Price Regulations in a Multi-unit Uniform Price Auction*

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Abstract

Inspired by recent regulations in the New York ICAP market, this paper examines the effect of price regulations on a multi-unit uniform price auction. General bid caps reduce the maximum price below the bid cap, but also the minimum potential market price below the cap. A bid cap only for the larger firms does not guarantee a market price below the cap. A sufficiently high bid floor only for relatively small firms destroys some or all pure strategy equilibria with equilibrium prices above the marginal costs. With a general bid floor this happens only with considerably larger bid floors.

Keywords: Electricity, Capacity Markets, Price Regulation.
JEL-Classification: D43, D44, L12, L13, L51

1 Introduction

This study aims at deriving in a very simple framework how different sorts of price regulations change the equilibrium price in a multi-unit uniform price auction. The choice of the framework as well as the different types of

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price regulations are motivated by similar price regulations in the New York Installed Capacity Market (NYICAP) which will be explained in more detail in the following section 2.

Multi-unit uniform price auctions are often used in electricity markets to determine market prices.[1] In a typical day-ahead wholesale market this usually means that providers of electricity bid the price quantity combinations into the market that reflect their willingness to provide electricity to the market at a particular hour of the next day. These providers are mainly electricity generating firms but this could also be retailers who bought more electricity on future markets for that particular hour than their customers most likely demand or pure traders. The demand side of the market could be represented by the system operator who confronts the aggregated supply schedules with a total demand forecast of a particular hour. It could, however, also be an aggregated demand schedule that the system operator constructs from price quantity bids representing the willingness to buy of, again electricity retail firms, large consumers of electricity, electricity generating firms which committed to higher production quantities in future markets for that particular hour than they most likely can or want to deliver, or pure traders.

Independent of the details of the particular market, the uniform price auction always means that the system operator determines the price where demand and supply is balanced. Finally all those providers who were willing to sell quantities below this balancing price will sell these quantities. Those providers who were willing to sell quantities exactly at the balancing price are rationed such that supply and demand equals. What makes the multi-unit auction a uniform price auction is that all traded quantities will be paid the balancing price, no matter whether the particular supplier demanded it or not. Those bidders who wanted to sell quantities at a higher price than the balancing price cannot sell anything.

The NYICAP is not a market for actual electricity like the day-ahead markets. It is an instrument introduced into the New York electricity market in order to compensate power generating firms for a presumed lack of incentives to invest in power generating capacity.

[1] The Electricity Pool in England and Wales before the reform in 2001, the Nord Pool in Scandinavia, the Spanish wholesale market, as well as the NYISO and the ERCOT market in the US are still or were organized as multi-unit uniform price auctions. See Bergman *et al.* (1999), Crampes and Fabra (2005), Newbery (2005), Hortaçsu and Puller (2008) and Zhang (2009).

The suboptimal level of investments in electricity generating capacity is usually motivated by the lack of informative scarcity signals implied in the unregulated electricity prices on wholesale markets (see e.g. Joskow (2008) and Cramton and Ockenfels (2012)) and might result in the *missing money problem*. On the one hand demand in wholesale electricity markets is very inelastic due to the fact that lots of consumers have contracts with their retailers where prices are fixed for longer periods.[2] On the other hand electricity cannot be stored which gives producers together with the inelasticity of demand an enormous amount of market power in periods with very high demand compared to the available capacity. In lots of electricity wholesale markets, especially in the US, system operators have implemented relatively low price caps which reduces the generators' market power but also the potential rents from capacity investments.

This paper does not aim at determining whether there is a missing money problem in the New York electricity market[3] nor, given that the problem exists, whether the chosen instrument in New York is the most efficient one to solve it with.[4] Here the much more modest ambition is to figure out in which way the outcome of a very simple multi-unit uniform price auction that resembles the NYICAP mechanism is influenced by price caps and price floors and how the effect is changed if the price cap or the price floor is only applied to a subset of firms active in the market instead of to the whole group of active firms. Motivated by the NYICAP example, this study focusses on a price cap only for the firms with a relatively large generating capacity in the market and a price floor only for the firms with a relatively small capacity in the market.

The theoretical analysis rests on a very simple model of a multi-unit uniform price auction without asymmetric information, first introduced by von der Fehr and Harbord (1993). The latter forms also the basis of the mainly empirical contribution by Schwenen (2012). He shows that the NYICAP data from 2006-2008 are in line with the conclusions of such an auction model. The particular version of the model used here is a linear version of the

[2]This will only change if more consumers get access to smart meters and are exposed to real-time prices.

[3]Léautier (2011) argues, for example, that the reduction in investments in capacity due to the missing money problem is small compared to the reduction in investments due to the lack of competition in the electricity market.

[4]See for a discussion of alternative instruments, e.g. Finon and Pignon (2008) and Cramton and Ockenfels (2012).

more general one presented in Moreno and Ubeda (2006) and Ubeda (2004). Section 3 introduces the main assumptions and identifies some characteristics of the market outcome without any price regulation. Section 4 focusses on the market outcome changes if the system operator either introduces general price caps or price caps only for firms with relatively large capacities, or if the system operator introduces general price floors or price floors only for firms with relatively small capacities.

The main results are that general bid caps reduce the whole range of equilibrium prices, also those below the cap, under usual circumstances. Only if total capacities are either very small or very large there is no effect. If total capacities are that small that the auctioneer always needs to balance supply and demand because even the highest possible monopoly price on the residual demand of the largest firm does not balance supply with demand, a price cap does not reduce the market price. The same is true if total capacities are very large and no firm's capacity is need to satisfy even the highest demand. Then the market price is zero and equal to the marginal cost of supplying capacity on the market. If bid caps only apply to relative large firms with large capacities their effect might be weakened if some intermediate firms are not restricted and can still set their monopoly price on the residual demand above the price cap. Sufficiently high general price floors applied to all firms destroy pure strategy equilibria at the lower end of the price range. With general bid floors the only remaining pure strategy equilibrium might be the one where all firms bid at the floor. With selective bid floors this is not an option. If, in this case, the floors are sufficiently high pure strategy equilibria no longer exist.

The final conclusions focus on which changes we expect to observe on the NYICAP market due to the changes of the regulation 2008. Multi-unit uniform price auctions are also commonly used in treasury auctions (see e.g. Brenner *et al.* (2009)), emission permit auctions (see e.g. Betz *et al.* (2010)) and even in order to place IPOs on financial markets (see e.g. Degeorge *et al.* (2010)). Therefore the final section 5 also discusses whether the findings of this study are relevant for these markets.

2 The New York Installed Capacity Market

The New York Installed Capacity Market (NYICAP) consists of three different regional markets for New York City, Long Island and the rest of the state of New York.[5] For each of these markets the New York ISO conducts auctions for installed capacity for each month of the capability year (starting on May 1 and ending on April 30) at least 15 days before that year starts. The final auction for each month called the *spot market* takes place at least 2 days before the start of the month. Each load serving entity in the state of New York needs to buy capacity requirements that reflect the forecasted peak-load demand of its customers plus a reserve. The load serving entities can buy these requirements directly from electricity generators or via forward auctions. However, if they are not covered 30 days before the month starts the NYISO buys on behalf of them on the *spot market*.

The NYISO does so by constructing a demand curve for capacity that is derived from the *minimum installed capacity requirement* that should ideally be priced with a reference price.[6] The latter should reflect the monthly *going forward cost* of a peaking facility adapted to whether the month is in the summer or the winter season. The *minimum installed capacity requirement* is derived from the forecasted peak-load in the particular region of the New York market in the considered capability year plus a reserve margin. The *going forward cost* is the annualized estimated cost of a peaking unit minus the estimated revenues that this peaking unit would generate on the New York electricity markets. Two points define the linear part of the demand for installed capacity in New York City: the reference point connecting the reference price and the *minimum installed capacity requirement* and the point connecting 112 % of the *minimum installed capacity requirement* with a price of zero. Additionally a maximum price, resembling a price cap, cuts this linear demand. The slope of the linear demand functions for the other regions in the NYISO market is a bit flatter in absolute terms because the second point combines 118 % of the minimum installed capacity and a price of zero, but all regional NYICAP markets have a linear demand which is cut off at a maximum price, meaning that the inverse demand becomes horizontal.

All generators who own generating capacity in the region can, after having the characteristics of their capacity registered with the NYISO, bid their capacity in the relevant regional NYICAP spot market as long as they have

[5] See the New York ISO's homepage www.nyiso.com and for another but less detailed summary of the rules Schwenen (2012).

[6] The argument for using a downward sloping function instead of a simple step function that only relates to the reference price and the targeted capacity is that the capacity price is less volatile over time when the load growth is uncertain. See, e.g. the simulation model in Hobbs *et al.* (2007)

not sold it already on a forward capacity market.[7] The NYISO demands the quantities according to the artificially constructed demand function minus the quantities that the load serving entities had already procured until the deadline via bilateral contracts or the forward market on behalf of the not yet covered load serving entities. The NYISO determines the price of the spot market via a uniform price auction. Those firms who sold capacities either via bilateral trade agreements with the load serving entities or in a future or the spot market for capacity are obliged to offer electricity equivalent to these capacities in the wholesale market for electricity. They can, however, always bid it at the highest possible price that is represented by the relevant price cap on the wholesale market.

The NYISO installed capacity market is not unique in the US. Similar set-ups also exist in other markets. Examples are the PJM and the New England capacity market.[8] They differ mainly by two characteristics, the timing of the auction and the demand. The auctions take place much longer in advance before the procured capacity needs to be offered on the wholesale electricity market.[9] In addition the demand function has more than one linear segment with different slopes and therefore more kinks.

Besides the general price cap inherent in the NYICAP demand function the NYISO had selective price cap regulations already before 2008. In 2008, after observing that the spot market price for capacity coincided close to always with this price cap since 2006 (see also the empirical paper by Schwenen (2012) on 2006-2008 data), the selective price caps were lowered, and selective price floors were newly introduced. In order to mitigate market power suppliers who are considered to be *pivotal* must now offer their capacity at a price not exceeding the default reference price. The latter is the reference price corrected by a monthly calculated factor that relates 80% of the load forecast to the the available capacity.[10] A firm is *pivotal* if its capacity is necessary for supplying the market with at least the reference quantity of

[7]These characteristics are for example typical maintenance times during a year where the facility does not work, etc., in order to determine for how much capacity a certain facility should count.

[8]See, e.g. Joskow (2008) and Hobbs *et al.* (2007). In Europe the UK is in the process of implementing such a market (Department of Energy and Climate Change (2013)).

[9]This is supposed to provide more incentives to enter the market with new capacity.

[10]Pivotal firms can apply for getting another firm specific bidding cap if they can prove to the ISO that their *going forward costs* deviate substantially from the average in the market.

capacity. The newly introduced price floor should prevent inefficient entry and applies therefore only to new capacity in the market. The latter cannot be supplied below 75% of the net cost of new entry, meaning the cost of a peaking facility minus the revenues that can be realized with it in the new market.[11]

In order to get a rough idea about how the market should respond to these regulations the following analysis uses a very simple model of a multi-unit uniform price auction with a certain linear demand as in the NYICAP market and with given capacities of the participating firms. First the potential equilibria for given capacities without any price regulations are derived. Then I introduce general price caps and identify the resulting changes in these equilibria. Afterwards I consider a price cap only for firms above a particular capacity and characterize again the resulting market equilibria. The latter pretty much resembles the selective price caps that were tightened in the NYICAP market after 2008. Then the paper analyzes a general price floor to all firms in the market. The latter does not exist in the NYICAP market, but later I compare it with a selective price floor that applies only to the sub-group of firms with small capacities. The latter reflects the NYICAP regulation, specifically targeted at new entrants, if new entrants enter on a small scale.

3 The Model and the Equilibria without Price Regulation

Consider a market with a set of $N = 1, 2, \ldots, n$ active firms with $n > 2$. Each firm $i \in N$ owns a certain amount of capacity K_i that it potentially can supply on the capacity market. For now it is assumed that supplying parts of the capacity or all of it on the market does not cause any costs.[12] Assume that the firms are indexed such that $K_i \geq K_{i+1}$. In addition define

[11] Again a firm that can prove that its entry cost substantially differs from this market wide determined index number can apply for an individually determined bidding floor.

[12] In the NYICAP market the cost could be the obligation to supply the capacity on the electricity wholesale market at least at the maximum possible price. As long as the marginal cost of producing electricity with the considered capacity does not exceed the price cap on the wholesale electricity market, withholding the capacity from the electricity market cannot be more profitable than supplying the capacity at the price cap. Only if the virtually calculated average running times used to calculate the maximum capacity of each provider on the capacity market is misspecified and does not allow the generating firm to

$\bar{K} = \sum_{i=1}^{n} K_i$ as the total capacity available, and $\bar{K}_{-i} = \bar{K} - K_i$ as the total capacity available if firm i does not supply its capacity K_i. Demand $D(p)$ for capacity is linear in the market price p with

$$D(p) = \alpha - \beta p \text{ and } \alpha, \beta > 0. \tag{1}$$

Each firm can submit a price bid $b_i \geq 0$ at or above which it is willing to supply its total capacity K_i to the market.[13] The auctioneer sorts all bids according to the demanded price in an ascending order and forms an aggregate supply function. The equilibrium price is the price at which the supply function equals the ex ante publicly known demand. All firms that bid their capacity at a price below the equilibrium price sell their total capacity. Those, which bid above, sell nothing, whereas the marginal firm(s) that bids(bid) the equilibrium price might be rationed in order to balance supply and demand.[14] The vector of all bids $(b_1, \ldots, b_n)$ needs to be a Nash equilibrium in order to determine an equilibrium price in the auction.

Note that if the firms could split their total capacity into $l \geq 1$ discrete pieces for which they could demand different minimum prices to supply them to the market, this would not change the potential equilibrium prices.[15] Given that the demand is certain and common knowledge, then, whatever the other firms bid in equilibrium, a single firm can never gain from withholding capacity and asking for a higher price if it is finally not one of the price setting firms. If it is not price setting and increases the bid on part of its capacity, it can potentially lose if it is substituted by another firm in the merit order and is bidding too high on part of its capacity to be necessary to satisfy demand. If it could gain by becoming price setting, then the situation is not an equilibrium to begin with because then it could also gain by bidding a higher price on its total capacity. If in the equilibrium the firm is a price setting marginal firm , then it does not matter whether it bids that marginal market clearing price on all its capacity and is rationed or on only part of its

fulfill the obligation, providing capacities would cause a cost because the firm would be punished by the NYISO.

[13]The Model is the same as in Moreno and Ubeda (2006) and Ubeda (2004) only with a more specified linear demand and, for now, without any possibility to invest in capacity.

[14]In the case of multiple marginal bidders we assume that they are rationed according to their relative share in the total capacity bid at the marginal price by the marginal bidders.

[15]See also Fabra *et al.* (2006) who prove this in their Lemma 2 for 2 competing firms and Schwenen (2012) who makes the intuitive argument presented here in a more formal way for $n > 2$ firms.

capacity and is not or less rationed. It is always less profitable to increase the price on part of the capacity such that another firm substitutes for that part of the capacity in the merit order than demanding just slightly less than that potentially substituting firm on one's total capacity.

When characterizing the potential market equilibria which depend on the capacities held by the firms it is worthwhile as in Moreno and Ubeda (2006) to split the firms according to their capacities in those who can potentially be price setting or marginal in an equilibrium and those who cannot. For the start define the two sets of firms

$$Q = \{j \in N | \bar{K}_{-j} < \alpha\} \text{ and } O = \{j \in N | \bar{K}_{-j} \geq \alpha\}. \tag{2}$$

Proposition 1 *The firms $j \in O$ can only set the marginal price in a Nash equilibrium if $Q = \emptyset$ and the equilibrium price is $p = b_j = 0$. If $Q \neq \emptyset$ then the price in any pure strategy Nash equilibrium needs to be strictly larger than the marginal costs of zero.*

Proof. Suppose $j \in O$ and $b_j > 0$ is the equilibrium price. This is only possible if the bids of all the other firms $i \in N$, $i \neq j$, satisfy $b_i \geq b_j$. However, then firm i has an incentive to undercut firm j slightly in a Bertrand fashion in order to increase its profit, no matter whether $i \in O$ or $i \in Q$. If on the other hand $j \in O$ and $b_j = 0$ then $p = b_j = 0$ cannot be an equilibrium price if there are still firms $i \in Q$. These firms always have an incentive by unilaterally bidding $\alpha > b_i > b_j = 0$ to increase the price to b_i and, thus, increase their profit. This argument would not change if the low bidding firm with $b_j = 0$ belonged to the set Q instead. ■

Not surprisingly, firms, whose capacity is so small that their capacity is not even needed to supply total demand at a price of zero ($j \in O$), can never be price setters in equilibrium as long as other firms have capacities large enough that they are necessary to satisfy demand at a price of zero.

Let us assume for the rest of the analysis that the firms' capacities are such that $Q \neq \emptyset$, implying

$$\bar{K} \geq K_j > \bar{K} - \alpha \text{ for some } j \in N. \tag{3}$$

The residual demand of firm $j \in Q$

$$D^r(p, \bar{K}_{-j}) = \max\{\alpha - \bar{K}_{-j} - \beta p, 0\}$$

is the demand left to firm j if all other firms in the market offer their total capacities and if demand would be efficiently rationed. For firm j's residual demand firm j's monopoly price is

$$p_j = \arg\max\{pD^r(p, \bar{K}_{-j})\} = \frac{\alpha - \bar{K}_{-j}}{2\beta}. \tag{4}$$

The price p_j is only feasible as an equilibrium price if $D^r(p_j, \bar{K}_{-j}) < K_j$ or

$$K_j > \alpha - \bar{K}. \tag{5}$$

If for none of the firms $j \in Q$ condition (5) holds, the only feasible equilibrium price is

$$\bar{p} = \{p | D(p) = \bar{K}\} = \frac{\alpha - \bar{K}}{\beta}. \tag{6}$$

The following proposition characterizes the potential Nash equilibria in the multi-unit-uniform-price auction.

Proposition 2 *If $Q \neq \emptyset$ then depending on the capacities of the firms $j \in Q$ we can distinguish between two different cases.*

(i) If $K_j \leq \alpha - \bar{K}$ for all $j \in Q$, there are infinitely many bidding equilibria in pure strategies where each firm $j \in N$ bids $b_j \leq \bar{p}$. The equilibrium price is $p = \bar{p}$ regardless of the firms bids and all firms j in N sell their total capacity K_j.

(ii) If $K_j > \alpha - \bar{K}$ for some $j \in Q$ there are potentially multiple equilibria in pure strategies where one of the firms $j \in P$ with

$$P = \{j \in Q | K_j \geq \frac{(\alpha - \bar{K})^2 + K_1^2}{2K_1}\} \tag{7}$$

bids $b_j = p_j$ as defined in (4) and sells potentially only part of its capacity necessary to satisfy the residual demand $D^r(p_j, \bar{K}_{-j})$. All other firms $i \in N \setminus j$ bid $b_i \leq \underline{b}_j$ with

$$\underline{b}_j = \frac{(\alpha - \bar{K} + K_j)^2}{4\beta K_j} < p_j = b_j \tag{8}$$

and sell their total capacity K_i. The equilibrium price is identical with the bid $b_j = p_j = p$ of the highest bidding firm j.

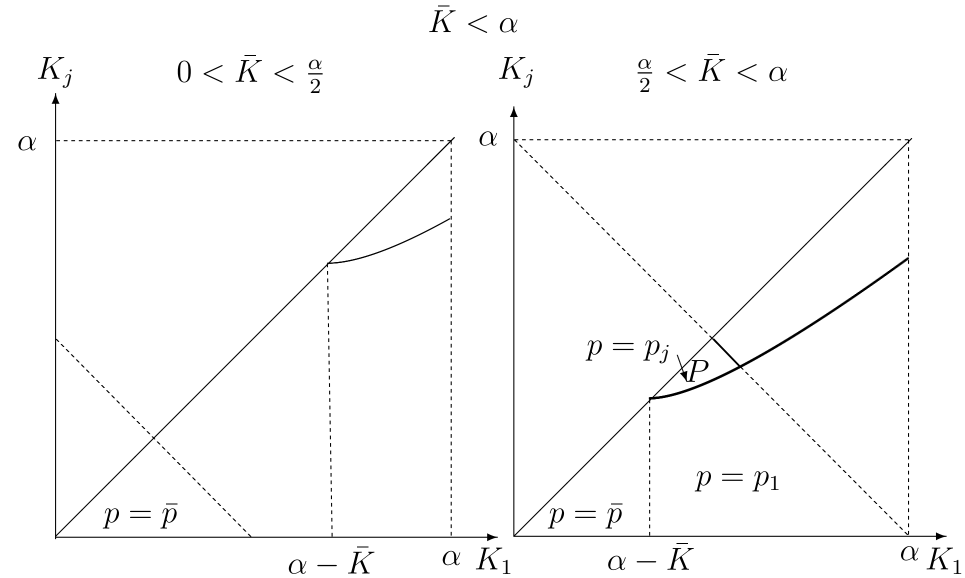

Figure 1: *Characterization of Potentially Price Setting Firms with* $\bar{K} < \alpha$

Proof. See the argument in Appendix A. ■

Proposition 2 implies that if all firms $j \in Q$ have a rather small capacity with $K_j \leq \alpha - \bar{K}$, then the equilibrium price is always determined by the balance of total available capacity and the demand for capacity and is given by $\bar{p}$ as defined in (6).[16] If some firms $j \in Q$ are larger with $K_j > \alpha - \bar{K}$, the firm with the largest capacity, firm 1 would then necessarily belong to P as defined in (7), meaning it could potentially be a price setting firm with $b_1 = p_1 = p$. However, whether firm 1 is the only possible price setting firm depends on the capacities of the other firms $j \in Q$. Given $K_1 > \alpha - \bar{K}$ and therefore also $1 \in P$, two different situations characterized in figure 1 and 2 can occur.

Suppose $\bar{K} < \alpha$ then all firms $j \in N$ also belong to Q and $O = \emptyset$. Note that if $0 < \bar{K} < \frac{\alpha}{2}$ we necessarily have $\alpha - \bar{K} > \frac{\alpha}{2} > K_1$ and neither firm 1 nor any other firm j with $K_j < K_1$ can be an element in P, and be price setting in equilibrium. This situation is sketched on the left-hand side of figure 1. With $\bar{K} < \alpha$ and $K_1 \geq K_j$ with $j \neq 1$ only those combinations of K_1 and K_j can occur which lie to the east of the upward sloping and to the

[16] In Figure 1 this is the case as soon as $K_1 \leq \alpha - \bar{K}$ which necessarily implies $K_j \leq K_1 \leq \alpha$ for all $j \in Q$.

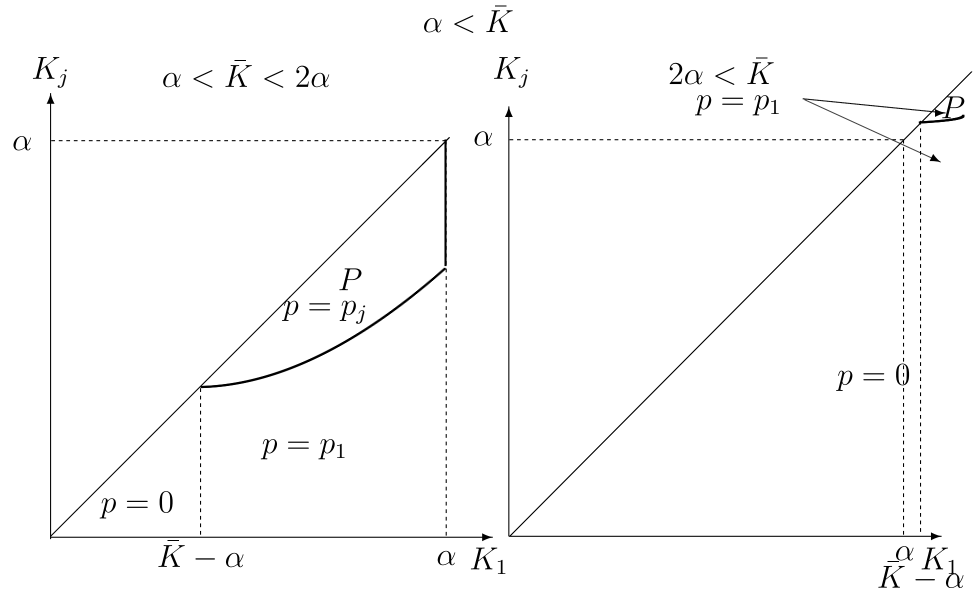

Figure 2: *Characterization of Potentially Price Setting Firms with* $\alpha \leq \bar{K}$

west of the downward sloping dashed 45^o-degree line. Thus, $P = \emptyset$ and the equilibrium price is necessarily $p = \bar{p}$. If $\frac{\alpha}{2} < \bar{K} < \alpha$ then firm 1 can be a price setter in equilibrium, given $K_1 > \alpha - \bar{K}$, and $1 \in P$. Whether other firms $j \in Q$ could also be price setters depends on their capacity relative to the capacity of firm 1. If their smaller capacity is relative close to firm 1's such that $j \in P$, they can also be price setting firms and we could potentially also observe $p = p_j$ as market equilibria. On the right-hand side in figure 1 all potentially price setting firms j with $K_j \leq K_1$ must have a capacity K_j above the bold curve, but since $\bar{K} < \alpha$ also implies $K_1 + K_j \leq \bar{K} < \alpha$ these firms capacities cannot be to the west of the dashed downward sloping 45^o-line.

Now suppose $\alpha \leq \bar{K} < 2\alpha$, then not all firms are necessarily an element of Q. If $Q = \emptyset$ this implies a relative even distribution of all capacities and proposition 1 applies with the equilibrium price being $p = 0$. However, if $Q \neq \emptyset$, then firm $1 \in Q$ and firm $1 \in P$ necessarily holds which means, firm 1 can be a price setter with $p = p_1$. In addition all those firms $j \in Q$ with a capacity $K_j \leq K_1$ such that it exceeds the bold curved line on the left-hand side of figure 2 can also be price setters which would result in an equilibrium price of $p = p_j$. Note that as soon as $K_1 > \alpha$ holds, no other firm $j \neq 1$ can be an element of Q and therefore P because necessarily $K_{-j} > \alpha$ and the equilibrium price can only be either $p = p_1$ if $K_1 > \bar{K} - \alpha$, or $p = 0$ if $K_1 \leq \bar{K} - \alpha$ and proposition 1 applies. These are the only two possible prices for $\bar{K} > 2\alpha$, the case illustrated on the right-hand side of figure 2.

Obviously, the potential market prices and highest potential bids in the market depend, on the one hand, on the level of the total capacity but, on the other hand, also on how unequally distributed the capacities of the different generators are. The following corollary characterizes the upper and lower limit of the market price and results mainly from proposition 1, the definition of P in equation (7) of proposition 2 and the definition of the the market price, given in equation (4).

Corollary 1 *Observed market prices depend on the total capacity and on the inequality of the capacities in the following way.*

(i) For $0 \leq \bar{K} < \frac{\alpha}{2}$ the market price observed should satisfy

$$p = \frac{\alpha - \bar{K}}{\beta} = \bar{p}.$$

(ii) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ the market prices observed should satisfy

$$p = \frac{\alpha - \bar{K}}{\beta} \text{ for } K_1 < \alpha - \bar{K} \text{ and}$$

$$p \in \left[\frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}, \frac{\alpha - \bar{K} + K_1}{2\beta} \right] \text{ otherwise.}$$

(iii) For $\alpha \leq \bar{K} < 2\alpha$ the market prices observed should satisfy

$$p = 0 \text{ for } K_1 < \bar{K} - \alpha,$$

$$p \in \left[\frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}, \frac{\alpha - \bar{K} + K_1}{2\beta} \right] \text{ for } \bar{K} - \alpha \leq K_1 < \alpha \text{ and}$$

$$p = \frac{\alpha - \bar{K} + K_1}{2\beta} \text{ otherwise.}$$

(iv) For $2\alpha \leq \bar{K}$ the market prices observed should satisfy

$$p = 0 \text{ for } K_1 < \bar{K} - \alpha \text{ and}$$

$$p = \frac{\alpha - \bar{K} + K_1}{2\beta} \text{ otherwise.}$$

Corollary 2 characterizes the non-marginal bids and follows from applying proposition 1 and the definition of P, given in equation (7), to the upper limit of the non-marginal bids, given in equation (8) of proposition 2.

Corollary 2 *The non-marginal bids in equilibrium need to satisfy the following conditions.*

(i) For $0 \leq \bar{K} < \frac{\alpha}{2}$ all bids can be non-marginal and need to satisfy

$$b_j \leq \frac{\alpha - \bar{K}}{\beta} \text{ for all } j \in N.$$

(ii) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ again all bids need to satisfy

$$b_j \leq \frac{\alpha - \bar{K}}{\beta} \text{ for } K_1 < \alpha - \bar{K}.$$

The non-marginal bids need to satisfy

$$b_i \leq \underline{b} \text{ with } \underline{b} \in \left[\frac{(\alpha - \bar{K} + K_1)^4}{8\beta K_1[(\alpha - \bar{K})^2 + K_1^2]}, \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}\right] \text{ otherwise.}$$

(iii) For $\alpha \leq \bar{K} < 2\alpha$ all bids are marginal and must satisfy

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$

The non-marginal bids need to satisfy

$$b_i \leq \underline{b}_j \text{ with } \underline{b}_j \in \left[\frac{(\alpha - \bar{K} + K_1)^4}{8\beta K_1[(\alpha - \bar{K})^2 + K_1^2]}, \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}\right] \text{ if } \bar{K} - \alpha \leq K_1 < \alpha$$

$$\text{and } b_i \leq \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1} = \underline{b}_1 \text{ otherwise.}$$

(iv) For $2\alpha \leq \bar{K}$ again all bids are marginal and must satisfy

$$b_j = 0 \text{ for } j \in N \text{ and } K_1 < \bar{K} - \alpha.$$

For $K_1 \geq \bar{K} - \alpha$ the non-marginal bids need to satisfy

$$b_i \leq \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}$$

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4 The Effect of Price Regulations on Market Outcomes

This section analyzes how different types of price regulations changes the market outcomes. First it focusses on maximum prices in the form of bid caps and later on price floors or minimum bids. In both cases the analysis starts with either general bid caps or general bid floors which apply to all firms in the market. Afterwards the focus shifts to selective price regulations, meaning bid caps only for firms with large capacities and bid floors only for firms with low capacities. The NYICAP market which inspired this study has general and selectively more stringent bid caps, but only selective and no general bid floors.[17]

4.1 The Effect of Maximum Prices

4.1.1 General Bid Caps

Obviously, a general bid cap can only have an effect if it forces at least some firms to change their bidding behaviour. From proposition 1 we know that if $Q = \emptyset$ then all firms $j \in O = N$ bid $b_j = 0$ and the equilibrium price is $p = 0$. This bidding behaviour would not be changed by any bid cap that forces all firms $j \in N$ to bid $b_j \leq \hat{b}$ with $\hat{b} \geq 0$. This obviously changes if $Q \neq \emptyset$ and proposition 2 would apply if there were no bid cap.

If $K_j \leq \alpha - \bar{K}$ for all firms $j \in Q$ then the bid cap might change the bidding behaviour if $\hat{b} < \bar{p}$ where the latter is the price, defined in equation (6), where total capacity balances with total capacity demand. It would however not change the market price because total supply and total demand cannot be balanced at any price $p = \hat{b} < \bar{p}$. The auctioneer needs to elevate the market price from the potentially highest bid $\hat{b}$ to $p = \bar{p}$ to balance total supply and demand. The same is true if some firms $j \in Q$ have a capacity with $K_j > \alpha - \bar{K}$ but the bid cap is still set such that $\hat{b} < \bar{p}$ then all firms bid at or below the threshold $\hat{b}$, but the final market price will always be $p = \bar{p}$.

Proposition 3 *If $Q = \emptyset$ and all firms $j \in N$ also satisfy $j \in O$, then a bid cap $\hat{b} \geq 0$ which only allows the firms to bid a price $b_j \leq \hat{b}$ for their total capacity, does not change the firms' bidding behaviour. All firms $j \in N$ still*

[17] See section 2 for more details on this.

bid $b_j = 0$ *and thus the market price is still* $p = b_j = 0$*. If* $Q \neq \emptyset$ *and* $\hat{b} \leq \bar{p}$ *the bid cap might reduce the bid of some bidders* $j \in Q$*, and it will always result in a market price of* $p = \bar{p}$ *as defined in (6).*

Proof. See the arguments above.■

If $K_j > \alpha - \bar{K}$ for some $j \in Q$ then a bid cap with $\hat{b} > \bar{p}$ does not only have potential to change the bidding behaviour but also to change the market outcome. For having an effect on the bidding behaviour and this way also on the market price it is necessary that the bid cap is either below the bid of the marginal bidder or changes the set of potentially marginal bidders P as given in (7) in proposition 2.

Proposition 4 *If* $Q \neq \emptyset$*, if some firms* $j \in Q$ *have a capacity* $K_j > \alpha - \bar{K}$ *and if the bid cap satisfies* $\hat{b} > \bar{p}$ *then the bid cap has only an impact on the bidding behavior of the firms if it constrains the potential bids of some firms* $j \in Q$ *meaning* $p_j > \hat{b} \Leftrightarrow K_j > 2\beta\hat{b} - (\alpha - \bar{K})$*.*

(i) If $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ *holds for all firms* $j \in Q$ *then there is a set of Nash equilibria where any of the firms* $j \in Q$ *with* $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ *bids* $b_j = \hat{b}$ *and all other firms* $i \in N \setminus j$ *set* $b_i \leq \hat{\underline{b}}_j$ *with*

$$\hat{\underline{b}}_j = \frac{\hat{b}(\alpha + K_j - \bar{K} - \beta\hat{b})}{K_j} < \underline{b}_j, \tag{9}$$

and with $\underline{b}_j$ *defined in equation (8). The market price will be* $p = \hat{b}$*.*

(ii) If only for some firms $j \in Q$ *the condition* $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ *holds, the equilibria described in (i) still exist, but in addition other Nash equilibria can exist. In these additional equilibria one of the other firms* $j \in Q$ *with* $K_j \leq 2\beta\hat{b} - (\alpha - \bar{K})$ *bids its monopoly price on the residual demand* $b_j = p_j < \hat{b}$ *and all the other firms* $i \in N \setminus j$ *bid* $b_i \leq \underline{b}_j$*. These equilibria exist if* $j \in \hat{P}$ *with*

$$\hat{P} = \left\{ j \in Q | K_j \geq \frac{2\beta\hat{b}(\alpha - \beta\hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K})}{K_1} \right\}. \tag{10}$$

(iii) If for all firms $j \in Q$ the condition $K_j \leq 2\beta\hat{b} - (\alpha - \bar{K})$ holds, then the bid cap has no effect on the firms bidding behaviour in equilibrium and proposition 2 still applies.

Proof. See the the proof in Appendix B.■

Note that the threshold that the capacity of an unconstrained firm j needs to exceed (meaning $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$) in order to be potentially price setting in case (ii) ($j \in \hat{P}$) is lower than the threshold that defined whether firm j could be a price setter without the bid cap ($j \in P$ defined in proposition 2). This results from other firms $i \in Q$ with $K_i > K_j$ finding it now less attractive to overbid firm j because they are restrained to the bid cap and can no longer bid the monopoly price on their residual demand because $p_i > \hat{b}$. Therefore firms $j \in Q$ with smaller capacities as before can be price setters in an equilibrium and, if they are, smaller equilibrium prices than without the price cap occur.

With $K_1 > 2\beta\hat{b} - (\alpha - \bar{K})$ and $p_1 > \hat{b} > \bar{p}$ firm 1 with the largest capacity is potentially constrained by the bid cap, but this also implies that $K_1 > \alpha - \bar{K}$. The latter excludes the case that $0 < \bar{K} < \frac{\alpha}{2}$ or case (i) as well as part of case (ii) from corollary 1. Corollary 3 characterizes the market prices that we can observe in this case.

Corollary 3 *If $Q \neq \emptyset$ and $K_1 \leq 2\beta\hat{b} - (\alpha - \bar{K})$ then a bid cap $\hat{b} > \bar{p}$ should not change the observed prices in the market as characterized in corollary 1. If, however, $Q \neq \emptyset$ and $K_1 > 2\beta\hat{b} - (\alpha - \bar{K})$ then a bid cap $\hat{b} > \bar{p}$ changes the observed market prices in the following way.*

(i) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ the market prices observed should satisfy

$$p \in \left[\hat{b}\frac{\alpha - \beta\hat{b} + K_1 - \bar{K}}{K_1}, \hat{b}\right].$$

(ii) For $\alpha \leq \bar{K} < 2\alpha$ the market prices observed should satisfy

$$p = 0 \text{ for } K_1 < \bar{K} - \alpha,$$

$$p \in \left[\hat{b}\frac{\alpha - \beta\hat{b} + K_1 - \bar{K}}{K_1}, \hat{b}\right] \text{ for } \bar{K} - \alpha \leq K_1 < \alpha \text{ and}$$

$$p = \hat{b} \text{ otherwise.}$$

(iii) For $2\alpha \leq \bar{K}$ the market prices observed should satisfy

$$p = 0 \text{ for } K_1 < \bar{K} - \alpha \text{ and}$$
$$p = \hat{b} \text{ otherwise.}$$

So, the bid cap does not only potentially reduce the upper limit of the equilibrium prices, but also reduces the lower limit even if the lower limit does not exceed the bid cap.

Since the threshold for non-marginal bids $\underline{\hat{b}}_j$, given in (9), is monotonously increasing in the capacity K_j of the price setting firm j $(\partial\underline{\hat{b}}_j/(\partial K_j) > 0)$ and now smaller firms can also be price setting the bid cap can also have an effect on the marginal bids, again without them exceeding the bid cap.

Corollary 4 *Given $Q \neq \emptyset$ and $K_1 > 2\beta\hat{b} - (\alpha - \bar{K})$ then a bid cap $\hat{b} > \bar{p}$ results in the following observed non-marginal bids.*

(i) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ the non-marginal bids need to satisfy

$$b_i \leq \underline{\hat{b}}_j \text{ with } \underline{\hat{b}}_j \in \left[\frac{\beta^2\hat{b}[2(\alpha - \bar{K} - \beta\hat{b}) + K_1]}{2\beta\hat{b}(\alpha - \beta\hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K})}, \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta\hat{b})}{K_1}\right].$$

(ii) For $\alpha \leq \bar{K} < 2\alpha$ the bids must satisfy

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$

The non-marginal bids need to satisfy

$$b_i \leq \underline{\hat{b}}_j \text{ with } \underline{\hat{b}}_j \in \left[\frac{\beta^2\hat{b}[2(\alpha - \bar{K} - \beta\hat{b}) + K_1]}{2\beta\hat{b}(\alpha - \beta\hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K})}, \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta\hat{b})}{K_1}\right]$$

if $\bar{K} - \alpha \leq K_1 < \alpha$ and

$$b_i \leq \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta\hat{b})}{K_1} = \underline{\hat{b}}_1 \text{ otherwise.}$$

(iii) For $2\alpha \leq \bar{K}$ the bids must satisfy

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$

The non-marginal bids need to satisfy

$$b_i \leq \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta\hat{b})}{K_1} = \underline{\hat{b}}_1 \text{ otherwise.}$$

4.1.2 Selective Bid Caps

Now, what happens if the bid cap is only selectively applied to firms with $K_j > \hat{K} > 0$ instead of to all firms participating in the auction?[18] Note that, if the bid cap $\hat{b}$ applies only to firms with $K_j > \hat{K}$ with $\hat{K} < \bar{K} - \alpha$, this would not change our results because this would mean it would still apply to all potentially price setting firms $j \in Q$. Thus, all our results summarized in proposition 3 and 4 as well as in corollary 3 and 4 would still hold.

Note also that only those firms with $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ are potentially constrained in their price setting behaviour by a bid cap $\hat{b} > \frac{\alpha - \bar{K}}{\beta} = \bar{p}$ because the constraint would not influence the behaviour of the firms j with $\hat{K} \leq K_j \leq 2\beta\hat{b} - (\alpha - \bar{K})$. This implies that if $\hat{K} \leq 2\beta\hat{b} - (\alpha - \bar{K})$ holds, then again nothing would change compared to the situation characterized in proposition 3 and 4 as well as in corollary 3 and 4.

Changes can only occur if $\hat{K} > 2\beta\hat{b} + \bar{K} - \alpha$ holds, meaning that not all firms with $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ will be constrained in their price setting behaviour. Firms with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ can, on the one hand, still bid their monopoly price on the residual demand, although $p_j > \hat{b}$ holds, and, on the other hand, their overbidding option, in case a firm with a smaller capacity places the highest bid, is still to set $b_j = p_j > \hat{b}$. The following proposition takes this into account.

Proposition 5 *Suppose $Q \neq \emptyset$ and that a bid cap $\hat{b} > \bar{p}$, defined in (6), is imposed only on firms $j \in N$ with a capacity $K_j > \hat{K}$.*

(i) This would not change the market equilibrium compared to proposition 4 if either $\hat{K} < 2\beta\hat{b} + \bar{K} - \alpha$ or if all the firms $j \in Q$ have a capacity $K_j > \hat{K}$.

(ii) If there are, however, firms $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ and at least firm 1 is constrained with $K_1 > \hat{K} > 2\beta\hat{b} - (\alpha - \bar{K})$, then the equilibria with firm $j \in Q$ with $K_j > \hat{K}$ bidding $\hat{b}$ and all the other firms $i \in N \setminus j$ bidding $b_i \leq \underline{\hat{b}}_j$ defined in (9) can still exist if there does not exist any other firm i with $K_i \in (2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \bar{K} - \alpha)}, \hat{K})$.

[18] Note that in the NYICAP market $\hat{K}$ is determined such that $\hat{K} = \bar{K} - \tilde{K} > \bar{K} - \alpha$ with $\tilde{K} \in (0, \alpha)$ being the reference quantity. The firms with $K_j > \hat{K} = \bar{K} - \tilde{K}$ cannot bid their capacity at a higher price than the reference price that satisfies $\tilde{K} = \alpha - \beta\hat{b} \Leftrightarrow \hat{b} = \frac{\alpha - \tilde{K}}{\beta}$.

In addition equilibria with firm $j \in Q$ with $2\beta\hat{b} + \bar{K} - \alpha < K_j < \hat{K}$ bidding its monopoly price on its residual demand $p_j > \hat{b}$ and all other firms $i \in N \setminus j$ bidding $b_i \leq \underline{b}_j$ defined in (8) exist if

$$K_j \geq \frac{K_i^2 + (\alpha - \bar{K})^2}{2K_i} \text{ for all } K_i \in (K_j, \hat{K}). \tag{11}$$

Equilibria with firm $j \in Q$ with $K_j \in [\max\{\bar{K} - a, 0\}, 2\beta\hat{b} + \bar{K} - \alpha]$ bidding its monopoly price on its residual demand $p_j \leq \hat{b}$ and all other firms $i \in N \setminus j$ bidding $b_i \leq \underline{b}_j$ can also exist if firm j's capacity satisfies not only condition (11) but also $K_j \in \hat{P}$ as defined in (10) of proposition 4.

(iii) If $K_1 < 2\beta\hat{b} - (\alpha - \bar{K})$ then the selective bid cap does not have any influence on the equilibrium behaviour of any of the firms and proposition 2 still applies.

Proof. See Appendix C. ■

Note that proposition 5 differs only in case (ii) from proposition 4. This has three major implications for the potential equilibrium. First, equilibria where the firms with the largest capacities bid the bid cap do not always exist even if $K_j > \hat{K}$ for some $K_j \in Q$. If there are firms $i \in Q$ with $K_i \in (2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \bar{K} - \alpha)}, \hat{K})$ these firms would have an incentive to overbid any firm $j \in Q$ with $K_j > \hat{K}$ that would bid $b_j = \hat{b}$. Second, prices above the bid cap can occur in equilibrium if there are firms $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$. They can be price setters in equilibrium with $p = p_j > \hat{b}$ given that condition (11) holds for all larger firms i which are not covered by the selective bid cap. Note that condition (11) is not a pure subset of P defined in (7) of proposition 2 because it includes also firms $j \notin P$ with smaller capacities since $K_i < \hat{K} < K_1$ holds. Third, prices below the bid cap might occur in equilibrium if, for some firms $j \in Q$ with $K_j \leq 2\beta\hat{b} + \bar{K} - \alpha$, both conditions, (11) and $K_j \in \hat{P}$, are satisfied. These two conditions ensure that neither the larger constrained firms $i \in Q$ with $K_i > \hat{K}$ nor the larger non constrained firms with $K_i \leq \hat{K}$ want to overbid firm j. Which of the two constraints is binding depends on the level of the bid cap $\hat{b}$ and the critical capacity level $\hat{K}$ above which firms are constrained.

Corollary 5 resembles corollary 1 and 3 and identifies the boundaries for the equilibrium prices.

Corollary 5 *Given $Q \neq \emptyset$ and some firms $j \in Q$ have a capacity $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ then a bid cap $\hat{b} > \bar{p}$ only applied to firms $j \in Q$ with $K_j > \hat{K} > 2\beta\hat{b} - (\alpha - \bar{K})$ results in the same observed market prices as in corollary 3 if there are no firms j with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$. If, however, such firms exist and if at least $K_1 > \hat{K}$ holds, the boundaries for the market price change in the following way.*

(i) For $\frac{\alpha}{2} \leq \bar{K} < 2\alpha$ the market prices observed should satisfy

$$p \in \left[\max\left\{\hat{b}\frac{\alpha - \beta\hat{b} + K_1 - \bar{K}}{K_1}, \frac{(\alpha - \bar{K} + \hat{K})^2}{4\beta\hat{K}}\right\}, \frac{\alpha - \bar{K} + \hat{K}}{2\beta}\right].$$

(ii) For $2\alpha \leq \bar{K}$ the market prices observed should satisfy

$$p \in \left[\max\left\{\hat{b}\frac{\alpha - \beta\hat{b} + K_1 - \bar{K}}{K_1}, \frac{(\alpha - \bar{K} + \hat{K})^2}{4\beta\hat{K}}\right\}, \frac{\alpha - \bar{K} + \hat{K}}{2\beta}\right]$$

$$\textit{for } \bar{K} - \alpha \leq \hat{K} < \alpha \textit{ and } p = \frac{\alpha - \bar{K} + \hat{K}}{2\beta} \textit{ if } \hat{K} > \alpha.$$

If $K_1 \leq \hat{K}$ holds, then corollary 1 still applies.

Since $\hat{b} < \frac{\alpha - \bar{K} + \hat{K}}{2\beta} < \frac{\alpha - \bar{K} + K_1}{2\beta}$ holds, it is possible to see a market price above the bid cap in the market, but it is strictly below the potential upper level in the case without any bid cap described in corollary 1. Given the restrictions $K_1 > \alpha$, $\hat{K} > 2\beta\hat{b} - (\alpha - \bar{K})$ and $\hat{b} > \bar{p}$, $p = 0$ cannot be an equilibrium any more. In addition $p = \frac{\alpha - \bar{K} + \hat{K}}{2\beta}$ can only be a unique equilibrium outcome if $\bar{K} \geq 2\alpha$ because otherwise K_1 and $\hat{K}$ cannot both exceed α.

4.2 The Effect of Price Floors

A general bid floor that is set at $b_f > 0$ forces all firms $j \in N$ to bid a price for their capacity that satisfies $b_j \geq b_f$. Obviously this destroys the equilibrium that we have identified in proposition 1 for the case that $Q = \emptyset$ in which all firms $j \in N$ bid $b_j = 0$ and the equilibrium price would be $p = 0$. However this is also the least likely case where bid floors are introduced because in this case none of the firms, even the largest, is necessary to supply even the maximum possible demand for capacity which implies $K_1 \leq \bar{K} - \alpha$ and it is hard to understand why a capacity market even exists.

Therefore let us now concentrate on the case $K_1 > \bar{K} - \alpha$ which ensures that without bid floors proposition 2 applies and the market outcome entails market power. If we first consider the case where $K_j \leq \alpha - \bar{K}$ holds for all $j \in Q$, then a bid floor with $b_f \leq \bar{p}$ might destroy some of the potential equilibria with $b_j < b_f$ of the unrestricted case, but would not change the outcome in equilibrium. The equilibrium price is still $p = \bar{p}$ determined by the auctioneer to balance demand with the totally available supply. Of course, if $b_f > \bar{p}$ holds, the market can never be balanced and all equilibria described in proposition 2 (i) would be destroyed, but again this does not seem to be a relevant scenario.

So, the case, where the price floor requesting $b_j \geq b_f$ really matters and might also be relevant, is where $K_1 > \alpha - \bar{K}$ and where the bid floor is chosen such that some of the equilibria described in proposition 2 (ii) would be destroyed because some price setting firms $j \in P$ with $\underline{b}_j > b_f$ have now an incentive to undercut the inframarginal bids which satisfy now necessarily $b_i \geq b_f > \underline{b}_j$. Note that the limit for non-marginal bids in any equilibrium is $b_i \leq \underline{b}_j$, defined in proposition 2 in equation (8), and that this limit increases with the capacity of the price setting firm $j \in P$ because

$$\frac{\partial \underline{b}_j}{\partial K_j} = \frac{K_j^2 - (\alpha - \bar{K})^2}{4\beta K_j^2} > 0$$

holds if $K_j \in Q$ and $K_j > \alpha - \bar{K}$. Thus, a bid floor potentially destroys the equilibria with smaller equilibrium prices where smaller firms are price setting. A bid cap might even destroy all equilibria described in 2 (ii) if it is chosen such that $b_f > \underline{b}_1$.

Proposition 6 *If $K_1 > \alpha - \bar{K}$, implementing a general bid floor which applies to all firm $j \in N$, destroys some pure strategy equilibria described in Proposition 2 (ii), if $b_f > \underline{b}_j$ for some firms $j \in P$ with $\underline{b}_j$ defined in equation (8). In the extreme case where $b_f > \underline{b}_1$ holds with*

$$\underline{b}_1 = \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1},$$

the pure strategy equilibria described in Proposition 2 (ii) disappear. How-

ever, as long as $\underline{b}_j < b_f \leq \bar{c}_j$ *holds*

$$\bar{c}_j \equiv \frac{\alpha}{2\beta} - \frac{\sqrt{(\bar{K} - K_j)K_j(\bar{K}K_j - (\alpha - \bar{K})^2)}}{2\beta K_j} < p_j,$$

new alternative pure strategy equilibria exist. In these equilibria all firms i *but one firm* $j \in P$ *with* $\bar{c}_j \geq b_f > \underline{b}_j$ *bid at the bid floor* $b_i = b_f$ *and firm* j *bids at its monopoly price on its residual demand* $b_j = p_j$ *defined in equation (4). If* $\bar{c}_1 < b_f$ *holds all firms bidding* b_f *is the unique equilibrium in pure strategies.*

Proof. See the arguments above for the destruction of the pure strategy equilibria from proposition 2 (ii). For the alternative pure strategy equilibria in case of $b_f > \underline{b}_1$, note that if all firms bid at the bid floor each firm achieves a profit of

$$b_f \frac{K_j(\alpha - \beta b_f)}{\bar{K}}.$$

If firm j bids its monopoly profit on its residual demand the profit is

$$\frac{(\alpha - \bar{K} + K_j)^2}{4\beta}.$$

The former exceeds the latter as long as $b_f \leq \bar{c}_j$ holds, meaning that the price setting firm j does not want to reduce its price to the price floor. The firms that bid the price floor do not want to overbid firm j as long as $j \in P$. If $b_f > \bar{c}_1$ holds than even firm 1 with the largest overbidding profit prefers to bid at the price floor instead of bidding its monopoly price on the residual demand. Since no firm wants to overbid if all firms i set $b_i = b_f$ and no firm can underbid, this is an equilibrium in pure strategies. Other bidding equilibria in pure strategies with $b_i = b > b_f$ do not exist because each firm has an incentive to underbid in order to sell its total capacity instead of a rationed share. ■

Selective bid floors have the same effect as general ones as long as $\underline{b}_1 > b_f > \underline{b}_j$ for some $j \in P$. They destroy some of the possible equilibria and make some lower equilibrium prices impossible. In the extreme case of $b_f > \underline{b}_1$, however, the selective price floor that does not apply to all firms in the market eradicates the pure strategy equilibria where all firms but at most one bid on the price floor. These equilibria disappear because with selective bid floors applied only to smaller firms in the market the potential price setting firms $j \in P$ for whom $\underline{b}_j < b_f \leq \bar{c}_1$ and $K_j > \breve{K}$ holds, are excluded from the bid floor and have an incentive to underbid the firms i with $K_i < \breve{K}$ to whom the price floor applies and who therefore need to obey $b_i \geq b_f > underline b_j$.

Proposition 7 *If $K_1 > \alpha - \bar{K}$, implementing a bid floor only to those firms $j \in N$ for which $K_j < \breve{K} < K_1$ destroys again some pure strategy equilibria described in Proposition 2 (ii), if $b_f > \underline{b}_j$ for some firms $j \in P$ with $\underline{b}_j$ defined in equation (8). In the extreme case where $b_f > \underline{b}_1$ holds with*

$$\underline{b}_1 = \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1},$$

all pure strategy equilibria described in Proposition 2 (ii) disappear. The alternative equilibria in pure strategies where all firms bid at the price floor but at most one larger firm do no longer exist as long as the price floor does not also apply to the price setting firm j.

Proof. See the arguments above. ■

5 Conclusions

Our theoretical analysis suggests that the reduction of selective bid caps in the NYICAP in 2008 should have reduced the upper limit of the range of observed equilibrium prices but not necessarily to the level of the selective bid caps. The same should hold for the inframarginal bids. At the same point in time the newly introduced selective price floors should have increased the lower level of the observed equilibrium prices in the auction, even if these lower limits lied way above the introduced price floors without them. Trivially there will be more inframarginal bids above the selective price floor.

So, both changes together should have compressed the observed equilibrium prices from both sides and, chosen adequately, should have reduced the deviations from the targeted reference price. Whether the expected revenues from the capacity market were increased or reduced depends on the chosen bid caps and bid floors, but taken together it is not clear from the outset whether the incentive to invest in new electricity capacity increased or decreased. The only effect of the reform could have been that the regulators more often than not hit the reference price.

Of course our model can also be applied to other multi-unit uniform price auctions. Theoretically it is known that these auctions are not necessarily efficient (see for example Ausubel *et al.* (2014)) and that reservation prices which are in our selling context equivalent to price caps can improve on the efficiency (see Bresky (2013)). In reality reservation prices are rarely used in treasury auctions although there is evidence for underpricing (see Kremer and Nyborg (2004) and Keloharyu *et al.* (2005)) and it seems clear not only from the analysis here that the prices of treasury bonds could be increased by using them. Betz *et al.* (2010) report that reserve prices are relatively common in emission permit auctions, meaning that here regulation authorities use a reserve price to ensure a sufficiently high price. I have not found evidence, yet, for too high prices in these auctions. If they existed, they could be fought by price caps which are in these buying auctions the equivalent to our price floors.

Appendix

A Proof of Proposition 2

A.1 Characterization of the Nash Equilibria for Less Capacity Constrained Firms

Assume $Q \neq \emptyset$ and consider first the case with $K_j > \alpha - \bar{K}$ for some $j \in Q$. This implies that these firms can potentially bid their monopoly price on the residual demand p_j and be price setting with $b_j = p_j = p$ because it is feasible. Note that $p_j > p_i$ if $K_j > K_i$. Firm j bidding $b_j = p_j = p$ and all other firms bidding lower can only be a Nash equilibrium if no low bidding firm wants to overbid and firm j does not want to underbid the second highest bid. Let us first consider overbidding by other firms. If $K_i < K_j$ then optimally overbidding $b_j = p_j$ implies $b_i = p_j + \varepsilon$ with $\varepsilon \to 0$ and a profit of $p_j D^r(p_j, K_{-i})$ because $p_i < p_j$. One can show that for all $i > j$ the profit from overbidding is smaller than the profit from either underbidding or matching $b_j = p_j$ if $K_j > \alpha - \bar{K}$. Now suppose $K_i > K_j > \alpha - \bar{K}$ then the optimal overbidding strategy for firm i would be to bid $b_i = p_i$. However, firm i only wants to overbid firm j with $b_j = p_j$ if the profit from doing so is higher than matching or underbidding firm j. Note that matching is always weakly dominated by undercutting. So firm i only has an incentive to overbid if

$$p_i D^r(p_i, K_{-i}) > p_j K_i \Leftrightarrow K_j < \frac{(\alpha - \bar{K})^2 + K_i^2}{2K_i}.$$

This implies that $b_j = p_j = p$ and all other firms bidding lower can only be an equilibrium strategy if the condition above holds for none of the firms with $K_i > K_j$. Since the right hand side of this inequality increases in K_i for $K_i > K_j > \alpha - \bar{K}$, this is ensured if it holds for the firm with the highest capacity K_1, meaning (7) holds. Now let us check under which condition underbidding the second lowest bid by firm j is more profitable for firm j. Suppose $b_i < b_j = p_j$ is the second highest bid. Firm j is only tempted to undercut if

$$b_i K_j > p_j D^r(p_j, K_{-j}) \Leftrightarrow b_i > \frac{(\alpha - \bar{K} + K_j)^2}{4\beta K_j}.$$

Thus, if (8) holds firm j does not have an incentive to undercut and the equilibria characterised in proposition 2 do exist for $K_j > \alpha - \bar{K}$.

A.2 Characterization of the Nash Equilibria for More Capacity Constrained Firms

Assume again that $Q \neq \emptyset$, but consider now the case with $K_j \leq \alpha - \bar{K}$ for all $j \in Q$. Then for all the potentially price setting firms $j \in Q$ the monopoly price on their specific residual demand is smaller than the price that balances total capacity with total demand, meaning $p_j \leq \bar{p}$. Suppose now that firm $i \in N$ bids the highest price $b_i \geq b_j$ for all $j \in N$ with $j \neq i$ for its capacity K_i. As long as $b_i \leq \bar{p}$ the equilibrium price would be $\bar{p}$ because the auctioneer needs to increase the price from b_i to $\bar{p}$ in order to balance supply and demand and firm i would sell all its capacity K_i at the price $p = \bar{p}$. None of the other firms $j \neq i$ would like to overbid firm i. For all $\bar{p} \geq b_j > b_i$ the auction price would not change and any firm j would still sell its total capacity at $p = \bar{p}$, whereas firm j would lose profits as soon as it sets $b_j > \bar{p} \geq b_i$ because then firm j would generate its monopoly profit on its residual demand which is due to $\bar{p} > p_j$ monotonously decreasing for all $p = b_j > \bar{p} > p_j$. In addition firm i with $b_i \leq \bar{p}$ has neither an incentive to underbid any of its competitors nor to increase its bid $b_i > \bar{p}$. In the first case nothing would change for firm i. It would still sell its total capacity at a price of $p = \bar{p}$. In the second case firm i would also decrease its profits because of the same argument made before for an overbidding firm j. Thus, there are infinitely many equilibria in pure strategies where all the firms $j \in N$ bid $b_j \leq \bar{p}$.

B Proof of Proposition 4

Assume again that $Q \neq \emptyset$ holds and that $K_j < \alpha - \bar{K}$ holds for some $j \in Q$. Assume in addition that the bid cap can potentially have an influence on the market price because $\hat{b} > \frac{\alpha - \bar{K}}{\beta} = \bar{p}$. Those firms j for which the monopoly price on their residual demand exceeds the bid cap $p_j > \hat{b} \Leftrightarrow K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ can at most bid $\hat{b}$ and they are willing to do so as long as

$$\hat{b} D^r(\hat{b}, K_{-j}) \geq b_i K_j \text{ for all } i \in N \setminus j.$$

This implies that all the other firms $i \in N \setminus j$ bid such that $b_i \leq \underline{\hat{b}}$ where $\underline{\hat{b}}$ is defined in equation (9) in proposition 4 and $\underline{b}$ in equation (8) in proposition 2.

Note that overbidding is not an option for all firms $i \in N \setminus j$ and matching the bid $b_j = \hat{b}$ would clearly reduce their profits. Thus, for all firms $j \in Q$ with $K_j > 2b\hat{b} - (a - \bar{K})$ bidding $\hat{b}$ and all other firms $i \in N \setminus j$ bidding $b_i \leq \underline{\hat{b}}$ are Nash equilibria. Now assume that the firm $j \in Q$ with $K_j \leq 2\beta\hat{b} - (\alpha - \bar{K})$ sets its monopoly price p_j on the residual demand. As long as there are no other firms $i \in N \setminus j$ for which the bid cap could potentially bind, meaning $K_i \leq 2\beta\hat{b} - (\alpha - \bar{K})$ for all $i \in N \setminus j$ the bid cap has no impact and proposition 2 still applies. However if there are other firms i with $K_i > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ then firm j bidding p_j and all other firms $i \in N \setminus j$ bidding $b_i < p_j$ can only be part of a Nash equilibrium if none of the firms with $K_i > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ has an incentive to overbid with $b_i = \hat{b}$ meaning

$$\hat{b} D^r(\hat{b}, K_{-i}) \leq p_j K_i \Leftrightarrow K_j \geq \frac{2\beta\hat{b}(\alpha - \beta\hat{b} + K_i - \bar{K}) - K_i(\alpha - \bar{K})}{K_i}.$$

When taking into account that the condition above needs to hold for all firms with $K_i > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ and that the right-hand side of the condition increases in K_i it becomes obvious that it must be satisfied for the largest potentially restrained bidder $K_i = K_1 > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ for the Nash equilibrium with $b_j = p_j$ and $b_i \leq \underline{b}$ to exist for $K_i > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ which implies (10), given in proposition 4.

C Proof of proposition 5

If $\hat{K} < 2\beta\hat{b} + \bar{K} - \alpha$ then the bid constraint $\hat{b}$ does not constrain a firm with capacity $K_i = \hat{K}$ because its monopoly price on the residual demand p_i does not exceed $\hat{b}$ but potentially some of the firms $j \in Q$ with $K_j > \hat{K}$. On the other hand if all the firms $j \in Q$ have a capacity $K_j > \hat{K}$ all of them are potentially constrained. In both cases the selective bid cap has the same effect as if it would not have been tied to the condition that $K_j \geq \hat{K}$ and proposition 4 would still apply.

If there are, however, firms $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ and at least $K_1 > \hat{K}$ then these firms j are not constrained by the bid cap, although they would be if $\hat{b}$ would not have been applied only selectively. These firms can still bid $b_j = p_j$ whereas at least firm 1 is constrained and can only bid $b_1 \leq \hat{b} < p_1$.

Of course all the firms $j \in Q$ with $K_j > \hat{K}$ can still be price setting in equilibrium if

$$\hat{b}D^r(\hat{b}, \bar{K}_{-j}) \geq b_i K_j \text{ for all } i \in N \setminus j \Leftrightarrow b_i \leq \underline{\hat{b}}_j$$

with $\underline{\hat{b}}_j$ defined in (9) and none of the firms $i \in Q$ with $K_i \in (2\beta\hat{b} + \bar{K} - \alpha, \hat{K})$ wants to overbid firm j by bidding $b_i = p_i > \hat{b}$.[19] The latter holds true if

$$\begin{aligned} \hat{b}K_i \geq p_i D^r(p_i, K_{-i}) \quad \Leftrightarrow \quad & K_i \leq 2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \bar{K} - \alpha)} \\ & \text{for all } i \text{ with } K_i \in (2\beta\hat{b} + \bar{K} - \alpha, \hat{K}). \end{aligned} \tag{12}$$

Thus if (12) holds, the equilibria where any of the firms j with $K_j > \hat{K}$ is bidding $\hat{b}$ and all the other firms $i \in N \setminus j$ bid $b_i \leq \underline{\hat{b}}_j$ still exists. Note that with $\hat{K} \leq 2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \bar{K} - \alpha)}$ condition (12) is necessarily satisfied.

In addition the unconstrained firms j with $K_j \in (2\beta\hat{b} - (\alpha - \bar{K}), \hat{K})$ could bid $b_j = p_j > \hat{b}$ and be price setting. This could only be an equilibrium if firm j could not generate more profits by undercutting another firm i's bid b_i, meaning all other firms need to bid $b_i \leq \underline{b}_j$ which is defined in (8). At the same point in time no other firm i with $K_i \in (K_j, \hat{K})$, should be

[19] Note that the firms $i \in Q$ with $K_i \notin (2\beta\hat{b} + \bar{K} - \alpha, \hat{K})$ do not want to overbid $b_j = \hat{b}$ because $p_i < \hat{b}$.

tempted to overbid firm j with $b_i = p_i$.[20] Thus, $b_j = p_j$ for firm $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ and $b_i \leq \underline{b}_j$ for all $i \in N \setminus j$ can only be an equilibrium if

$$p_j K_i \geq p_i D^r(p_i, K_{-i}) \Leftrightarrow K_j \geq \frac{K_i^2 + (\alpha - \bar{K})^2}{2K_i} \text{ for all } K_i \in (K_j, \hat{K}). \quad (13)$$

Note that the latter inequality is increasing on the right-hand side in K_i and is necessarily satisfied if $K_j > \frac{\hat{K}^2 + (\alpha - \bar{K})^2}{2\hat{K}}$.

Also firms $j \in Q$ with $\hat{K} > 2\beta\hat{b} - (\alpha - \bar{K}) \geq K_j$ can potentially be price setting and bid $b_j = p_j \leq \hat{b}$ in equilibrium. In this case the other firms $i \in N \setminus j$ need to bid again $b_i \leq \underline{b}_j < p_j$. However, now all firms $i \in Q$ with $K_1 \geq K_i > K_j$ could potentially overbid firm j with either $b_i = \hat{b}$ if $K_i \in [\hat{K}, K_1]$ or with $b_i = p_i$ if $K_i \in (K_j, \hat{K})$. In an equilibrium they should not have an incentive to do so. The latter implies not only that(13) needs to hold but also

$$p_j K_i \geq \hat{b} D^r(\hat{b}, K_{-i})$$
$$\Leftrightarrow K_j \geq \frac{2\beta\hat{b}(\alpha - \beta\hat{b} + K_i - \bar{K}) - K_i(\alpha - \bar{K})}{K_i} \text{ for all } K_i \in [\hat{K}, K_1].$$

This restriction is again increasing in K_i and is therefore equivalent with $K_j \in \hat{P}$ as defined in (10).

References

Ausubel, L. M., P. Cramton, M. Pycia, M. Rostek and M. Weretka (2014), ‘Demand Reductions and Inefficiency in Multi-unit Auctions’, forthcoming in *The Review of Economic Studies*.

Bergman, L., G. Brunekreeft, C. Doyle, N.-H. M. von der Fehr, D. M. Newbery, M. Pollitt and P. Règibeau (eds.) (1999), *A European Market for Electricity?*, London and Stockholm: CEPR and SNS.

Betz, R., S. Seifert, P. Cramton and S. Kerr (2010), ‘Auctioning Greenhouse Gas Emission Permits in Australia’. *The Australian Journal of Agricultural and Resource Economics*, 54, pp. 219–238.

[20]Note that the firms i with $K_i \in [\hat{K}, K_1]$ cannot overbid firm j's bid $b_j = p_j$ and, as before no firm with $K_i < K_j$ will ever be tempted to overbid.

Brenner, M., D. Galai and O. Sade (2009), 'Sovereign Debt Auctions: Uniform or Discriminatory'. *Journal of Monetary Economics*, 56, pp. 267–274.

Bresky, M. (2013), 'Revenue and Efficiency in Multi-unit Uniform-price Auctions'. *Games and Economic Behavior*, 82, pp. 205–217.

Crampes, C. and N. Fabra (2005), 'The Spanish Electricity Industry: Plus ça change…'. *The Energy Journal*, 26(European Energy Liberalisation Special Issue), pp. 127–153.

Cramton, P. and A. Ockenfels (2012), 'Economics and Design of Capacity Markets for the Power Sector'. *Zeitschrift für Energiewirtschaft*, 36, pp. 113–134.

Degeorge, F., F. Derrien and K. L. Womack (2010), 'Auctioned IPOs: The US Evidence'. *Journal of Financial Economics*, 98, pp. 177–194.

Department of Energy and Climate Change (2013), 'Electricity Market Reform: Capacity Market - Detailed Design Proposals'. Presented to the Parliament by the Secretary of State for Energy and Climate Change.

Fabra, N., N.-H. von der Fehr and D. Harbord (2006), 'Designing Electricity Auctions'. *The Rand Journal of Economics*, 37(Spring), pp. 23–46.

von der Fehr, N.-H. and D. C. Harbord (1993), 'Spot Market Competition in the UK Electricity Industry'. *The Economic Journal*, 103, pp. 531–546.

Finon, D. and V. Pignon (2008), 'Electricity and Long-term Capacity Adequacy: The Quest for Regulatory Mechanism Compatible with Electricity Market'. *Utilities Policy*, 16, pp. 143–158.

Hobbs, B. F., M.-C. Hu, J. G. Iñón and S. E. Stoft (2007), 'Dynamic Analysis of a Demand Curve-based Capacity Market Proposal: The PJM Reliability Pricing Model'. *IEEE Transactions on Power System*, 22(1), pp. 3–14.

Hortaçsu, A. and S. L. Puller (2008), 'Understanding Strategic Bidding in Multi-unit Auctions: A Case Study of the Texas Electricity Spot Market'. *The Rand Journal of Economics*, 39, pp. 86–114.

Joskow, P. L. (2008), 'Capacity Payments in Imperfect Electricity Markets: Need and Design'. *Utilities Policy*, 16, pp. 159–170.

Keloharyu, M., K. G. Nyborg and K. Rydqvist (2005), 'Strategic Behavior and Underpricing in Uniform Price Auctions: Evidence from Finnish Treasury Auctions'. *The Journal of Finance*, 60, pp. 1865–1902.

Kremer, I. and K. G. Nyborg (2004), 'Underpricing and Market Power in Uniform Price Auctions'. *Review of Financial Studies*, 17, pp. 849–877.

Léautier, T.-O. (2011), 'The Visible Hand: Ensuring Optimal Investment in Electric Power Generation'. IDEI Working Paper 605, Toulouse School of Economics, Toulouse.

Moreno, D. and L. Ubeda (2006), 'Capacity Precommitment and Price Competition Yield the Cournot Outcome'. *Games and Economic Behavior*, 56, pp. 323–332.

Newbery, D. (2005), 'Electricity Liberalisation in Britain: The Quest for a Satisfactory Wholesale Market.' *The Energy Journal*, 26(European Energy Liberalisation Special Issue), pp. 43–70.

Schwenen, S. (2012), 'Strategic Bidding in Multi-unit Auctions with Capacity Constrained Bidders: The New York Capacity Market'. EUI Working Paper RSCAS 2012/62, European University Institute, Robert Schuman Centre for Advanced Studies, Florence.

Ubeda, L. (2004), 'Capacity and Market Design: Discriminatory versus Uniform Auctions', mimeo, Universidad Carlos III de Madrid and Purdue University.

Zhang, N. (2009), 'Generators' Bidding Behavior in the NYISO Day-ahead Wholesale Electricity Market'. *Energy Economics*, 31, pp. 897–913.