* \leq \tilde{\alpha} + \varepsilon, \\ \frac{1}{2}(\varepsilon - p^* + \frac{3}{2})^2 & \text{if } \tilde{\alpha} + \varepsilon \leq p^* \leq \varepsilon + \frac{3}{2} \\ 0 & \text{if } p^* > \varepsilon + \frac{3}{2}. \end{cases} \tag{27}$$

Solving for the the optimal p_j^* yields

$$p_j^* = \frac{3 + 2\tilde{\alpha} + 4\varepsilon}{8} - \frac{K_i}{3 - 2\tilde{\alpha}} \text{ if } 0 < K_i < D_0'', \tag{28}$$

with $p_j^* < \tilde{\alpha} + \varepsilon < r$. D_0'' is defined as

$$D_0'' = D(0, r, \tilde{\alpha}, \varepsilon) = \frac{1}{8}(3 - 2\tilde{\alpha})(3 + 2\tilde{\alpha} + 4\varepsilon). \tag{29}$$

Note that independent of the specific case that we are looking at $p_j^* < \tilde{a} + \varepsilon$ and $\frac{\partial p_j^*}{\partial K_i} < 0$ always holds. Thus, all consumers with real-time pricing have a positive demand at p_j^* and it is never located at the non-linear part of the demand function, see figure 2. The equilibrium $(p_i, p_j) = (0, p_j^*)$ does always exist for this case. In addition condition (17) is usually satisfied for a range of $0 \leq p_i \leq \bar{p}_i$ where $p_i = \bar{p}_i$ satisfies the condition with equality. $\bar{p}_i$ is unique because one can show that the left-hand side of (17) is a convex increasing or single peaked function with at most one point of discontinuity at $p_i = r$ for all $p_i < p_j^*$. Given that the condition is never satisfied for $p_i = p_j^*$ and always for $p_i = 0$ there exists a unique $0 \leq \bar{p}_i < p_j^*$ such that condition (17) is satisfied for all $0 \leq p_i \leq \bar{p}_i$. Thus we have multiple Nash equilibria with $(p_i, p_j) = (p_i, p_j^*)$ and $0 \leq p_i \leq \bar{p}_i$. They are all pay-off equivalent and result in a unique auction price $p^* = p_j^*$.

A.3 Case (iii), (iv) and (v): $K_i \leq K_j < D(0, r, \tilde{\alpha}, \varepsilon)$

Here both firms are capacity constrained and both firms have an incentive to deviate from $p_i = p_j = 0$ because both firm can benefit from a positive residual demand. Given that the rival sticks to a price of zero each firm has an incentive to set

$$p_j^* = \arg\max_p \{p[D(p, r, \tilde{\alpha}, \varepsilon) - K_i]\} \text{ or } p_i^* = \arg\max_p \{p[D(p, r, \tilde{\alpha}, \varepsilon) - K_j]\}. \qquad (30)$$

Like in case (ii) in subsection A.2 this might even be a best response for a positive price of one's rival as long as (17) or the equivalent condition for firm i choosing p_i^* holds. Since both firms are capacity constrained, bidding a price above p_j^* or p_i^* is potentially profitable for both firms. Therefore the Nash equilibria with either $(p_i, p_j) = (p_i, p_j^*)$ with $p_i \leq \bar{p}_i < p_j^*$ and $\bar{p}_i$ implicitly defined in (17) or $(p_i, p_j) = (p_i^*, p_j)$ with $p_j \leq \bar{p}_j < p_i^*$ and $\bar{p}_j$ implicitly defined in the equivalent to (17) can only exist as long as the low-bidding firm does not have an incentive to bid above the high-bidding firms price level.

Note that p_j^* and p_i^* are still defined by either (19), (24), or (28) or the equivalent equations for p_i^* depending on the retail price level. And no matter which definition applies we can show that $p_j^* \geq p_i^*$ as long as $K_j \geq K_i$. If $(p_i, p_j) = (p_i, p_j^*)$ with $p_i \leq \bar{p}_i < p_j^*$ holds and the total capacity in the market is sufficient to satisfy $D(p_j^*, r, \tilde{\alpha}, \varepsilon)$, the best $p_i > p_j^*$ would be $p_i = p_j^* + \epsilon$ with $\epsilon \to 0$ for the low-capacity firm. Then firm i's profit would be $(p_j^* + \epsilon)[D(p_j^* + \epsilon, r, \tilde{\alpha}, \varepsilon) - K_j]$ and this does never exceed the profit $p_j^* K_i$ that it would achieve with $p_i \leq \bar{p}_i < p_j^*$. Thus the equilibrium with the low-capacity firm bidding low with $p_i \leq \bar{p}_i$ and the high-capacity firm bidding high with $p_j = p_j^* > \bar{p}_i$ always exist for $K_i \leq K_j < D(0, r, \tilde{\alpha}, \varepsilon)$ as long as $K_i + K_j \geq D(p_j^*, r, \tilde{\alpha}, \varepsilon)$ holds. The latter condition is equivalent to $K_i > S_i$ with

$$S_i = \begin{cases} \frac{1}{4}(\frac{3}{2}-\tilde{\alpha})(3+2\tilde{\alpha}+4\varepsilon)-2K_j & \text{if } 0 \leq K_j < K_1, \\ 1+\varepsilon-r-K_j & \text{if } \max\{0,K_1\} \leq K_j < K_2, \\ 1+\varepsilon-\left(\tilde{\alpha}-\frac{1}{2}\right)r-2K_j & \text{if } \max\{0,K_2\} \leq K_j < D_0, \end{cases} \tag{31}$$

if $0 \leq r \leq \frac{1}{2}+\varepsilon$,

$$S_i = \begin{cases} \frac{(3+2\varepsilon-2r)^2}{8}-K_j & \text{if } 0 \leq K_j < K_2', \\ \frac{1}{8}\left(9+12\varepsilon+4\varepsilon^2+4r^2-8(\tilde{\alpha}+\varepsilon)r\right) & \\ \quad -2K_j & \text{if } \max\{0,K_2'\} \leq K_j \\ & \quad < D_0', \end{cases} \tag{32}$$

if the retail price fulfills $\frac{1}{2}+\varepsilon < r \leq \tilde{\alpha}+\varepsilon$ and

$$S_i = \frac{1}{8}\left(\frac{3}{2}-\tilde{\alpha}\right)(3+2\tilde{\alpha}+4\varepsilon)-2K_j \text{ if } 0 \leq K_j < D_0'', \tag{33}$$

if $\tilde{\alpha}+\varepsilon < r$.

If we now consider the other potential equilibrium with $(p_i,p_j) = (p_i^*,p_j)$ with $p_j \leq \bar{p}_j < p_i^*$, this equilibrium only exists if the high capacity firm j does not have an incentive to deviate to a price with $p_j > p_i^*$. Since $p_j^* \geq p_i^*$ holds, the optimal deviation for the high capacity firm is given by its p_j^* that is defined in (19), (24), or (28), depending on the relevant retail price level. Checking the profits from choosing $p_j^* > p_i^*$ reveals that this deviation is not beneficial if $K_i \geq \underline{K_i}$ with

$$\underline{K_i} = \begin{cases} \frac{1}{8}\left(9+12\varepsilon-4\tilde{\alpha}(\tilde{\alpha}+2\varepsilon)\right. & \\ \quad \left.-4\sqrt{K_j(9+12\varepsilon-8K_j-4\tilde{\alpha}(\tilde{\alpha}+2\varepsilon))}\right) & \text{if } 0 \leq K_j < K_1 \\ \max\left\{\frac{1}{8}\left(9+12\varepsilon-4\tilde{\alpha}(\tilde{\alpha}+2\varepsilon)\right.\right. & \\ \quad \left.\left.-8\sqrt{2K_j r(3-2\tilde{\alpha})}\right), 1+\varepsilon-K_j-r\right\} & \text{if } \min\{0,K_1\} \\ & \quad \leq K_j < K_2 \\ \max\left\{\frac{1}{8}(3-2\tilde{\alpha})(3+2\tilde{\alpha}+4\varepsilon)\right. & \\ \quad -\sqrt{K_j(2+2\varepsilon-2K_j-r(2\tilde{\alpha}-1))}, & \\ \quad \left.\frac{2K_j^2+2(3-2\tilde{\alpha})(1+\varepsilon-r)r-K_j(2+2\varepsilon-r(\tilde{\alpha}-1))}{2r(3-2\tilde{\alpha})}\right\} & \text{if } \min\{0,K_2\} \\ & \quad \leq K_j < D_0 \end{cases} \tag{34}$$

if $0 \leq r < \frac{1}{2} + \varepsilon$,

$$\underline{K_i} = \begin{cases} \frac{3+2\varepsilon-2r}{8} - K_j & \text{if } 0 \leq K_j < K_2' \\ \frac{8K_j^2+(3-2\tilde{\alpha})(3+2\varepsilon-2r)^2 r - K_j\left((3+2\varepsilon)^2-8r(\tilde{\alpha}+\varepsilon)+4r^2\right)}{2r(3-2\tilde{\alpha})} & \text{if } K_2' \leq K_j < K_3 \\ \frac{1}{8}\left\{9 - 8\tilde{\alpha}r + 4[\varepsilon(3+\varepsilon) - 2\varepsilon r + r^2] \right. & \\ \quad +4\sqrt{K_j}\left(9 + 12\varepsilon + 4\varepsilon^2 - 8K_j + 4r^2\right. & \\ \quad \left.\left. -8(\tilde{\alpha}+\varepsilon)r\right)^{1/2}\right\} & \text{if } \max\{K_2', K_3\} \\ & \quad \leq K_j < D_0' \end{cases} \tag{35}$$

if $\frac{1}{2} + \varepsilon \leq r < \tilde{\alpha} + \varepsilon$ and

$$\begin{aligned} \underline{K_i} &= \frac{1}{8}\left(9 + 12\varepsilon - 4\tilde{\alpha}(\tilde{\alpha} + 2\varepsilon)\right. && (36) \\ &\quad \left. -4\sqrt{K_j(9 + 12\varepsilon - 8K_j - 4\tilde{\alpha}(\tilde{\alpha}+2\varepsilon))}\right) \text{ if } 0 \leq K_j < D_0'', \end{aligned}$$

if $r > \tilde{\alpha} + \varepsilon$. The parameter K_3 in (35) is only relevant as long as $K_2 > K_3 > D_0'$ holds and is defined as

$$\begin{aligned} K_3 &= \frac{1}{16}\left(9 + 12\varepsilon + 4\varepsilon^2 - 8\tilde{\alpha}r - 8\varepsilon r + 4r^2\right. && (37) \\ &\quad \left. + \sqrt{-128(3-2\tilde{\alpha})^2 r^2 + ((3+2\varepsilon)^2 - 8(\tilde{\alpha}+\varepsilon)r + 4r^2)^2}\right). && (38) \end{aligned}$$

One can also show that $S_i \leq \underline{K_i}$ for the relevant ranges of K_j. Thus, the two types of equilibrium with either $(p_i, p_j) = (p_i, p_j^*)$ and $p_i \leq \bar{p}_i < p_j^*$ or $(p_i, p_j) = (p_i^*, p_j)$ and $p_j \leq \bar{p}_j < p_i^*$ exist for $K_j \geq K_i \geq \underline{K_i}$. For $\min\{K_j, \underline{K_i}\} > K_i \geq S_i$ only the equilibria with $(p_i, p_j) = (p_i, p_j^*)$ and $p_i \leq \bar{p}_i < p_j^*$ exist. For $K_i < S_i$ the total capacity in the market does not satisfy the total demand at p_j^* any more. The system operator will set the market clearing price $\hat{p}$. Both firms bid a price that does not exceed the anticipated market clearing price because this would reduce their profit.

B Proof of proposition 2

The retailers will always compete the retail price down to a level where $r \leq p^*$ is ensured for all $\varepsilon \in [0, 1]$ due to the Bertrand competition among them. From equations (19), (24) and (28) and the definition of $\hat{p}$ from case (v) in Proposition 1 it is obvious that $\frac{\partial p^*}{\partial \varepsilon} \geq 0$ for all $\varepsilon \in [0, 1]$ if $p^* \geq 0$. Thus $r \leq p^*$ for all ε implies that $r \leq p^*$ for $\varepsilon = 0$.

If $K_i < K_2$ or $K_i < K_2'$ holds for $\varepsilon = 0$, then the retail price will always satisfy $r \leq p^*$. Alternatively, $r \leq p^*$ might also occur for either $K_2 \leq K_i < S_i$, or $K_2' \leq K_i < S_i$, if the system operator needs to set the price $\hat{p}$ such that it exceeds the retail price for $\varepsilon = 0$.

Let us first assume that K_i and K_j are large enough that the system operator does not need to set a price $\hat{p}$ for $K_i > K_2$ or $K_i > K_2'$ such that it exceeds the retail price, then $K_i \leq K_2$ or $K_i \leq K_2'$ for the smallest $\varepsilon = 0$ is sufficient to ensure that $r \leq p^*$ for all $\varepsilon \in [0, 1]$. Taking into account p^* from either (19) or (24) and solving these inequalities for r yields

$$r < r' = \begin{cases} 3 - \tilde{\alpha} - \frac{1}{2}\sqrt{27 - 4\tilde{\alpha}(6 - \tilde{\alpha}) + 8K_i} & \text{if } 0 \leq K_i < \frac{1}{2}(\tilde{\alpha} - \frac{1}{2}), \\ \frac{2(1-K_i)}{5-2\tilde{\alpha}} & \text{if } \frac{1}{2}\left(\tilde{\alpha} - \frac{1}{2}\right) \leq K_i < 1. \end{cases} \tag{39}$$

The split occurs because for $K_i < \frac{1}{2}(\tilde{\alpha} - \frac{1}{2})$ the retail price threshold r_1 exceeds $\frac{1}{2}$ where, given $\varepsilon = 0$, the retail market is no longer fully covered and the parameter K_2' instead of K_2 becomes relevant. For $K_j \geq K_i \geq 1$ the only possible outcome for the retail competition is $r = 0$.

Let us now assume that K_i and K_j are not large enough to avoid the case that the system operator needs to set a price $\hat{p} \geq r$ for some $K_2 < K_i \leq K_j$ or $K_2' < K_i \leq K_j$ if $\varepsilon = 0$. The system operator price, given a fully covered retail market, is

$$\hat{p} = \frac{2\left(1 - K_i - K_j - r(\tilde{\alpha} - \frac{1}{2})\right)}{3 - 2\tilde{\alpha}}. \tag{40}$$

The system operator price, given a partially covered retail market, is

$$\hat{p} = \frac{9 - 8K_i - 8K_j - 8\tilde{\alpha}r + 4r^2}{12 - 8\tilde{\alpha}}. \tag{41}$$

In order to ensure $\hat{p} \geq r$

$$r \leq r'' = \begin{cases} \frac{3}{2} - \sqrt{2(K_i + K_j)} & \text{if } 0 \leq K_i < \frac{1}{2} - K_j, \\ 1 - K_i - K_j & \text{if } \frac{1}{2} - K_j \leq K_i < 1 - K_j, \end{cases} \tag{42}$$

must hold.

Note that we do not need to consider the case where the retail market is uncovered for $\varepsilon = 0$ because this implies that the retail price is too high for a positive demand of the retail consumers and would be competed downward by the retail firms. In addition r'' is only relevant if $r' < r''$ for the given $K_i \leq K_j$. Checking for which $K_i \leq K_j$ the inequality holds yields the definition of $\bar{r}$ in Proposition 2.

C Welfare derivation

C.1 Critical shocks

The critical demand shock, ε^*, that decides on whether the wholesale price is at or above the retail price can de derived as follows. Because ε^* determines if K_i is smaller or larger than K_1, we set K_1 from equation (20) equal to K_i and solve for ε. This yields

$$\varepsilon^* = -\frac{\tilde{\alpha}}{2} + \frac{2(K_i + 2r)}{3 - 2\tilde{\alpha}} + \frac{\sqrt{2}\sqrt{(3-2\tilde{\alpha})^2(2\tilde{\alpha}-1)r(2\tilde{\alpha}+4K_i+4r-3)}}{(3-2\tilde{\alpha})^2} - \frac{3}{4}. \tag{43}$$

For zero retail prices the critical shock is then

$$\varepsilon_z^* = \frac{8K_i + 4\tilde{\alpha}^2 - 9}{12 - 8\tilde{\alpha}}. \tag{44}$$

Inserting the intermediate and the high retail prices from proposition 2 in equation (43) yields the respective critical shocks ε_i^* and ε_h^*.

C.2 Conditions for market clearing

For the welfare analysis to be tractable, we assume that the firms can always clear the market and the SO never has to intervene. For retail prices of zero this is given through the capacity requirements that have to hold for the retail prices to be zero.

For intermediate and high retail prices capacities can be too low and the SO has to set the market clearing price for some demand shocks. To calculate welfare when the firms clear the market at their bids we define minimum capacity endowments such that the firms are able to play $p^* = r$ or $p^* = p_j^*$.

Intermediate retail price are always below $\frac{1}{2}$ and hence below $\frac{1}{2} + \varepsilon$ and therefore the retail market is always fully covered. Capacities have to fulfill $K_i + K_j \geq D(r \mid \varepsilon = 1)$, where $r < \frac{1}{2} + \varepsilon$, and $K_i + K_j \geq D(p_j^* \mid \varepsilon = 1)$. Rearranging the first condition yields

$$K_i \geq \frac{8 + 2\tilde{\alpha}(K_j - 2) - 5K_j}{3 - 2\tilde{\alpha}}. \tag{45}$$

Because $r < p_j^*$ and hence $D(r \mid \varepsilon = 1) > D(p_j^* \mid \varepsilon = 1)$, whenever equation (45) is satisfied, the firms can also cover the market at all optimal prices higher than the retail price.

Likewise, for high retail prices, capacities have to fulfill $K_i + K_j \geq D(r \mid \varepsilon = 1)$ and $K_i + K_j \geq D(p_j^* \mid \varepsilon = 1)$. When $\varepsilon = 1$, $r < \frac{1}{2} + \varepsilon = \frac{3}{2}$ always holds for high retail prices and the retail market is always fully covered. When $r < \frac{1}{2} + \varepsilon$ and all retail customers demand electricity the firms capacities have to satisfy $K_i + K_j \geq D(r \mid \varepsilon = 1)$ what implies

$$K_i \begin{cases} \geq \tilde{\alpha} - K_j + \frac{1}{2}\sqrt{4(\tilde{\alpha}-4)\tilde{\alpha} - 8K_j + 23} & \text{if } K_i > -\frac{1}{2}(\tilde{\alpha}-6)\tilde{\alpha} - \frac{23}{8} \\ \leq \tilde{\alpha} - K_j - \frac{1}{2}\sqrt{4(\tilde{\alpha}-4)\tilde{\alpha} - 8K_j + 23} & \text{if } K_i < -\frac{1}{2}(\tilde{\alpha}-6)\tilde{\alpha} - \frac{23}{8} \end{cases} \tag{46}$$

If the firms can cover all demand at the retail price when the retail market is fully covered, they can also cover the reduced demand for a partially covered retail market and the wholesale market at optimal prices above the retail price. Figure 8 illustrates the area that equations (45) and (46) define.

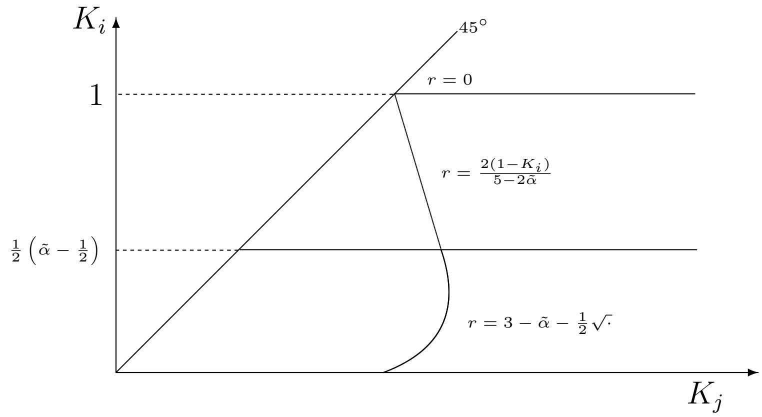

Figure 8: *Combinations of K_i and K_j that ensure $p = r$ or $p = p_j^*$.*

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