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Procurement Auctions**

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Abstract

The paper analyzes the excess entry hypothesis for sealed-bid first-price public procurement auctions. The hypothesis is proved analytically for any feasible combination of bid preparation cost and bid evaluation cost when the bidders face a rectangular cost density function and confirmed in numerical simulations based on a family of flexible cost density functions. The excess entry hypothesis implies that the procurer may reduce both his own cost and the social cost by imposing a positive fee on the bids.

Sequential search is a superior strategy to a public procurement auction whether or not the procurer imposes an optimal fee on the bids.

Keywords: Excess entry, Public procurement auctions, Optimal fee, Sequential search

JEL codes: D21, D43, D44, L13, L51,

1 Introduction

According to the seminal papers by Mankiw and Whinston 1986 [15] and Suzumura and Kiyono 1987 [20], entry costs may cause the market equilibrium number of firms to exceed the socially efficient number of firms in oligopolistic markets for homogeneous final goods.¹ Following Suzumura and Kiyono the result is dubbed *the (second-best) excess entry theorem*.²

In Mankiw and Whinston's setting the sufficient conditions for this to happen are: (a) firms face identical cost functions and act symmetrically; (b) total output increases in the number of firms; (c) the output per firm declines as the number of firms increases; (d) the postentry (equilibrium) price exceeds marginal costs.³

¹For recent surveys, see Suzumura 2012 [21] and Fujita 2016 [6]

²Second-best because the social planner is assumed to take oligopolistic pricing as granted rather than enforcing the marginal-cost principle.

³In effect, conditions (b) to (d) say that the demand function is declining and that the individual firm's producer's surplus is insufficient to cover its total costs (inclusive fixed costs)

The hypothesis analyzed in this paper is that the excess entry theorem also holds in the case of ordinary sealed-bid first-price public procurement auctions under similarly, in this case seemingly innocuous, conditions: (a') firms face the same *ex ante* cost density function and act symmetrically; (b') the expected winning bid decreases in the number of bidders; (c') the bidders' expected profit declines as the number of bidders increases; (d') the winning bid exceeds the winning bidder's cost. The analytical problem is that in the case of a public procurement auction there are no simple analogies to the market demand function and the producers cost function in the ordinary goods market.

The excess entry theorem is of little practical relevance in ordinary goods markets because, in general, it is not feasible to reduce the number of firms to the socially efficient number without raising the price of the good and harming consumers. On the contrary, in public procurement auctions the procurer may appropriate the efficiency gain and reduce both private and social costs by imposing a fee on the bids. The analysis indicates that the potential saving is significant.

An ordinary public procurement auction is just one way of organizing procurement. An alternative is sequential search. In sequential search (as defined in this paper), the procurer is supposed to peg a price and ask a randomly selected potential bidder if he will accept the offer. If the supplier addressed declines, the procurer continues until he finds a supplier that accepts. The price offered must be sufficiently high to induce the supplier to pay bid preparation costs and learn his production costs. As shown below, sequential search is a superior search strategy to a public procurement auction. This finding is consistent with the empirical fact that private sector procurers, who are free to choose the mode of contract they find most suitable, make little use of public procurement auctions (literature referred to below).

The present analysis is deliberately made as simple as possible. It falls within the independent private value paradigm and is restricted to the case of perfect symmetry. Potential bidders are assumed not to know their production cost before they have paid a common (sunk) bid preparation cost and entry fee, if any. However, they (and the procurer) know the (common, restricted) cost density function. These assumptions imply that each bidder's price is a monotonically increasing function of his production cost; that any potential bidder that has decided to pay the bid preparation cost and the entry fee, if any, also submits a bid even if his realized cost and price may be high and his probability of winning be correspondingly low; and that the bidder that realizes the lowest production cost also offers the lowest price, wins the auction and (in market equilibrium) earns a profit equal to the bid preparation cost paid by his competitors.

if the price equals marginal costs and, consequently, that the second-best (market equilibrium) solution is inferior to the (infeasible) first-best solution. In technical terms, the cost function satisfies (i) $C'(z) > 0$ for all $z > 0$, (ii) either $C(0) > 0$ or $\lim_{z \rightarrow 0^+} C''(z) := C''(0) < 0$ and (iii) $C''(z_i) > \mu \cdot p'(z_i + \sum_{j \neq i} z_j)$, where z_i is the i -th firm's output, p the market price, and μ the coefficient of conjectural variations, $\mu = \partial Q / \partial z_i$ ($i = 1, 2, \dots, n$) (Suzumura and Kiyono 1987) [20]

For simplicity, I assume that the procurer determines the number of bidders that are invited to submit bids and that this number (≥ 2) is made known to the bidders. The number of invited bidders is determined as the number of bidders consistent with the condition that the bidders' *ex ante* expected profit is non-negative. These assumptions ensure that the invited bidders will actually bid, and that no bid exceeds the cost density function's upper support. The outcome of the game is defined in terms of (i) the market equilibrium number of bidders and the associated expected private and social costs, (ii) the efficient number of bidders and the associated expected private and social costs, and (iii) the optimum (cost-minimizing) fee.

As to public procurement auctions the paper relates to Riley and Samuelson 1981 [17] (as to the derivation of the bid function) and to Samuelson 1985 [18] (as to the impact of bid preparation costs on the efficient number of bidders). A crucial difference is that Samuelson assumes that the bidders know their costs *before* incurring bid preparation costs and, consequently, only pay bid preparation costs and submit a bid if their production cost is below a certain value (increasing in the entry cost and number of competitors), which makes the supplier indifferent between accepting the invitation or abstaining. The analysis of the relation between entry costs and the optimal entry fee has a parallel in French and McComick 1984 [5], although their model differs significantly from this one.

McAfee and McMillan 1988 [14] analyze the *sequential direct search mechanism* introduced by Stigler in his seminal paper on the economics of information [19]. In their paper, the imperfection is solely due to the procurer's cost of searching for potential suppliers: The procurer continues searching until the search cost exceeds the expected gain from addressing one more potential supplier. Providers' bid preparation cost is implicitly assumed of no significance. In my setting, the most important factor is the providers' (bid preparation) cost; consequently, the price offered must be sufficiently high to make it worthwhile for potential providers to consider the offer.

In section 2, I develop the general setup. However, the general model does not allow me to derive conclusions regarding the efficiency of public procurement auctions. The pivotal element is the cost density function. In section 3, I solve the model analytically under the mathematically simplifying assumption that the cost density function is rectangular. In section 4, I solve the model numerically for a family of trigonometric (multi-humped) density functions. I select this family of density functions because the one-humped density function looks very much like the normal density function (and satisfies the essential condition of being bounded by a lower and an upper support), the two-humped density functions may be considered an approximation to the case in which the bidders (randomly) fall in two groups (low and high costs), and the many-humped density function approximates the discontinuous version of the rectangular density function. Numerical solutions based on other mathematical specifications of the density function (not reported here) do not qualitatively affect the results. Section 5 concludes the paper.

2 The general setup

2.1 Strategic games

In the following, I consider and compare two procurement strategies: (i) public procurement auction and (ii) sequential search.

The time line in the public procurement auction case is as follows: At time t_0 the procurer invites n potential bidders to request a description of the project in question⁴. The access to the tender documents may depend on their paying a fee, z . The potential bidders are informed of the number of rivals. The number of invited potential bidders, n , and the fee, z , if any, are determined by the procurer on basis on his *ex ante* information of the potential bidders' production cost density function, $f(c)$, their (common) bid preparation cost, u , and his own bid evaluation cost, v , so as to minimize his expected total net costs. At time t_1 , each of the invited bidders 'draws' his production cost, c , and determines the bid $b(c)$ that maximizes his expected profit. At time t_2 , the procurer 'opens the envelopes', evaluates the bids and awards the contract to the bidder, who has submitted the lowest bid.

In the sequential search case, the procurer at time t_0 , pegs a price, \hat{b} , and asks a potential supplier selected at random if he is willing to deliver the project at that price. The offered price must be sufficiently high to make it worthwhile for the supplier to pay the bid preparation cost and learn his production cost given the probability that his realized cost may be higher than the offered price. At time t_1 , the supplier learns his production cost, accepts the offer if his cost is below the offered price, and declines if his cost exceeds the offered price. In the latter case, the procurer selects another potential supplier at random and asks the same question. The game continues until a supplier accepts the offer.

Note that I focus on determining the optimal procurement strategy and, consequently, on the privately and socially best *ex ante* expected outcome, which due to the rationality and information assumptions made is identical to the optimal equilibrium outcome.

2.2 Derivation of the bid function

Each bidder pegs his bid, b , to maximize his expected profit, $E(\Pi)$, given his production cost, c , his bid preparation cost (common to all bidders), u , and the probability that his $n - 1$ competitors have higher realized costs, $G(c, n)$,

⁴As in most of the literature, it is (unrealistically) assumed that the number of bidders is continuous. Alternatively, we might assume that the procurer pegs the number of bidders to the integer just below the estimated market equilibrium number of bidders. A more satisfactory solution to the integer problem is to assume that the buyer issues an open invitation to suppliers to gauge their interest, announces the number of potential (i.e. interested) bidders and assumes that the potential bidders adopt a common mixed strategy regarding whether to participate in the auction as in, e.g., Li and Zheng 2009 [11]. However, this solution is less efficient, as the expected private and social costs are increasing in the number of potential bidders. See Levin and Smith 1994 [10]

$$E(\Pi) = (b - c) \cdot G(c, n) - u \quad (1)$$

$$G(c, n) \equiv \Pr_{i=[2, n]}(c_i > c) = (1 - F(c))^{n-1}; \quad G(\underline{c}, n) = 1; \quad G(\bar{c}, n) = 0$$

where \underline{c} is the lower support and \bar{c} is the upper support of the common cost density function $f(c)$.

The first-order profit maximization condition is

$$\frac{\partial E(\Pi)}{\partial b} = G(c, n) + (b - c) \cdot g(c, n) \cdot \frac{\partial c}{\partial b} = 0$$

or (as b is a monotonically increasing function of c)⁵

$$\frac{\partial b}{\partial c} \cdot G(c, n) + (b - c) \cdot g(c, n) = 0 \quad (2)$$

The bid function is derived as the solution to eq. (2)

$$\begin{aligned} \frac{\partial b}{\partial c} \cdot G(c, n) + b \cdot g(c, n) &= c \cdot g(c, n) \\ \frac{\partial (G(c, n) \cdot b)}{\partial c} &= \frac{\partial (G(c, n) \cdot c)}{\partial c} - G(c, n) \end{aligned}$$

$$G(c, n) \cdot b = G(c, n) \cdot c - \int G(c, n) dc$$

$$b(c, n) = c - \frac{1}{G(c, n)} \cdot \int G(c, n) dc \quad (3)$$

which, as $G(\bar{c}, n) = 0$, also may be written as

$$b(c, n) = c - \frac{1}{G(c, n)} \cdot \int_{\bar{c}}^c G(x, n) dx \quad (4)$$

2.3 The market equilibrium number of bidders

The *ex ante* expected lowest cost is

$$E(c_1(n)) = \frac{\int_{\underline{c}}^{\bar{c}} (c \cdot G(c, n) \cdot f(c)) dc}{\int_{\underline{c}}^{\bar{c}} G(c, n) \cdot f(c) dc} = n \cdot \int_{\underline{c}}^{\bar{c}} (c \cdot G(c, n) \cdot f(c)) dc \quad (5)$$

⁵This is a common assumption in the literature; see e.g. Krishna 2009 [9]. A formal proof is provided in McAfee and McMillan 1987b [13]. The proof is based on two steps: First, one bidder's response to a particular decision rule that he arbitrarily conjectures his rivals to be using. Second, the Nash requirement that the conjecture decision rule is consistent with optimizing behavior by the other bidders.

because, by symmetry, each of the n bidders is equally likely to realize the lowest cost, and consequently,

$$\int_{\underline{c}}^{\bar{c}} G(c, n) \cdot f(c) dc = \frac{1}{n}$$

The *ex ante* expected lowest bid is

$$\begin{aligned} E(b_1(n)) &= \frac{\int_{\underline{c}}^{\bar{c}} \left(c - \frac{1}{G(c, n)} \cdot \int_{\bar{c}}^c G(x, n) dx \right) G(c, n) \cdot f(c) dc}{\int_{\underline{c}}^{\bar{c}} G(c, n) \cdot f(c) dc} \\ &= E(c_1(n)) - n \cdot \int_{\underline{c}}^{\bar{c}} \left(\int_{\bar{c}}^c G(x, n) dx \right) \cdot f(c) dc \end{aligned} \quad (6)$$

and the *ex ante* expected profit for all n competitors

$$E(\Pi(n)) = E(b_1(n)) - E(c_1(n)) - n \cdot u \quad (7)$$

$$= n \cdot \int_{\underline{c}}^{\bar{c}} \left(\left(- \int_{\bar{c}}^c G(x, n) dx \right) \cdot f(c) \right) dc - n \cdot u \quad (8)$$

The market equilibrium number of bidders, $n_{eq}(u)$, is the solution to $E(\Pi(n, u)) = 0$.

2.4 Social costs and the efficient number of bidders

The expected total social cost is the expected lowest production cost plus the bidders' bid preparation cost, u , and the procurer's bid evaluation cost, v ,

$$E(C_S) = E(c_1(n)) + n \cdot (u + v) \quad (9)$$

The efficient number of bidders, n_S^* , is the solution to $E'(C_S(n)) = 0$.

2.5 Procurer's cost

The procurer's total expected cost is

$$E(C_B) = E(c_1(n)) + n \cdot v \quad (10)$$

The procurer's preferred number of bidders, n_B^* , is the solution to $E'(C_B(n)) = 0$. The preferred number of bidders may be large, in fact infinite, if the procurer has no bid evaluation costs. However, the procurer cannot coerce potential bidders to submit a bid if their *ex ante* expected profit is negative. Consequently, the highest *feasible* number of bidders is n_{eq} and the associated lowest feasible cost in the case of ordinary public procurement auctions is $E(C_B(n_{eq})) = E(C_S(n_{eq}))$.

2.6 Sequential search

An alternative to a public procurement auction is to peg a price, \hat{b} , select a potential supplier at random and ask whether he is willing to deliver at that price. The procurer's cost of finding and addressing a potential supplier is \hat{v} . The search cost \hat{v} may differ from the bid evaluation cost in public procurement auctions, v . However, for simplicity, in the following I shall assume that $\hat{v} = v$. If the supplier declines after having learned his production cost, the procurer moves to the next supplier and repeats the process, until a finds one that accepts the offer.

The procurer's expected cost is

$$E(\hat{C}) = \hat{b} + E(\hat{n}) \cdot \hat{v} \quad (11)$$

where $E(\hat{n})$ is the expected number of potential suppliers he must address before he finds one that realizes a cost below the price offered, $c \leq \hat{b}$. The probability $\Pr(c \leq \hat{b}) = F(\hat{b})$, and the expected number of trials is $E(\hat{n}_1) = \frac{1}{F(\hat{b})}$.

The unrestricted cost-minimizing offered price⁶ is the solution to

$$\begin{aligned} \frac{\partial E(\hat{C}(\hat{b}_u))}{\partial \hat{b}_u} &= 1 - \frac{f(\hat{b}_u)}{F(\hat{b}_u)^2} \cdot \hat{v} = 0 \\ \frac{F(\hat{b}_u)^2}{f(\hat{b}_u)} &= \hat{v} \end{aligned} \quad (12)$$

However, if \hat{b}_u is low, then the potential suppliers' expected profit may be negative. If so, they will abstain from considering the offer, and the procurer must offer $\hat{b}_r > \hat{b}_u$. The restricted offered price \hat{b}_r is the solution to $E(\hat{\Pi}(\hat{b})) = 0$,

$$\begin{aligned} E(\hat{\Pi}(\hat{b})) &= (\hat{b} - E(\hat{c})) \cdot F(\hat{b}) - u \\ &= \left(\hat{b} - \frac{\int_0^{\hat{b}} (c \cdot f(c)) dc}{\int_0^{\hat{b}} f(c) dc} \right) \cdot F(\hat{b}) - u \\ &= \hat{b} \cdot F(\hat{b}) - \int_0^{\hat{b}} (c \cdot f(c)) dc - u \end{aligned} \quad (13)$$

The expected private costs and expected social costs are identical

$$E(\hat{C}_{B_r}) = \hat{b}_r + n_{\hat{b}_r} \cdot \hat{v} = E(\hat{C}_{S_r}) = E(\hat{c}(\hat{b}_r)) + n_{\hat{b}_r} \cdot (u + \hat{v}) \quad (14)$$

as $E(\hat{\Pi}(\hat{b}_r)) = 0$

⁶The subscript u denotes unrestricted and the subscript restricted.

I want to compare the social outcomes of the three procurement strategies: (a) ordinary procurement auction, (b) procurement auction *cum* optimal fee, and (c) sequential search. However, to do so I need to specify the cost density function.

3 Rectangular cost density function

To compare the social outcomes of the three procurement strategies considered: (a) an ordinary procurement auction, (b) a procurement auction *cum* optimal fee, and (c) sequential search, I need to specify the cost density function.

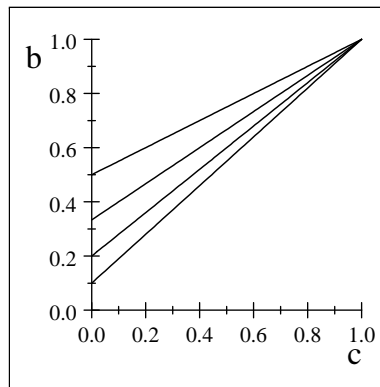
For simplicity and with no loss of generality I assume that $\underline{c} = 0$ and $\bar{c} = 1$ and, consequently, that the spread $s = \bar{c} - \underline{c}$ is normalized to 1. With this normalization I obtain

$$f(c) = 1; F(c) = c; G(c, n) = (1 - c)^{n-1}; \int G(x, n) dx = \frac{-1}{n} \cdot (1 - c)^n$$

3.1 Public procurement auctions

The bid function reduces to

$$b(c, n) = \frac{1}{n} + \frac{n-1}{n} \cdot c \tag{15}$$



$b(c); n = (2, 3, 5, 10)$

Figure 1: The bid function, rectangular cost density function

The expected lowest cost and lowest bid are, respectively,

$$\begin{aligned}
E(c_1(n)) &= n \cdot \int_0^1 (c \cdot (1-c)^{n-1} \cdot 1) dc \\
&= n \cdot \left(\frac{1}{n} - \frac{1}{n+1} \right) = \frac{1}{n+1} \\
E(b(c, n)) &= \frac{1}{n} + \frac{n-1}{n} \cdot E(c_1(n)) \\
&= \frac{1}{n} + \frac{n-1}{n} \cdot \frac{1}{n+1} = \frac{2}{n+1}
\end{aligned}$$

The zero-profit restriction

$$E(\Pi(n, u)) = E(b(c, n)) - E(c_1(c, n)) - n \cdot u = \frac{1}{n+1} - n \cdot u = 0$$

implies that the market equilibrium number of bidders is

$$n_{eq} = \sqrt{\frac{1}{4} + \frac{1}{u}} - \frac{1}{2} \quad (16)$$

The total expected social cost is

$$E(C_S(n)) = E(c_1) + n \cdot (u + v) = \frac{1}{n+1} + n \cdot (u + v) \quad (17)$$

The corresponding efficient (cost-minimizing) number of bidders and minimum social cost are

$$n_S^* = \sqrt{\frac{1}{u+v}} - 1 \quad (18)$$

$$E(C_S^*(n_S^*)) = 2 \cdot \sqrt{u+v} - (u+v) \quad (19)$$

Comparing equations 16 and 18, respectively 17 and 19, implies

Proposition 1 *If the cost density function, $f(c)$, is rectangular, then the excess entry hypothesis*

$$\begin{aligned}
n_{eq} &> n_S^* \\
E(C_S(n_{eq})) &> E(C_S^*(n_S^*))
\end{aligned}$$

holds true for any feasible⁷, non-negative values of the bid preparation cost, u , and the bid evaluation cost, v .

⁷The condition for the existence of a competitive market solution, $n_{eq} \geq 2$, is $u \leq \frac{1}{6}$. Hence, the model implies that the number of public procurement auctions is likely to be rather small. If the spread is narrow compared to the bidders' bid preparation cost, there may be no or only one bidder. The condition for an efficient market solution, $n_S^* \geq 2$, is stricter, $u + v \leq \frac{1}{9}$.

The excess entry hypothesis implies that it is profitable for the procurer to reduce the number of bidders by imposing a *positive* fee, z , on the bidders. The optimal fee, z^* , is derived from the zero expected profit condition:

$$\begin{aligned} E(\Pi(n_S^*, (u+z))) &= \frac{1}{n_S^* + 1} - n_S^* \cdot (u+z) = 0 \\ z^* &= \frac{1}{(n_S^* + 1) \cdot n_S^*} - u \\ &= \frac{v + u \cdot \sqrt{u+v}}{1 - \sqrt{u+v}} > 0 \end{aligned}$$

Proposition 2 *If $f(c)$ is rectangular, then the procurer may reduce both private and social costs by imposing an optimal fee, z^* , on the bidders. The optimal fee is positive for any feasible non-negative values of bid preparation cost, u , and bid evaluation cost, v , and is increasing in both u and v .*

Figure 2 illustrates the solution of the model for various combinations of bid preparation cost, u , and bid evaluation cost, v . The curves depict (from above) the procurer's expected cost, $E(C_B(n))$, the expected social cost, $E(C_S(n))$, and the bidders' expected profit, $E(\Pi(n))$, as functions of n . In all three cases, the *sum* of bid preparation cost and bid evaluation cost is identical, $u+v = 0.075$, and, consequently, so is the expected social cost.

The market equilibrium solution $E(C_B(n_{eq})) = E(C_S(n_{eq}))$ is marked by a dot, the efficient solution $E(C_S(n_S^*)) = E(C_B(n_S^*, z^*))$ is marked by a circle, and the procurer's preferred solution $E(C_B(n_B^*))$ (infeasible if $n_B^* > n_{eq}$) is marked by a box. The crosses indicate the expected number of searches and the resulting expected total cost if the procurer adopts a sequential search strategy. The derivation is given below.

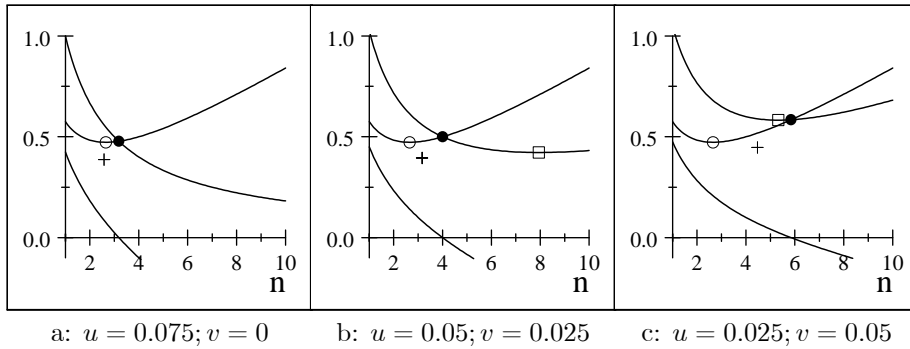


Figure 2: Solution of the model for various combinations of u and v .
Rectangular cost density function

Figure 2a illustrates the case in which the bidders' bid preparation costs are relatively high and the procurer pays no bid evaluation cost, $u = 0.075, v = 0$.

In this case the market equilibrium number of bidders is $n_{eq} = 3.19$ and only somewhat higher than the efficient number of bidders, $n_S^* = 2.65$. As $v = 0$, the procurer would like to see many more bidders, in fact $n_B^* \rightarrow \infty$. It is natural to conjecture that the procurer in this case might benefit from subsidizing the bidders, thereby attracting more bidders, intensifying the competition among the bidders and reducing his cost. This conjecture is erroneous. On the contrary, the optimal strategy is to impose a fee, $z^* = 0.028$, on the bidders. The fee will, of course, reduce the number of bidders and raise the lowest expected bid, however by less than the fees collected (the move from the solution indicated by a dot to the solution indicated by a circle). Figure 2b illustrates the standard case, i.e., the bid evaluation costs are positive but significantly smaller than the bid preparation costs, $u = 0.05$, $v = 0.025$. The overall picture is very much the same as in figure 2a. The market equilibrium number of bidders is higher, as is the potential cost reduction by imposing a (higher) optimum fee, $z^* = 0.053$. Figure 2c illustrates the unlikely case, $v = 2 \cdot u = 0.05$. In this case, $z^* = 0.078$ is quite high, as is the gain, $E(C_S(n_{eq})) - E(C_S(n_S^*)) = 0.11$. Notice that in this case the number of bidders that minimizes the procurer's expected costs is *less* than the market equilibrium number of bidders, $n_B^* < n_{eq}$, and consequently, the procurer may reduce his cost (marginally) by *directly* limiting the number of bidders. However, it is more profitable to *indirectly* reduce the number of bidders by imposing a (high) fee on the bids.

3.2 Sequential search

In sequential search, the procurer pegs a price, \hat{b} , and asks a randomly selected potential supplier whether he is willing to deliver at that price. If not, the procurer moves on to the next potential supplier and repeats the process, until his offer is accepted.

The procurer's expected cost is

$$E(\hat{C}_B) = \hat{b} + E(\hat{n}) \cdot \hat{v} = \hat{b} + \frac{1}{F(\hat{b})} \cdot \hat{v} = \hat{b} + \frac{1}{\hat{b}} \cdot \hat{v}$$

The unrestricted optimal offered price, \hat{b}_u , is the solution to

$$\frac{\partial E(\hat{C}_B(\hat{b}))}{\partial \hat{b}} = \frac{\partial (\hat{b} + \frac{\hat{v}}{\hat{b}})}{\partial \hat{b}} = 0$$

from which

$$\hat{b}_u = \sqrt{\hat{v}} \text{ and } E(\hat{n}_u) = \frac{1}{F(\hat{b})} = \frac{1}{\hat{b}_u} = \frac{1}{\sqrt{\hat{v}}}$$

Provided a potential supplier realizes production cost $\hat{c}_1 \leq \hat{b}$, then his expected production cost is

$$E(\hat{c}_1) = \frac{\int_0^{\hat{b}} c dc}{\int_0^{\hat{b}} dc} = \frac{1}{2} \cdot \hat{b}_u = \frac{\sqrt{\hat{v}}}{2}$$

The expected costs to the procurer and society and the suppliers' expected profit are, respectively,

$$\begin{aligned}
E\left(\hat{C}_B\left(\hat{b}_u\right)\right) &= \hat{b}_u + E\left(\hat{n}_u\right) \cdot \hat{v} = \sqrt{\hat{v}} + \frac{\hat{v}}{\sqrt{\hat{v}}} \\
E\left(\hat{C}_S\left(\hat{b}_u\right)\right) &= E\left(\hat{c}_1\right) + E\left(\hat{n}_u\right) \cdot (u + \hat{v}) = \frac{\sqrt{\hat{v}}}{2} + \frac{u + \hat{v}}{\sqrt{\hat{v}}} \\
E\left(\hat{\Pi}\left(\hat{b}_u\right)\right) &= \left(\hat{b}_u - E\left(\hat{c}_1\right)\right) - E\left(\hat{n}_u\right) \cdot u = \frac{\sqrt{\hat{v}}}{2} - \frac{u}{\sqrt{\hat{v}}} > 0 \text{ iff } \hat{v} \geq 2 \cdot u
\end{aligned}$$

The non-negative profit restriction implies that b_u is only feasible in the unlikely case that the procurer's cost of finding and addressing a potential supplier, \hat{v} , is more than twice the supplier's cost of calculating his production cost, u .

In the 'normal' case, $\hat{v} \leq 2 \cdot u$, the procurer must offer $b_r > b_u$ to attract potential suppliers.

The offered price, b_r , is the solution to

$$\begin{aligned}
E\left(\hat{\Pi}\right) &= \left(\hat{b} - E\left(\hat{c}_1\right)\right) - E\left(\hat{n}\right) \cdot u \\
&= \left(\hat{b} - \frac{1}{2} \cdot \hat{b}\right) - \frac{1}{\hat{b}} \cdot u = 0
\end{aligned}$$

from which

$$\begin{aligned}
\hat{b}_r &= \sqrt{2u} \\
E\left(\hat{n}_r\right) &= \frac{1}{\hat{b}_r} = \frac{1}{\sqrt{2u}}
\end{aligned}$$

3.3

$$E\left(\hat{C}_S\left(\hat{b}_r\right)\right) = E\left(\hat{C}_B\left(\hat{b}_r\right)\right) = \hat{b}_r + E\left(\hat{n}_r\right) \cdot \hat{v} = \sqrt{2u} + \frac{\hat{v}}{\sqrt{2u}} = \frac{2u + v}{\sqrt{2u}}$$

Comparing the three procurement strategies

The isocost functions $E\left(C_S(u, v)\right)$ related to the three procurement strategies are, respectively,

$$\begin{aligned}
E\left(C_S^*\right) &= \frac{1}{\left(\sqrt{\frac{1}{u+v}} - 1\right) + 1} + \left(\sqrt{\frac{1}{u+v}} - 1\right) \cdot (u + v) = C \\
E\left(C_S\left(n_{eq}\right)\right) &= \frac{1}{\left(\sqrt{\frac{1}{4} + \frac{1}{u} - \frac{1}{2}}\right) + 1} + \left(\sqrt{\frac{1}{4} + \frac{1}{u} - \frac{1}{2}}\right) \cdot (u + v) = C \\
E\left(\hat{C}_S\left(\hat{b}_u\right)\right) &= \frac{\sqrt{\hat{v}}}{2} + \frac{u + \hat{v}}{\sqrt{\hat{v}}} = C \wedge E\left(\hat{C}_B\left(\hat{b}_r\right)\right) = \sqrt{2u} + \frac{\hat{v}}{\sqrt{2u}} = C
\end{aligned}$$

where C is any feasible cost.

The isocost functions for $C = 0.5$ are shown in figure 3.⁸

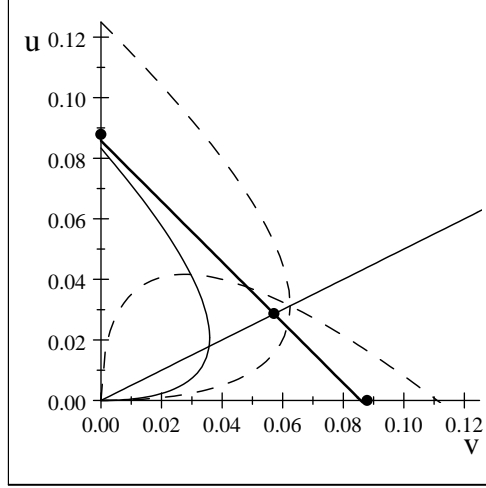


Figure 3: Isocost curves for different procurement strategies

The thick downward-sloping line depicts the combinations of u and v for which the socially efficient solution $E(C_S^*(u+v)) = E(C_S(n_{eq}(u+z^*), u+v)) = C$. The thin curved solid line depicts the combinations for which the market solution, $E(C_S(n_{eq}(u), u+v)) = C$, if $z = 0$. Any point on curved line is to the left of the thick line indicating that the market equilibrium is inferior to the socially efficient equilibrium, as lower values of u and v result in the same total social cost, C . The broken curves depict the combinations of u and v for which, respectively, $E(\hat{C}_S(\hat{b}_u(\hat{v}), u+v)) = C$ and $E(\hat{C}_B(\hat{b}_r(u), u+v)) = C$. If $u \leq \frac{\hat{v}}{2}$, the optimal search strategy is to offer \hat{b}_u , and if $u \geq \frac{\hat{v}}{2}$ the optimal

⁸The corresponding $u(v)$ -functions are

$$\begin{aligned}
 E(C_S^*) &: u = 0.085786 - v \\
 E(C_S(n_{eq})) &: u = \left\{ \begin{array}{l} \frac{1}{4(0.5)-8} \left(4v + (0.5) \left((0.5) + \sqrt{-8v + 2(0.5)v + (0.5)^2 + v^2} \right) - (0.5)v - 2(0.5)^2 \right), \\ -\frac{1}{4(0.5)-8} \left(-4v - (0.5) \left((0.5) - \sqrt{-8v + 2(0.5)v + (0.5)^2 + v^2} \right) + (0.5)v + 2(0.5)^2 \right) \end{array} \right\} \\
 E(\hat{C}_S(\hat{b}_u)) &: u = (0.5)\sqrt{v} - \frac{3}{2}v \\
 E(\hat{C}_B(\hat{b}_r)) &: u = \left\{ \begin{array}{l} -\frac{1}{2}v - \frac{1}{4}(0.5)\sqrt{-4v + (0.5)^2 + \frac{1}{4}(0.5)^2}, \\ -\frac{1}{2}v + \frac{1}{4}(0.5)\sqrt{-4v + (0.5)^2 + \frac{1}{4}(0.5)^2} \end{array} \right\}
 \end{aligned}$$

strategy is to offer \hat{b}_r . In both cases, any combination of u and v is to the right of the $E(C_S^*)$ line. This fact implies that sequential search as search strategy is superior to a socially efficient public procurement auction (an auction where the procurer imposes an optimal fee), as total costs, C , are the same even though bid preparation cost, u , and bid evaluation cost, v , are higher.

Proposition 3 *If $f(c)$ is rectangular, then sequential search is a superior strategy to a public procurement auction for any feasible non-negative combination of bid preparation cost, u , and bid evaluation cost, v .*

Qualitatively, the broken isocost curves illustrating the sequential search strategy must have the same shapes regardless of the cost density function. At the origin, $(u = 0, v = 0)$, searching costs nothing and $\hat{n} \rightarrow \infty$. A slight increase in \hat{v} reduces the optimal number of searches leaving a relatively considerable 'room within the budget' for an increase in u . If \hat{v} is already relatively high, a further increase will squeeze the budget left for u . At some value of \hat{v} , u is reduced to zero. That is, $E(\hat{C}_S(\hat{b}_u(\hat{v}), u + v)) = C$ must be shaped as an inverted U along the v -axis. Similar reasoning implies that $E(\hat{C}_B(\hat{b}_r(u), u + v)) = C$ must have the shape of an inverted U along the u -axis. The two curves must intersect at some point along the line defined by $E(\Pi(\hat{b}_r(u), u + v)) = 0$.

These general properties allow us to conclude that if $E(\hat{C}) < E(C_S^*)$ for all three combinations of u and v at the thick, solid $E(C_S^*(u + v)) = C$ isocost curve indicated by dots, then sequential search is a superior strategy to public procurement auction for *any* feasible combination of u and v . That conclusion holds for *any* feasible cost density function, $f(c)$.⁹

4 A family of multi-humped density functions

4.1 Public procurement auctions

The cost density function is defined as

$$f(c) = 1 - \cos 2\theta\pi c \quad (20)$$

where $\theta \geq 0$ is an *integer* defining the number of humps. The rectangular distribution is a special case, $\theta = 0$.

⁹ $f(c) > 0; F(c = 0); F(\bar{c}) = 1$

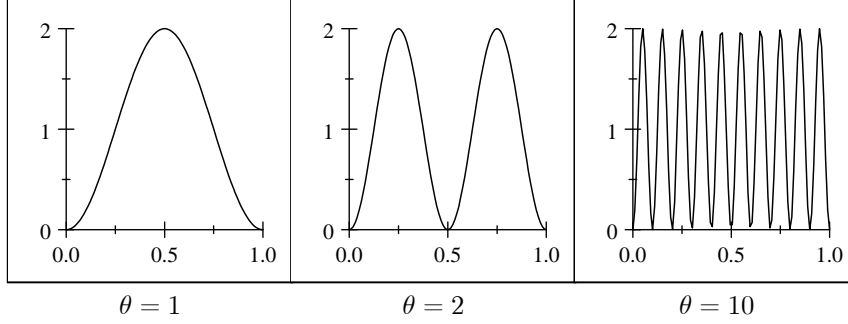


Figure 4. Density functions, $\theta = 1, 2, 10$.

The corresponding distribution function, probability function, and bid function are

$$F(c) = c - \frac{\sin 2\theta\pi c}{2\theta\pi} \quad (21)$$

$$G(c, n) = \left(1 - c + \frac{\sin 2\theta\pi c}{2\theta\pi}\right)^{n-1} \quad (22)$$

$$b(c, n) = c - \frac{\int_1^c \left(1 - x + \frac{\sin 2\theta\pi x}{2\theta\pi}\right)^{n-1} dx}{\left(1 - c + \frac{\sin 2\theta\pi c}{2\theta\pi}\right)^{n-1}} \quad (23)$$

The bid functions are depicted in figure 5 below. As $\theta \rightarrow \infty$, the distribution function and bid functions approximate the corresponding functions in the case in which the density function is rectangular.

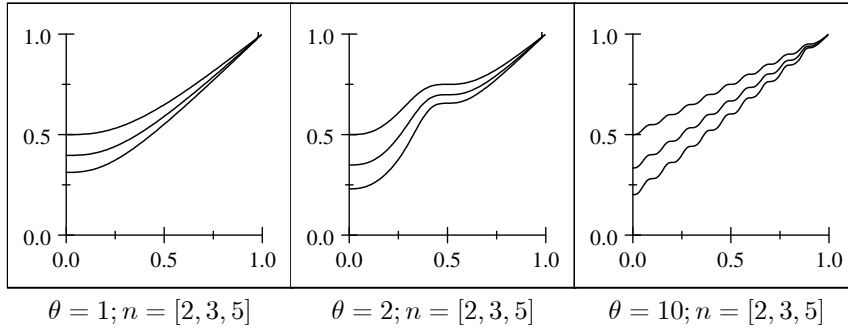


Figure 5 The bid function, trigonometric density function

The bidders' total profit and social cost are, respectively,

$$\begin{aligned} E(\Pi(n, u)) &= n \cdot \int_0^1 ((b(c) - c) \cdot G(n, c) \cdot f(c)) dc - n \cdot u \\ &= -n \cdot \int_0^1 \left(\left(\int_1^c \left(\left(1 - x + \frac{\sin 2\theta\pi x}{2\theta\pi} \right)^{n-1} \right) dx \right) \cdot (1 - \cos 2\theta\pi c) \right) dc - n \cdot u \end{aligned} \quad (24)$$

$$\begin{aligned} E(C_S(n, u + v)) &= n \cdot \int_0^1 (c \cdot G(n, c) \cdot f(c)) dc + n \cdot (u + v) \quad (\text{E(CS(tri))}) \\ &= n \cdot \int_0^1 \left(c \cdot \left(1 - c + \frac{\sin 2\theta\pi c}{2\theta\pi} \right)^{n-1} \cdot (1 - \cos 2\theta\pi c) \right) dc + n \cdot (u + v) \end{aligned}$$

As above, the market equilibrium number of bidders, $n_{eq}(u)$, is the solution to $E(\Pi(n, u)) = 0$. The socially efficient number of bidders, $n_S^*(u, v)$, is the solution to $\frac{\partial E(C_S(n, u + v))}{\partial n} = 0$.

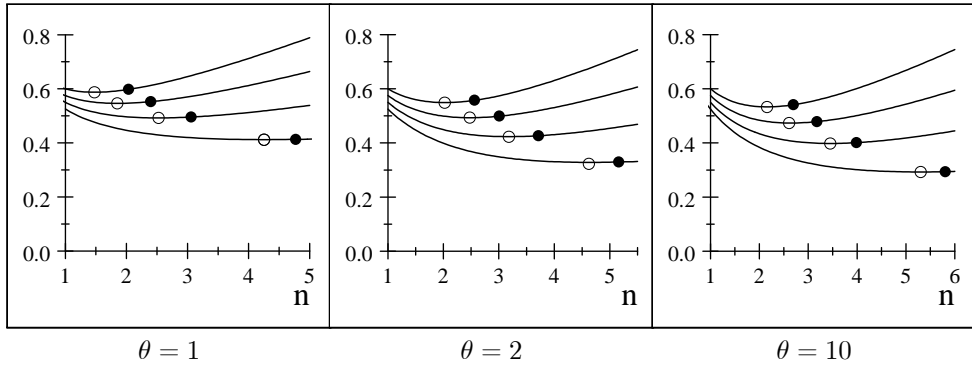


Figure 6. Expected social cost, efficient solution and market equilibrium

The curves in figure 6 depict the expected social cost, $E(C_S(n))$, for (from below) $u = [0.025, 0.05, 0.075, 0.1]$. The circles indicate the socially efficient solutions, and the dots indicate the market equilibrium solutions. In all three cases (and in cases not reported), the circles are to the left of and below the corresponding dots as the excess entry theorem predicts.

Proposition 4 *If the cost density function is trigonometric, then the excess entry theorem*

$$\begin{aligned} n_{eq}(\theta, u) &> n_S^*(\theta, u + v) \\ E(C_S(n_{eq}(\theta, u))) &> E(C_S(n_S^*(\theta, u + v))) \end{aligned}$$

holds and, consequently, it is possible to reduce private and social costs by imposing a positive fee, z , on the bidders,

$$\begin{aligned} z^*(\theta, u, v) &> 0 \\ E(C_B(n_{eq}(\theta, u + z^*))) &= E(C_S(n_{eq}(\theta, u + z^*))) = E(C_S(n_S^*(\theta, u + v))) \\ &< E(C_B(n_{eq}(\theta, u))) \end{aligned}$$

for any integer number of humps, θ , and any feasible non-negative combinations of bid preparation cost, u , and bid evaluation cost, v .

4.2 Sequential search

The expected number of searches, \hat{n} , and the procurer's expected cost are, respectively,

$$E(\hat{n}) = \frac{1}{F(\hat{b})} = \frac{1}{\hat{b} - \frac{\sin 2\theta\pi\hat{b}}{2\theta\pi}} \quad (25)$$

$$\begin{aligned} E(\hat{C}_B(\hat{b})) &= \hat{b} + E(\hat{n}) \cdot \hat{v} \\ &= \hat{b} + \frac{\hat{v}}{\hat{b} - \frac{\sin 2\theta\pi\hat{b}}{2\theta\pi}} \end{aligned} \quad (26)$$

The unrestricted cost-minimizing offered price, $\hat{b}_u(\hat{v})$, is the solution to $E'(\hat{C}_B(\hat{b})) = 0$.

However, the minimum feasible offered price must be sufficiently high to make a potential supplier's expected profit non-negative. The restricted offered price, $\hat{b}_r(u)$, is the solution to

$$\begin{aligned} E(\hat{\Pi}(\hat{b})) &= (\hat{b} - E(\hat{c}_1)) - \hat{n} \cdot u \\ &= \left(\hat{b} - \frac{\int_0^{\hat{b}} (c \cdot (1 - \cos 2\theta\pi c)) dc}{\hat{b} - \frac{\sin 2\theta\pi\hat{b}}{2\theta\pi}} \right) - \frac{u}{\hat{b} - \frac{\sin 2\theta\pi\hat{b}}{2\theta\pi}} = 0 \end{aligned}$$

In both cases, the expected social cost is

$$\begin{aligned} E(\hat{C}_S(\hat{b})) &= E(\hat{c}_1) + \hat{n} \cdot (u + \hat{v}) \\ &= \frac{\int_0^{\hat{b}} (c \cdot (1 - \cos 2\theta\pi c)) dc}{\hat{b} - \frac{\sin 2\theta\pi\hat{b}}{2\theta\pi}} + \frac{u + \hat{v}}{\hat{b} - \frac{\sin 2\theta\pi\hat{b}}{2\theta\pi}} \end{aligned}$$

Figure 7 serves to demonstrate that sequential search is superior to a public procurement auction.¹⁰

¹⁰The ten-humped case is not shown, as it is very similar to the rectangular cost density function.

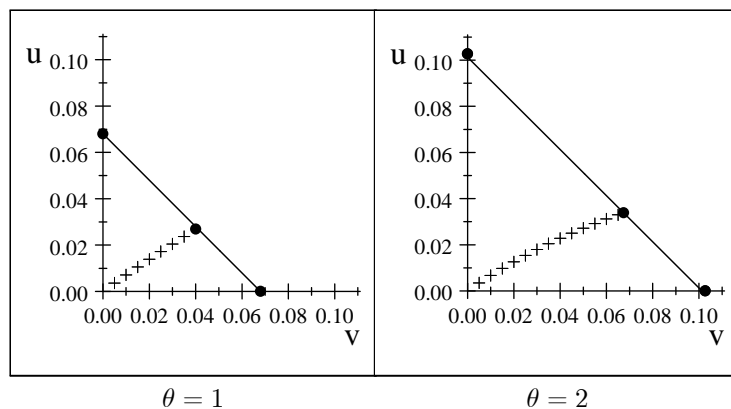


Figure 7: The social isocost function and combinations of \hat{v} and u for which the procurer's best strategy is $\hat{b}_u(\hat{v})$ or $\hat{b}_r(u)$.

The downward-sloping lines are isocost functions determined by the condition $E(C_S^*(\theta, u+v)) = C(\theta)$. The slope of the line is -1 . The $C(\theta)$ is the maximum social cost consistent with efficient competitive equilibrium, i.e. $n_S^*(\theta, u+v) = 2$. In the one-humped case, the maximum value of $u+v$ is 0.068, and in the two-humped case, the value is 0.10127. The upward-sloping sequences of crosses delimit the combinations of u and v where $\hat{b}_u(\hat{v})$ is the optimal search strategy (below) and $\hat{b}_r(u)$ is the optimal search strategy (above).

As discussed in section 3.3 above, we only need to compare $E(\hat{C}_S)$ and $E(C_S^*)$ at the three points on the linear $E(C_S^*)$ isocost curves depicted by dots. In all three cases $E(\hat{C}_S)$ is smaller than $E(C_S^*)$ ¹¹. Consequently, we may conclude that sequential search is always a superior strategy to a public procurement auction *cum* an optimal (positive) fee, z^* , and more so compared to a less efficient ordinary public procurement auction.

Proposition 5 *If the cost density function, $f(c)$, is trigonometric, then sequential search is a superior procurement strategy to a public procurement auction for any integer number of humps, θ , and feasible non-negative combinations of, u and $\hat{v} = v$.*

¹¹The numerical results are

	$\theta = 1$			$\theta = 2$		
v	0.04070	0	0.06800	0.06740	0	0.10127
u	0.02729	0.06800	0	0.03387	0.10127	0
$E(C_S^*)$	0.5326	0.5326	0.5326	0.5517	0.5517	0.5517
$E(\hat{C}_S)$	0.5301	0.48870	0.4962	0.4943	0.4525	0.4729

The first column refers to the acombination of u and v , where $\hat{b}_u = \hat{b}_r$, the second column refers to the case where $v = 0$, and the third column to the case where $u = 0$.

Figure 8 illustrates the standard case, i.e. $u = 0.05$, and $v = 0.025$. The curved lines depicts the expected social costs, $E(C_S(\theta, n, u + v))$ for $\theta = [1, 2, 10]$. The optimal fee, z^* is the solution to $E(\Pi(n_S^*, (\theta, (u + z^*)))) = 0$. The dots indicate the market equilibrium, the circles indicate the efficient solution, and the diamonds denote the expected social costs, $E(\hat{C}_S(\hat{b}_r(u)))$ in the case of sequential search. The vertical lines mark the corresponding values of n_{eq} , n_S^* and \hat{n} .

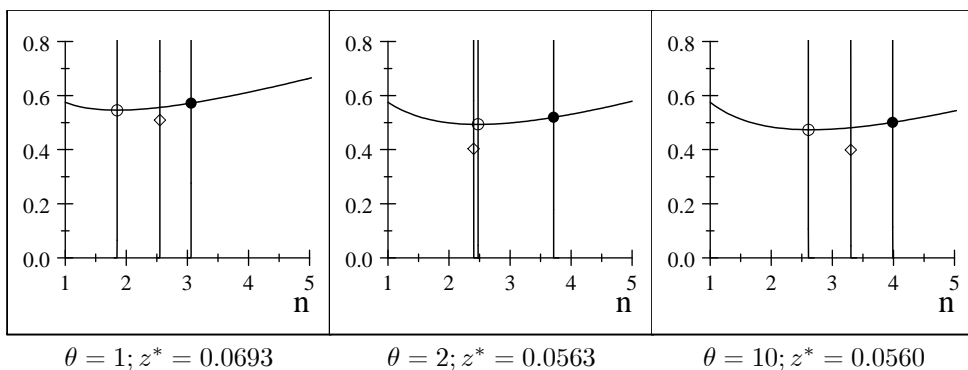


Figure 8 Social costs in public procurement auctions and sequential search for various values of θ .

5 Conclusion

Under the simplifying assumption that the cost density function is rectangular, I have analytically proved *the excess entry hypothesis* for any feasible non-negative combination of bid preparation cost and bid evaluation cost.

The hypothesis implies

- that an ordinary public procurement auction is an inefficient procurement strategy and
- that the procurer may reduce his own costs and social costs by imposing a *positive* fee on the bids.

In addition I proved

- that sequential search is a superior strategy to a public procurement auction for any feasible non-negative combination of bid preparation cost and bid evaluation cost.

I found it not possible to prove analytically that these results hold in general as the general model implies integrals that have no analytical solution. As an alternative, I explored the properties of the three procurement strategies by numerically solving the optimization problem for a flexible family of trigonometric

cost density functions. The simulations confirm the strong results derived analytically in the rectangular case and *suggest* (but do not prove) that they do hold in general.¹²

Although simple, the model conforms well to a number of empirical observations: (a) The number of bidders in public procurement auctions is often quite low¹³, (b) the costs associated with public procurement auctions are relatively high¹⁴, and (c) in general, private sector procurers, who are free to choose the mode of contract they find most suitable, make little use of public procurement auctions¹⁵.

The model and these empirical observations indicate that public procurement auction, as required by, e.g., EU legislation is an inefficient mode of contracting and that the sought for transparency, equality of treatment and economic integration come with a significant cost¹⁶.

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¹²Various second-order polynomials, the normal distribution and the exponential distribution not reported produced the same pattern.

¹³Most EU advertised tenders receive between 4 and 6 bids with an average of 5.4 bids. One in five tenders receives only one bid. There are large differences across countries varying from 8.8 in Spain to 2.1 in Slovakia. [4]. See also la Cour and Ølykke 2017 [3]. Li and Zheng 2009 [11] analyze a dataset consisting of 553 projects with a total of 1606 bids. On average, an auction had approximately 11 *potential* bidders, but the average number of bids was fewer than 3.

¹⁴According to the European Commission 2011 [4], the average cost of running each procedure is approximately €28000. 75 percent of the total cost is incurred by suppliers as the cost of preparing tenders. The cost of the procurement process may represent a high percentage of the total value of the contract, particularly at the lower end of contract value. At the lowest threshold in the directives, €125000, total costs can amount to between 18 and 29 percent of the contract value. At €390000, the median contract value, costs reach between 6 and 9 percent.

Li and Zheng 2009 [11] report from a study based on procurement auctions offered by Texas Department of Transportation that entry cost is approximately 13 percent of the private production cost and 8 percent of the winning bid.

Hansen et al. 2017 [7] estimate that total bid preparation costs (all tenderers) range from 5 percent of the contract value in eldercare to 45 percent in consultancy.

¹⁵Bajari et al. 2002 [2] report that 97 percent of public sector construction projects in Northern California were procured using competitive bidding during the period 1995-2000, compared to only 18 percent in the private sector.

¹⁶Sue Arrowsmith (2012) [1] argues that the directives seek solely promote the internal market and rejects the broader conceptions, including that they replicate in the public market the competitive process of the private market and that they seek a value for the taxpayer's money.

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